New scheme of formation of bistatic laser guide star

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The bistatic optical arrangement is proposed to form an artificial guide star for the ground-based telescopes. The efficiency of correction for the wave-front tilts is calculated for a laser guide star in the form of intersecting lines. An attempt is undertaken to correct for the focal and angular non-isoplanatism.

Introduction

Laser guide stars[1–7] are among most important elements of modern adaptive optics systems. It is well known that there exists a rather complicated problem on determining the total wave front tilt by use of signals from a laser guide star (LGS).[6–8] Let us consider here some approaches to solution of this problem within already known arrangements of the LGS formation as well as new ones.

In our analysis we shall use information available on the altitude distribution of the intensity, C_n^2, and the outer scale of atmospheric turbulence κ_0^{-1}. The jitter of the image of a natural star (total tilt of the wave front) is described by the vector $\varphi_{\rm pl}(R_0)$, which characterizes fluctuations of the angular position of the centroid of the image formed by a plane wave in the focal plane of a telescope. The determined angular position of the LGS image is described by the vector $\varphi_{\rm m}$.

Optimal correction algorithm

In Refs. 5, 9, and 10 it was proposed to apply the so-called optimal correction algorithm. In this algorithm, the measured LGS positions (vector $\varphi_{\rm m}$) are used to correct for the jitter of the image of a natural star $\varphi_{\rm pl}(R_0)$. Having multiplied the vector $\varphi_{\rm m}$ by specially chosen coefficient A, to provide minimum for the following functional, we obtain

$$e^2 = \langle(\varphi_{\rm pl} - A\varphi_{\rm m})^2\rangle. \qquad (1)$$

It is easy to show that the minimum of the functional (1) characterizing the level of residual jitter in the natural star image equal to

$$e^2_{\rm min} = \langle(\varphi_{\rm pl} - A\varphi_{\rm m})^2\rangle_{\rm min} =$$
$$= \langle(\varphi_{\rm pl})^2\rangle \left\{1 - \frac{\langle(\varphi_{\rm pl}\ \varphi_{\rm m})^2\rangle}{\langle(\varphi_{\rm pl})^2\rangle\ \langle(\varphi_{\rm m})^2\rangle}\right\}, \qquad (2)$$

occurs at the coefficient

$$A = \frac{\langle(\varphi_{\rm pl}\ \varphi_{\rm m})\rangle}{\langle(\varphi_{\rm m})^2\rangle}. \qquad (3)$$

One can introduce a new concept of the relative efficiency of the correction for tilts using the following definition:

$$\beta^2 = \frac{e^2_{\rm min}}{\langle(\varphi^2_{\rm pl})\rangle} = \frac{\langle(\varphi_{\rm pl} - A\varphi_{\rm m})^2\rangle_{\rm min}}{\langle(\varphi^2_{\rm pl})}.$$

The optimal coefficient A minimizing the functional (1) is calculated by Eq. (3), or based on the data on the altitude distribution of the turbulence intensity C_n^2 and the outer scale of turbulence κ_0^{-1}, or based on the data of direct measurements with the use of the LGS and a sufficiently bright natural star. Here $\langle \ldots \rangle$ means averaging over an ensemble of turbulent fluctuations, $\langle(\varphi_{\rm pl})^2\rangle$ is the variance of fluctuations of the absolute value of $\varphi_{\rm pl}(R_0)$.

It should be noted that the proposed optimal algorithm of the correction for tilts *partially compensates for the cone non-isoplanatism*. As known, this non-isoplanatism is caused by the fact that the laser guide star generates a spherical wave front, whereas the wave front of a natural star is plane. Let us demonstrate what actually can be obtained with the use of the optimal correction algorithm. Toward this end, let us compare two values of the residual variance of fluctuations of the wave front tilts [calculated by Eqs. (1) and (2)] under the condition that measured data on the LGS position coincide with the value of jitter of the point-like guide source, namely, $\varphi_{\rm m} = -\varphi_{\rm sp}$. So we obtain

$$e^2 = \langle(\varphi_{\rm pl} + \varphi_{\rm sp})^2\rangle,$$

$$e^2_{\rm min} = \langle(\varphi_{\rm pl})^2\rangle \left\{1 - \frac{\langle\varphi_{\rm pl}\ \varphi_{\rm sp}\rangle^2}{\langle(\varphi_{\rm pl})^2\rangle\ \langle(\varphi_{\rm sp})^2\rangle}\right\}.$$

