Selective laser cavity with compound diffraction gratings of unequal lengths

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Some peculiarities of a laser cavity with the diffraction gratings of unequal lengths are considered for the cases of homogeneous and inhomogeneous filling of the gratings with radiation. The conditions, under which the system efficiency is high, are determined. It is also shown that the non-cophased position of gratings of both equal and different lengths does no affect the selectivity parameters of a laser.

1. Some principal features of a laser cavity, the selectivity of which is provided for by two identical diffraction gratings, have been discussed in Ref. 1. In Ref. 1 the gratings are considered to be arranged in such a way that they are the in-phase prolongation of each other. Some modifications of the laser cavity are proposed. These cavities actually use a single grating but with an optical delay line inserted on the path of a beam incident onto the “far” half of the grating.

The identity of two gratings (or two parts of the same grating), their in-phase arrangement, and homogeneity of a light beam over its cross section are some idealization, while an actual situation is always different than an idealized one. Estimation of their influence on the performance of the system under discussion is the purpose of this paper.

2. Apparently, light beams also interfere in the case of not identical diffraction gratings. However, as one grating becomes shorter, the influence of beam interference on the formation of the diffraction peak becomes weaker. So, our purpose is to determine what parameters of the system ensure its sufficiently high performance.

Considering the situation has shown that in this case it is worth using the parameter W corresponding to the gap between the gratings in spite of the parameter M corresponding to the distance between the gratings' centers. The parameter W is the inter-grating gap measured in units of a grating constant. Assume that the total number of grooves on both gratings is $2N$, and the widths of the gratings are in a L:1 ratio. (The case $L = 0$ corresponds to one grating with $2N$ grooves; the case $L = 1$ corresponds to the system of two identical gratings each with N grooves. It was just this latter system that was considered in Ref. 1.)

The expression for intensity of the diffracted radiation has now a more complicated form than that given in Ref. 1:

$$I = A^2 \frac{\sin^2 u}{u^2} \frac{4}{\sin^2 \nu} \left[\sin^2 N \frac{L-1}{L+1} \nu \cos^2 N\nu + \right. \\ \left. + \sin \frac{2N}{L+1} \nu \sin \frac{2NL}{L+1} \nu \cos^2 (N+W)\nu \right], \tag{1}$$

where A^2 is proportional to the power per one groove of a grating;

$$u = \frac{\pi}{\lambda} \cos\varphi \sin 2\varphi d; \;\; \nu = \frac{\pi}{\lambda} 2 \sin\varphi d,$$

φ is the angle of incidence onto a grating operating in the autocollimation mode; d is the grating constant.

As an example, Fig. 1 shows the dependence $I(\nu)$ for $N = 4000$ and $W = 4000$. One can see that as L increases, the width of the central peak increases smoothly, the interference oscillations of intensity in wings decrease, and the function tends to the form corresponding to the case with a single grating of the width $2N$. This circumstance allows us to remove the line of demarcation between the beams from the center and thus improve the reliability of the laser system. Some ideas on the optimal position of the line of demarcation are given below for a spatially-bounded beam.

3. Deviations from the cophased arrangement of "two" gratings in an actual laser are caused by imperfections in the system manufacture, dispersion of the delay line in tunable lasers, change in the delay value at rotation of the grating in the process of changing the selective cavity tuning frequency. Therefore, these deviations practically cannot be eliminated.

Using a traditional approach,[2] it is easy to obtain the energy distribution of the diffracted radiation. For the case of identical gratings

$$I = A^2 \frac{\sin^2 u}{u^2} \left(\frac{\sin N\nu}{\sin \nu} \right)^2 4 \cos^2 (M+\varepsilon)\nu , \tag{2}$$

where $(M + \varepsilon)d$ is the "separation" between centers of "two" gratings; M is integer and larger than N, $0 \le \varepsilon < 1$.

To estimate the influence of $\varepsilon \ne 0$, let us rewrite

$$\cos (N+\varepsilon)\nu = \cos N\nu \cos \varepsilon\nu - \sin N\nu \sin \varepsilon\nu \tag{3}$$

and find within which limits, $\pm\Delta\nu_{\text{op}}$, the parameter ν can vary. The grating reflection is maximum at $\sin \nu = 0$ and $\nu = 0 \pm k\pi$, where k is the diffraction order. The halfwidth of the diffraction peak $\Delta\nu$ is determined by the number of grooves N (Ref. 2):

$$\Delta\nu = \pi / N , \tag{4}$$

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and the operating range $\Delta\nu_{op}$ of the diffraction angles is several times narrower because a laser can operate at the relative coefficient of reflection from the grating no less than 70–80%. Since $N \approx 5000$ (Ref. 1), $\Delta\nu_{op} \approx 10^{-5}\,\pi$ and the contribution of $\cos\varepsilon\nu$ and $\sin\varepsilon\nu$ to Eq. (2) is always insignificant at any M. Similar conclusion is valid for not identical gratings. Thus, the out-of-phase arrangement of the gratings has practically no effect on the performance of the system.

