Probabilistic approach to the simulation of problems of rational nature management

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Mathematical simulation of the distribution of anthropogenic air pollutants is based on the stochastic approach, which enables one to detect zones with dangerous concentrations of pollutants and to estimate duration of their effect on the environment. It is suggested to take into account climatic features of the examined regions with the help of the transition probability functions.

To estimate the level of pollution of the surface atmospheric layer and of the underlying surface (soil, vegetation, water reservoirs, and currents of water) with anthropogenic admixtures emitted by elevated and surface extended sources, including waste dumps of ore-mining enterprises and ash dumps of thermal power plants, we consider that the admixture is passive, i.e., that it is transported with the velocity of the medium and has no noticeable effect on the dynamic properties of this medium. In other words, in the linear approximation the effect of the admixture on the velocity field can be neglected considering that the turbulence of the medium is also independent of the admixture concentration. Mathematical models used for the description of the spreading of substances in media are based mainly on the semi-empirical equations of transport and turbulent diffusion of an admixture in homogeneous media

$$\frac{\partial s}{\partial t} + \frac{\partial u_i s}{\partial x_i} + \alpha s = F + \nu_{ij} \frac{\partial^2 s}{\partial x_i \partial x_j} \tag{1}$$

or anisotropic media

$$\frac{\partial s}{\partial t} + \frac{\partial u_i s}{\partial x_i} + \alpha s = F + \frac{\partial}{\partial x_i} \nu_{ij} \frac{\partial s}{\partial x_j} . \tag{2}$$

To describe the spreading of admixtures in the inhomogeneous media, the direct (second) Kolmogorov equation[1]

$$\frac{\partial s}{\partial t} + \frac{\partial u_i s}{\partial x_i} + \alpha s = F + \frac{\partial^2 \nu_{ij} s}{\partial x_i \partial x_j} , \tag{3}$$

can be used or, transforming the last term on the right side of Eq. (3), we obtain

$$\frac{\partial s}{\partial t} + \frac{\partial u_i s}{\partial x_i} + \alpha s = F + \frac{\partial}{\partial x_i} \nu_{ij} \frac{\partial s}{\partial x_j} + \frac{\partial}{\partial x_i} s \frac{\partial \nu_{ij}}{\partial x_j} . \tag{4}$$

In Eqs. (1)–(4), $i, j = \overline{1, 3}$ are the serial numbers of the coordinate, t is time, u_i is the velocity component of the medium along the coordinate x_i, s is the admixture concentration, α specifies the degree of non-conservativity of the admixture, $F = F(t, x_i)$ is the function that describes the sources of the examined admixture, ν_{ij} is the tensor of the turbulent diffusion coefficients. Equations (1)–(4) are written in the tensor form; therefore, the sum is taken over doubly repeated subscripts in monomials within the limits of their variations.

Comparing Eqs. (2), (3), and (4), we see that Eq. (2) is a particular case of Eq. (3) or, which is the same, of Eq. (4), the last term of which comprises the information about the inhomogeneity of the medium.

In the literature, various methods are suggested for closing Eqs. (1)–(3), from simpler ones (use of the observational data on the parameters of the medium) to more complicated (a solution of the systems of hydrothermodynamics equations for each nodal point of the calculation grid). In any case, solutions of Eqs. (1)–(3) give, as a rule, estimates of the absolute pollutant concentration for an individual realization of the behavior of the medium (for example, either typical, averaged, or unfavorable conditions for the spreading of admixtures or the parameters of the medium calculated for a given time from the equations of hydrothermodynamics).

However, for many practical problems of interest are the zones of dangerous concentrations of the compounds from the viewpoint of not only their excess over the norms established for them (for example, over their maximum permissible concentrations), but also of their long-term effect on the natural medium. Just the long-term effect of pollutants creates real threat to the most vulnerable objects and contributes to the origin of cumulative effect, which can lead to delayed negative consequences and irreversible deviations from the natural equilibrium. Therefore, from my viewpoint, of definite interest are the mathematical models capable to detect the zones of hazardous effects on the natural medium with consideration for all climatic peculiarities of the examined region.