Note that the optimal correction takes place at

$$A = \frac{\langle\varphi_{\rm pl}\ \varphi_{\rm sp}\rangle}{\langle(\varphi_{\rm sp})^2\rangle}.$$

Thus, this coefficient proportionally increases the data measured using a spherical wave so that become closer to the data obtained with a plane wave. To be compensated is the difference in fluctuations for the plane and spherical waves due to focal non-isoplanatism. In this connection, it should be noted that the term

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“laser guide star” is a poor choice; the term “laser beacon” is more correct. It is simply impossible to create a guide source generating a plane wave (i.e., guide star).

Bistatic optical arrangement for the LGS formation

It was many times noted in the literature[11–14] that bistatic arrangement for forming the LGS is most efficient from the viewpoint of performing correction for tilts. Figure 1 shows two versions of the arrangement proposed that employ additional laser sources (version *a* from Refs. 11 and 12) and additional telescopes (version *b* from Ref. 13). The size of the primary telescope aperture is R_0, the size of the additional telescope aperture (or illuminator) is R_a, and the spacings between the additional telescopes and the axis of primary telescope in two mutually orthogonal directions are respectively ρ^y and ρ^z. For these versions of the bistatic arrangement[13,15] the angular position of the LGS image is described as

$$\varphi_m = \varphi_{lb} + \varphi_{ss} , \tag{4}$$

where φ_{lb} are random angular shifts of the centroid of a focused laser beam; φ_{ss} are random angular shifts of the image of the “secondary source,” namely, the LGS.

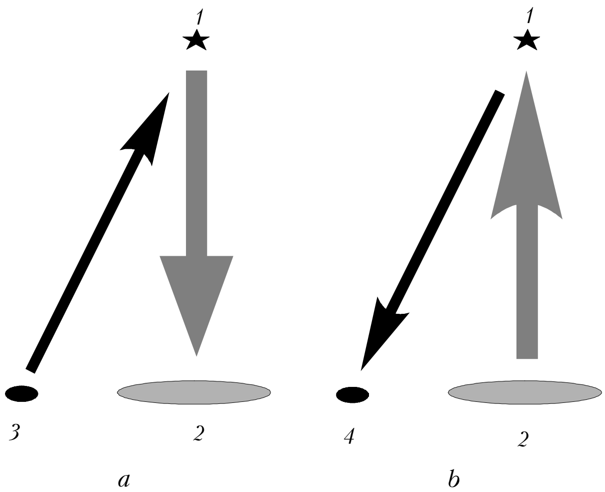

Fig. 1. Bistatic optical arrangement for the formation of a laser guide star: laser guide star (*1*), primary telescope (*2*), additional laser source (*3*), additional telescope (*4*).

For the limiting bistatic mode,[10,14] the signal variance (4) is

$$<(\varphi_m)^2> = <(\varphi_{lb})^2> + <(\varphi_{ss})^2>. \tag{5}$$

The visible dimensions of the secondary source (the LGS formed) are determined by the displacements of the observation point from the axis of the primary telescope, namely,

$$a_b^y = \rho^y\, l_b / X, \quad a_b^z = \rho^z\, l_b / X, \tag{6}$$

where X is the altitude, at which the LGS is formed, l_b is the LGS length. For the Rayleigh star, the LGS length l_b is determined by the length of caustic of the beam focusing system and equals 2 to 5 km; a sodium star (at X = 100 km) has the length determined by the natural thickness of the atmospheric sodium layer and roughly equals to 10 km. From that it can be easily understood that as far as the relative spacing between the telescope and the laser source (or additional telescope) is small, the guide source is seen by the primary telescope as a point.