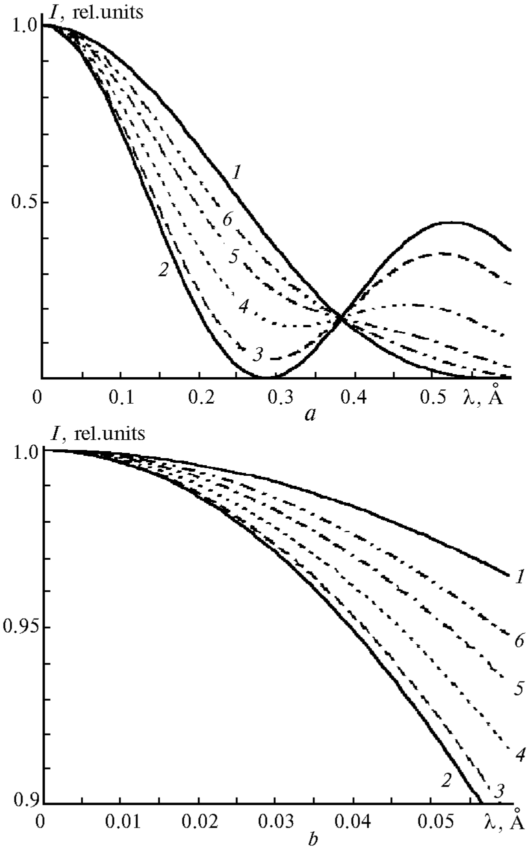

Fig. 1. Calculated dependence of the shape of the instrumental function of a "split" diffraction grating with the parts that are not identical (*a*) and its central area (*b*): $L = 0$ (*1*), 1(*2*), 2(*3*), 4 (*4*), 8 (*5*), and 16 (*6*).

4. Estimate now the influence of deviation from the cophased arrangement at rotation of the grating in order to tune the lasing wavelength. Using Fig. 2, it is easy to relate the optical delay l to other parameters of the system:

$$l = (M + \xi - N) d \sin\varphi \tag{5}$$

or

$$M + \xi = \frac{l}{d \sin\varphi} + N. \tag{6}$$

Hereinafter we take N and M constant and ξ varying widely.

Taking into account the equation of grating $2d\sin\varphi = k\lambda$, we finally derive

$$M + \xi = \frac{2l}{k\lambda} + N. \tag{7}$$

Thus, the variation of ξ at wavelength tuning by $\Delta\lambda$ is equal to

$$\Delta\xi = \frac{2l\Delta\lambda}{k\lambda^2} = (M - N)\frac{\Delta\lambda}{\lambda}. \tag{8}$$

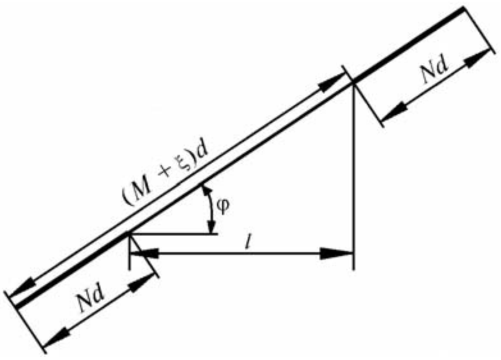

Fig. 2. Geometry of the compound grating.

For a 4-methylumbellypheron dye laser ($\lambda = 460$ nm, $\Delta\lambda = 30$ nm, $M = 4000$, $N = 8000$, Ref. 1), we have $\Delta\xi \approx 260$. However, even that large deviation from the in-phase arrangement gives no drastic consequences because $\cos(\Delta\xi\, \Delta\nu_{op})$ and $\sin(\Delta\xi\, \Delta\nu_{op})$ in Eq. (2) are equal to 0.997 and 0.082, respectively.