The main prerequisites for the suggested models are that at different time periods definite types of air mass motion are encountered in the atmosphere of the given region, which can be considered stationary during some characteristic periods (for example, meteorological observations at fixed hours at stationary stations and posts). After each stationary period, a new observation

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is carried out, i.e., the air mass motion is reorganized instantaneously, and a new stationary condition is realized, whose duration is determined by the time interval between two adjacent observations. Because the time of reorganization of atmospheric circulation is much smaller in comparison with the lifetime of the atmospheric motion of definite type, we can assume that it happens instantaneously. Thus, the system transforms from state to state as time passes.

In addition, the long-term observations of the hydrometeorological parameters can be considered as an ensemble of climatic characteristics for the given region. Because their realizations were recorded in different years, they can be considered statistically independent. This approach allows one to overcome the difficulties, connected with the non-ergodic character of natural phenomena, by averaging over the realizations rather than over time. Thus, the observations at fixed hours at meteorological stations and posts can be considered as realizations of a random function, whereas the long-term observations - as a set or an ensemble of all realizations of this random function.

By averaging over all realizations, we already obtain the climatic norm.

In other words, variations (increments) acquired by new states of the system during non-intersecting time intervals $T \gg \tau$ (where τ is the Lagrangian scale of time) are practically uncorrelated.[2] Therefore, the random sequence of states with independent increments can be considered as the Markovian process without aftereffects (the Markov chain), as if the system does not remember its previous states. As is known, the transition probability density for the Markov chain obeys the integral Smolukhovskii equation,[3,4] whose solution for definite classes of random processes is reduced to the direct Kolmogorov differential equation:

$$\frac{\partial p}{\partial t} + \frac{\partial[A(t,\,x)\,p]}{\partial x} = \frac{\partial^2[B(t,\,x)\,p]}{\partial x^2}\,. \qquad (5)$$

For the examined region D in Eq. (5), $p = p(t_0,\,x_0;\,t,\,x)$ is the probability density of system transition from the state x_0 to the state x during the time period from t_0 to t, which satisfies the conditions $p \ge 0$ and

$$\int\limits_D p(t_0,\,x_0;\,t,\,x)\mathrm{d}x = 1, \qquad (6)$$

A is the average rate of systematic change of the parameter x, B is the intensity of oscillations about the average.

It is natural that in Eq. (5) the admixture decay coefficient can be introduced together with its sources, and the system state can be considered as a function of many variables.

Following Ref. 5, in Eq. (5) we convert to the phase coordinate s:

$$\frac{\partial p}{\partial t} + \frac{\partial[A(t,\,x)\,p]}{\partial s} = \frac{\partial^2[B(t,\,x)\,p]}{\partial s^2}\,, \qquad (7)$$

where $p = p(t,\,s)$; $A = \dfrac{\partial \bar{s}}{\partial t}$; $B = \dfrac{1}{2}\overline{\left(\dfrac{\partial s'}{\partial t}\right)^2}$; $s' = s - \bar{s}$.

Here, the bar atop denotes averaging, and the prime denotes the deviation from the average. In this case, averaging is performed over the areas of the examined regions. The necessity of introduction of 1/2 in the coefficient B was easily proved in Refs. 3, 4, and 6.

The technique for closing Eq. (7), suggested in Ref. 5, is based on the method of recursive enclosures,[1] where the equation for the average admixture concentration was derived as a particular case for different meteorological conditions. The equations for A and B are solved numerically with the use of the method of fictitious regions in the Cartesian coordinate system. In so doing, the boundary condition of the third kind is specified on the lower boundary of the region, which allows one to take into account the reflection and absorption of the admixture by the underlying surface; the conditions of the first or second kind are specified on the upper and side boundaries of the calculation region as functions of the direction of the wind velocity vector with respect to this calculation region. The initial conditions, in accordance with the available information, can be $\bar{s} = s_0$ or $\bar{s} = s_{\mathrm{bg}}$, where s_0 and s_{bg} are the given concentration or its background value, respectively.