Let us consider the case of formation of a guide star with the use of two additional laser sources (Fig. 1*a*). It can be shown according to Ref. 14 that for a point-like guide source, the signal of the LGS position, namely, the signal of the form

$$\varphi_m = \varphi_{lb} + \varphi_{sp}$$

can be used in correcting for tilts.

The latter equation was derived from Eq. (4) by replacing φ_{ss} by φ_{sp}, where φ_{sp} is the angular image of the point-like source which is at the altitude X. As a result, the limiting relative efficiency of correction with the use of such a signal is

$$\beta^2 = <(\varphi_{pl} - A\varphi_m)^2>_{min} / <(\varphi_{pl})^2> = \left\{1 - \frac{K^2(X)}{1 + <(\varphi_{pl})^2> / <(\varphi_{sp})^2>}\right\} , \tag{7}$$

where

$$K(X) = \frac{<\varphi_{pl}\, \varphi_{sp}>}{\sqrt{<(\varphi_{pl})^2>}\,\sqrt{<(\varphi_{sp})^2>}}.$$

The function $K(X)$ is the normalized correlation of the image jitter of a plane wave $\varphi_{pl}(R_0)$ coming from infinity and the spherical wave φ_{sp} emitted from the point at the altitude X. For the model of turbulence from Ref. 16, the function $K(X)$ was calculated in Ref. 17 for different altitudes of the point-like guide star (see Table 1).

Table 1

X, km	$K(X)$	$K^2(X)$	$1 - K^2(X)$
1	0.65	0.42	0.58
2	0.72	0.52	0.48
3	0.75	0.56	0.44
4	0.79	0.62	0.38
10	0.84	0.71	0.29
20	0.88	0.77	0.23
40	0.89	0.79	0.21
100	0.90	0.81	0.19

For the model of the atmosphere with an infinite outer scale

$$<\varphi_{lb}^2> / <\varphi_{sp}^2> = (R_a / R_0)^{-1/3}.$$

Thus, it is seen that for β^2 from Eq. (7) to be minimum, the size of the source R_a should be as large as possible. So we can formulate *a particular recommendation*. The laser guide star has to be created by the biggest telescope in an observatory.[18,19] This telescope initiates a point-like guide star for the smaller telescopes. The separation between telescopes in this case should not exceed 1 to 2 km.

Laser guide star as an extended object

As the separation between the primary and the additional telescopes increases, the guide star is seen as an extended object. Let us consider the arrangement *a* (Ragazzoni, Refs. 11 and 12). From the primary telescope the guide star is seen as a segment a straight line with the length determined by Eq. (6). For the LGS in the form of an extended object, the following differential scheme of signal processing can be proposed. A wave front sensor of the primary telescope measures the random angular position of the LGS image. Upon integration of the signal over the whole LGS image, we obtain

$$\varphi_1 = \varphi_{lb} + \varphi_{ss},$$

and integration only over the central part of the LGS line gives

$$\varphi_2 = \varphi_{lb} + \varphi_{sp}.$$

The difference between the signals φ_1 and φ_2 is

$$\Delta = \varphi_{ss} - \varphi_{sp}. \tag{8}$$

This difference signal is used as a correcting one for fluctuations of the total tilt of the wave front. As a result of this correction (with the use of two guide stars in the form of two orthogonal lines) the level of residual fluctuations of the wave front tilt is characterized by the value

$$\beta^2 = 1 - \frac{K^2(X)}{1 + \langle(\varphi_{ss})^2\rangle / \langle(\varphi_{sp})^2\rangle} . \tag{9}$$