5. Influence of inhomogeneity in the radiation field is considered for a beam with the Gaussian intensity distribution over the cross section (the position of the radiation beam on the diffraction grating is schematically shown in Fig. 3):

$$E(x, y) = E_0 \exp\left(-\frac{x^2 + y^2}{4\rho^2}\right). \tag{9}$$

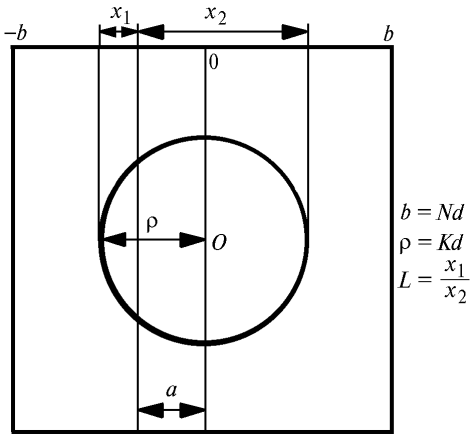

Fig. 3. Position of the radiation beam centered at the point *O* with respect to the diffraction grating.

The spectral distribution of the diffracted radiation intensity is expressed through the integral of probability Erf(x):

$$I \propto \exp(-8K^2\nu^2)\left\{\mathrm{Erf}\left(\frac{N}{2K} + i2K\nu\right) - \mathrm{Erf}\left(\frac{1-L}{2(1+L)} + i2K\nu\right) + \right.$$

$$\left. + \exp(iW\nu)\left[\mathrm{Erf}\left(\frac{N}{2K} - i2K\nu\right) + \mathrm{Erf}\left(\frac{1-L}{2(1+L)} + i2K\nu\right)\right]\right\} \times$$