The derivatives with respect to the spatial variables are approximated by the interpolation method, and the derivatives with respect to time are approximated by the method of two-cyclic complete separation. For numerical realization of the finite difference analogs, the nonmonotonic pass technique is used.

Equation (7) is also solved numerically given that the condition of the probability measure is fulfilled. Numerical solutions of Eq. (7) were verified using the well-known analytical solutions obtained with some simplifications of physical processes. In particular, the analytical solution

$$p(s) = \frac{C}{B}\exp\int\limits_0^s \frac{A(\xi)\mathrm{d}\xi}{B(\xi)}\,,$$

suggested for the stationary equation in Ref. 3, was in good agreement. Here, the constant C was determined from normalization condition (6).

The direct simulation of Eq. (7) with the closures suggested in Ref. 5 is rather cumbersome. In some particular cases, for example, when we proceed to the Gaussian random fields of long-term meteorological observations, their complete statistical description is reduced only to calculations of the first initial and second central moments. However, following Ref. 2, for any random field with finite moments of the first two orders, it is always possible to find the Gaussian field with the same moments. In this case, to detect the zones with dangerous pollution, one can consider not all ensemble of the random fields of meteorological observations, but only those, which contribute to the occurrence of the enhanced concentrations with the probability of occurrence of the conditions favorable to them. In so doing, the solution must be obtained in the moving coordinate system rotating with the wind. Such particular approaches were realized in Refs. 7–9.

It should be noted that the theoretical laws of distribution simplify a solution of the formulated problem only partly, providing the most optimal detection of the zones with dangerous pollution. However, their knowledge is not obligatory, because the method itself allows one to establish the laws of distributions with the minimum discrepancy with respect to their approximation from the empirical data.

The method can be generalized for heavy admixtures possessing their own sedimentation velocity. In this case, one can estimate not only the dust content in the atmosphere from sources of different type (high-altitude smoke stacks, dumps, and so on) and draw contour lines for the regions with hazardous enhanced concentrations, but also calculate the amount of the admixture accumulated over a definite time interval on the underlying surface taking into account the probability of realization of all meteorological conditions during this time interval. Thus, the zones of enhanced pollution by various compounds are evaluated as integral functions of the climatic peculiarities of the given region.

As an example of calculation, the regions of enhanced pollution of Irkutsk by nitrogen oxides in December are shown in Fig. 1. The origin of coordinates is at the center of the city, where the stationary station of atmospheric pollution monitoring is located. For convenience, the probability of the excess over the established norms is normalized to the number of hours comprised in one month (a probability of 0.1 corresponds to 72 h).

The most dangerous situation is observed in the northwest part of the city, where the population more than half a month breathes the air, in which the concentrations of this compound exceed even the maximum permissible norms. In the calculations of the cases of the excess over the maximum permissible average daily concentrations (which are more rigorous in comparison with their maximum instantaneous values), probability of excess reaches almost unity, and the regions of dangerous pollution significantly extend.

Consideration of climatic peculiarities of the region for the mathematical models allows one to detect more stable zones of enhanced level of pollution, which must attract particular attention of the experts in different fields to make optimal decisions.

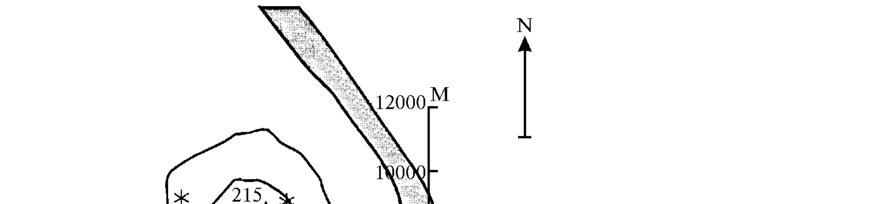

Fig. 1. Probability of excess of the MPC = 0.085 mg/m^3 of nitrogen oxide in December (Irkutsk): sources of admixture (∗); local maxima (◊).

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