Here $\langle(\varphi_{ss})^2\rangle / \langle(\varphi_{sp})^2\rangle$ is the ratio of the variance of the image jitter of the extended object with the length a_b to that of the image of a point-like object.[13,17] Let us denote this ratio as the function

$$\langle(\varphi_{ss})^2\rangle / \langle(\varphi_{sp})^2\rangle = f(b, a) \tag{10}$$

and introduce the following normalization:

$$b = a_b / R_0, \quad a = R_a / R_0, \quad c^2 = 2\kappa_0^{-2} R_0^{-2}.$$

The calculated data on the function $f(b, a)$ from Ref. 20 are shown in Figs. 2 and 3 as six fragments for the altitudes of 10 and 100 km, respectively. The left-hand column corresponds to $a = 0.1$, and the right-hand one to $a = 1$. The first, second, and third rows correspond to the aperture of the primary telescope of $R_0 = 1$, 4, and 10 m. The outer scale of turbulence κ_0^{-1} in each fragment was set as a function (see Ref. 20) denoted as C and E or was equal to 3, 10, 100, and 1000 m. The latter case practically corresponds to turbulence of the Kolmogorov type, i.e., the case of the infinite outer scale.

As a result, using our calculations (see Table 1 and Figs. 2 and 3) and Eqs. (7), (9), and (10), we can obtain the relative efficiency of the tilt correction by the Ragazzoni scheme. With new designations Eq. (9) can be written as

$$\beta^2 = 1 - \frac{K^2(X)}{1 + f(b, 1)} . \tag{11}$$

The bistatic optical arrangement with the additional laser source (see Fig. 1*a*) has some disadvantages, namely:

– if the separation between the primary telescope and the illuminating one is small, then the illuminating source of as large as possible size should be used,

– to make use of the LGS extension, the separation between the axes of the telescope and the illuminating source should be (for $X = 100$ km and $2R_0 = 8$ m) about 40 km.

Let us analyze now the alternative arrangement (Fig. 1*b*) that uses two additional telescopes.[13] In this case, the position of the image of the guide star is measured in two additional telescopes. The valid signal measured in the aperture R_a is written as $\varphi_m = \varphi_{lb} + \varphi_{ss}$. As a result of the optimal correction, we have

$$\beta^2 = \left\{1 - \frac{\langle\varphi_{pl}\, \varphi_{lb}\rangle^2}{\langle(\varphi_{pl})^2\rangle\, \langle(\varphi_m)^2\rangle}\right\} . \tag{12}$$

By comparing Eqs. (11) and (12) it can easily be understood that the efficient correction in the arrangement version *a* can be obtained only if the entire aperture R_0 is illuminated (focusing laser beam is generated by full aperture of the primary telescope), i.e., if $\varphi_{lb} = -\varphi_{sp}$, and the quality of correction in this case is characterized by the following value:

$$\beta^2 = 1 - \frac{K^2(X)}{1 + f(b, a)} . \tag{13}$$

The comparison of this equation with the Eq. (11) shows that the same efficiency can be obtained in both *a* and *b* versions (Fig. 1) only under the condition that $R_a = R_0$, i.e., if the primary and additional telescopes have the same apertures.

Finally, the following conclusions can be drawn:

– the major disadvantage of the version with two additional telescopes is the requirement that the star should be generated by the full aperture because of the phosphorescence of optical elements of the telescope,[8]

– three identical (large) telescopes are needed,

– the separation between the axes of the additional telescopes should be (for $X = 100$ km and $2R_0 = 8$ m) more than 40 km.