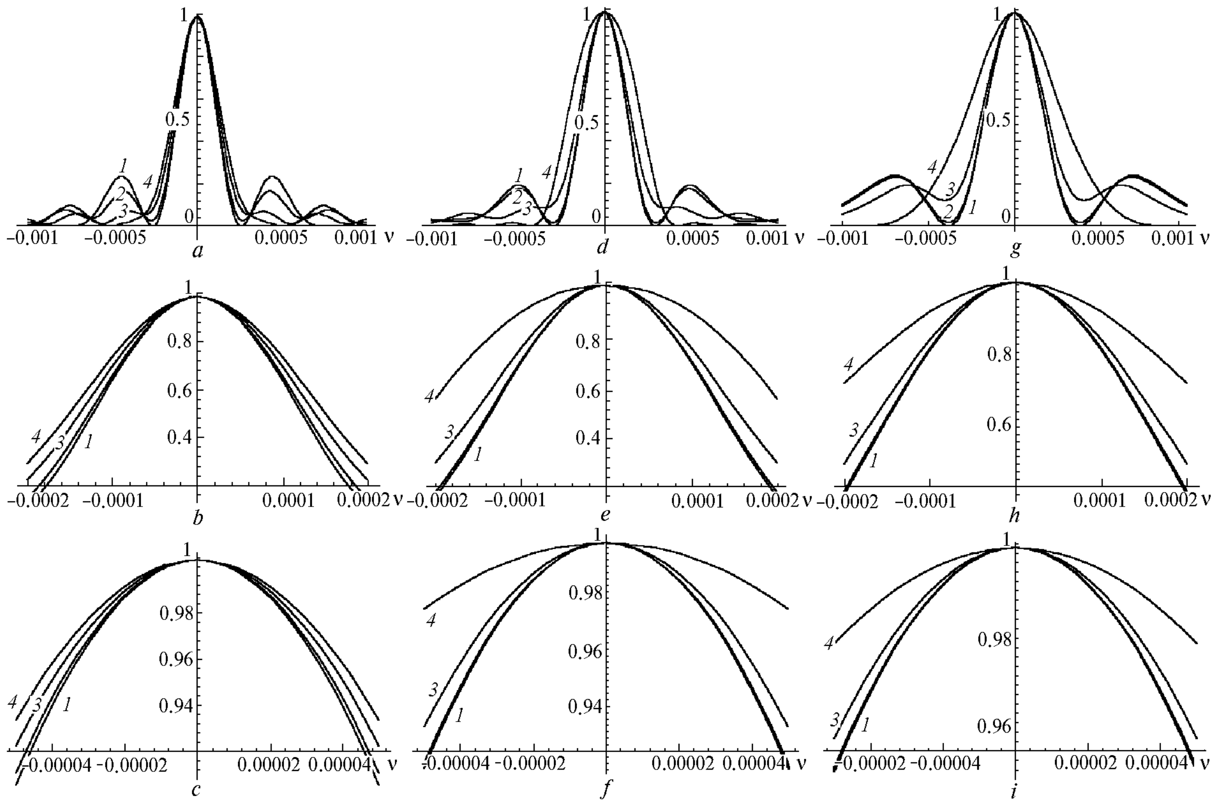

Fig. 4. The shape of the instrumental function (*a*, *d*, *g*) and its central part in different scales: $N = 4000$, $W = 4000$, $K = 16000$ (practically homogeneous beam, *a*–*c*), 2000 (*d*–*f*), and 1000 (*g*–*i*). $L = 1$ (*1*), 0.5 (*2*), 0 (*3*), and −0.5 (*4*).

$$\cdot\times\left\{\mathrm{Erf}\left(\frac{N}{2K}-i2K\nu\right)-\mathrm{Erf}\left(\frac{1-L}{2(1+L)}-i2K\nu\right)+\right.$$

$$\left.+\exp(-iW\nu)\left[\mathrm{Erf}\left(\frac{N}{2K}+i2K\nu\right)+\mathrm{Erf}\left(\frac{1-L}{2(1+L)}-i2K\nu\right)\right]\right\}. \qquad (10)$$

The calculated results are shown in Fig. 4. Inhomogeneity of the spatial distribution plays the role of an apodizing diaphragm and results in widening of the central peak and faster fall–off of the secondary maxima. The smaller the beam diameter, the more significant the widening and fall–off. Note also that the smaller the beam diameter (the smaller number of grating grooves participate in the formation of the diffraction pattern), the greater is narrowing of the central peak due to the increase in W. Unequal size of the gratings yields the same results as in the case of a homogeneous beam. They are the widening of the central peak and decrease of interference oscillations in the wings.

The analysis of plots (see Fig. 4) has allowed us to find the relation between the relative widening $\Delta_d = \Delta(a \neq 0)/\Delta(a = 0)$ of the central peak $\Delta(a)$ near the maximum (this widening is responsible for the width of the lasing spectrum and the accuracy of wavelength tuning) and the position of the grating "cut" line (the value of a) and the fraction Δ_Q of the total beam energy, which falls on the smaller part. From Fig. 5 it follows that if only 15% of the total energy falls on the smaller grating, then the central peak (at the level $0.99I_{\max}$) is widened by no more than 10–15%. This widening allows the “cut” line to be positioned sufficiently far from the beam axis in the zone, where the intensity of radiation is two to three times less than the maximum (curves of $1/e$ and $1/e^2$

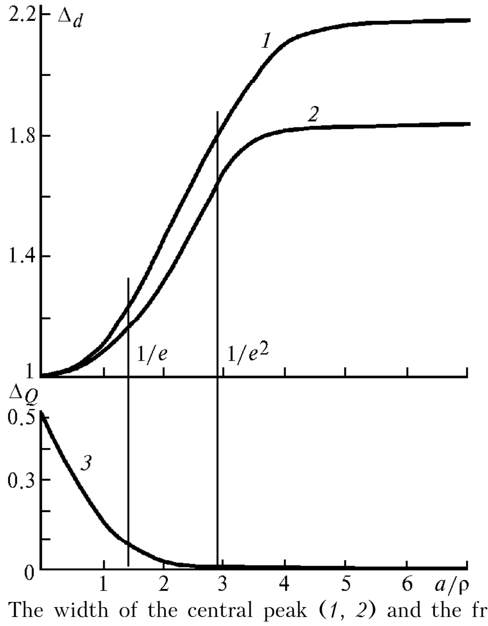

Fig. 5. The width of the central peak (*1*, *2*) and the fraction of the total energy (*3*) falling onto the smaller grating vs. the position of the "cut" line. $N = 4000$, $W = 4000$, $K = 2000$ (*1*) and 1000 (*2*).

indicate the values of a/ρ, at which the irradiance drops respectively by e and e^2 times).

6. Thus, the selective laser cavity with compound diffraction gratings of different lengths possesses an additional (in comparison with the cavity described in Ref. 1) positive property, namely, far higher reliability, because the line of demarcation between the beams incident on the gratings is shifted from the central area to the periphery, where the irradiance is lower.

The non-cophased arrangement of both identical and not identical gratings has no marked influence on the performance of the cavity considered.

Acknowledgments

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References

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