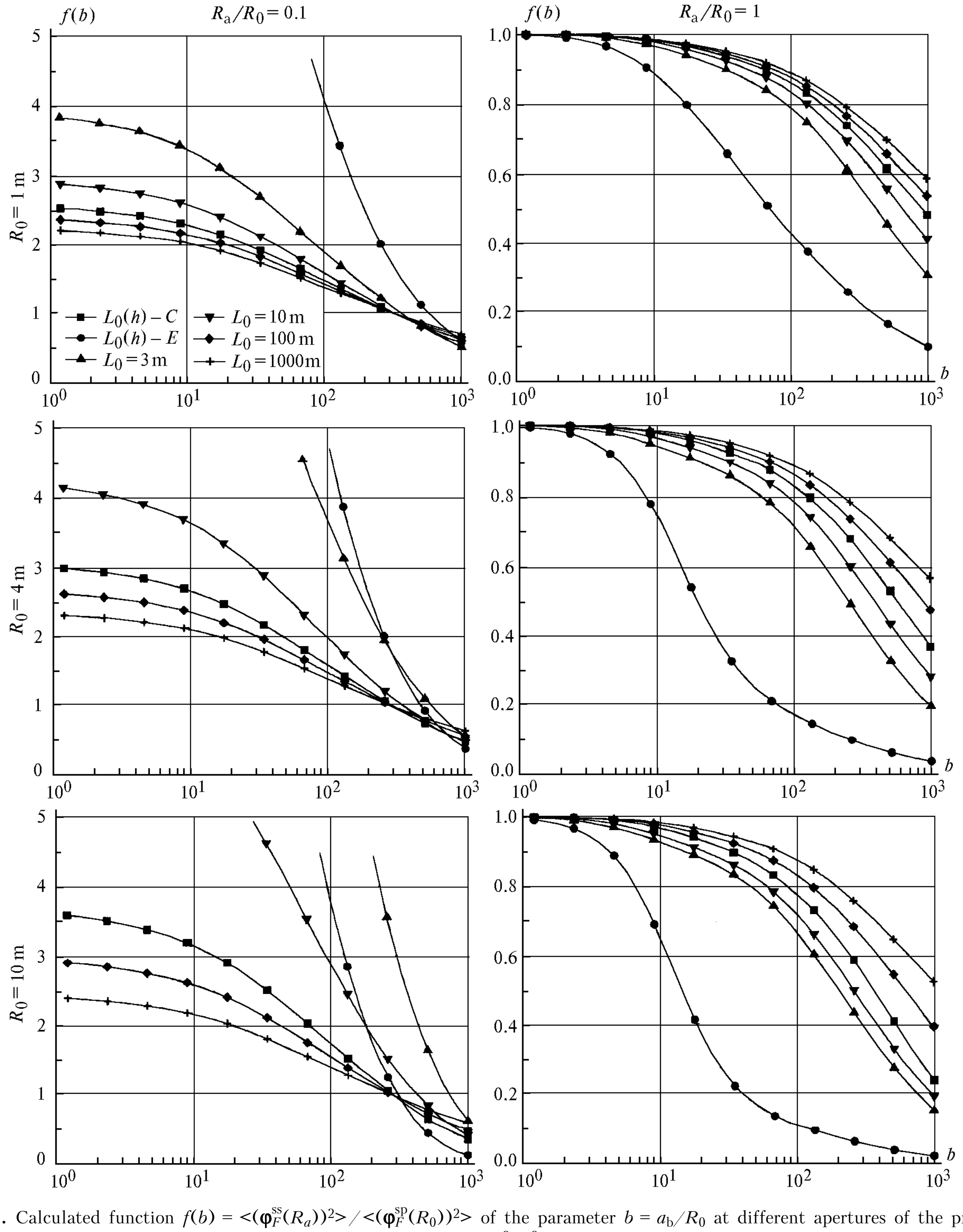

Fig. 2. Calculated function $f(b) = \langle(\varphi_F^{ss}(R_a))^2\rangle / \langle(\varphi_F^{sp}(R_0))^2\rangle$ of the parameter $b = a_b/R_0$ at different apertures of the primary telescope, different values of the parameters $a = R_a/R_0$ and $c^2 = 2\kappa_0^{-2}R_0^{-2}$, and the altitude of the guide star $X = 10$ km.

Thus, some advantages of the bistatic optical arrangement employing two additional remote laser sources over the version of the arrangement with two additional telescopes are obvious. However, this version has some disadvantages as well:

– to provide an acceptable level of correction, the separation of the illuminating source from the axis of the primary telescope should be more than 40 km,

– the primary telescope and the two additional illuminating sources, whose radiation forms an image in the form of two intersecting lines in the primary telescope, should be aligned highly accurate (within 100″).

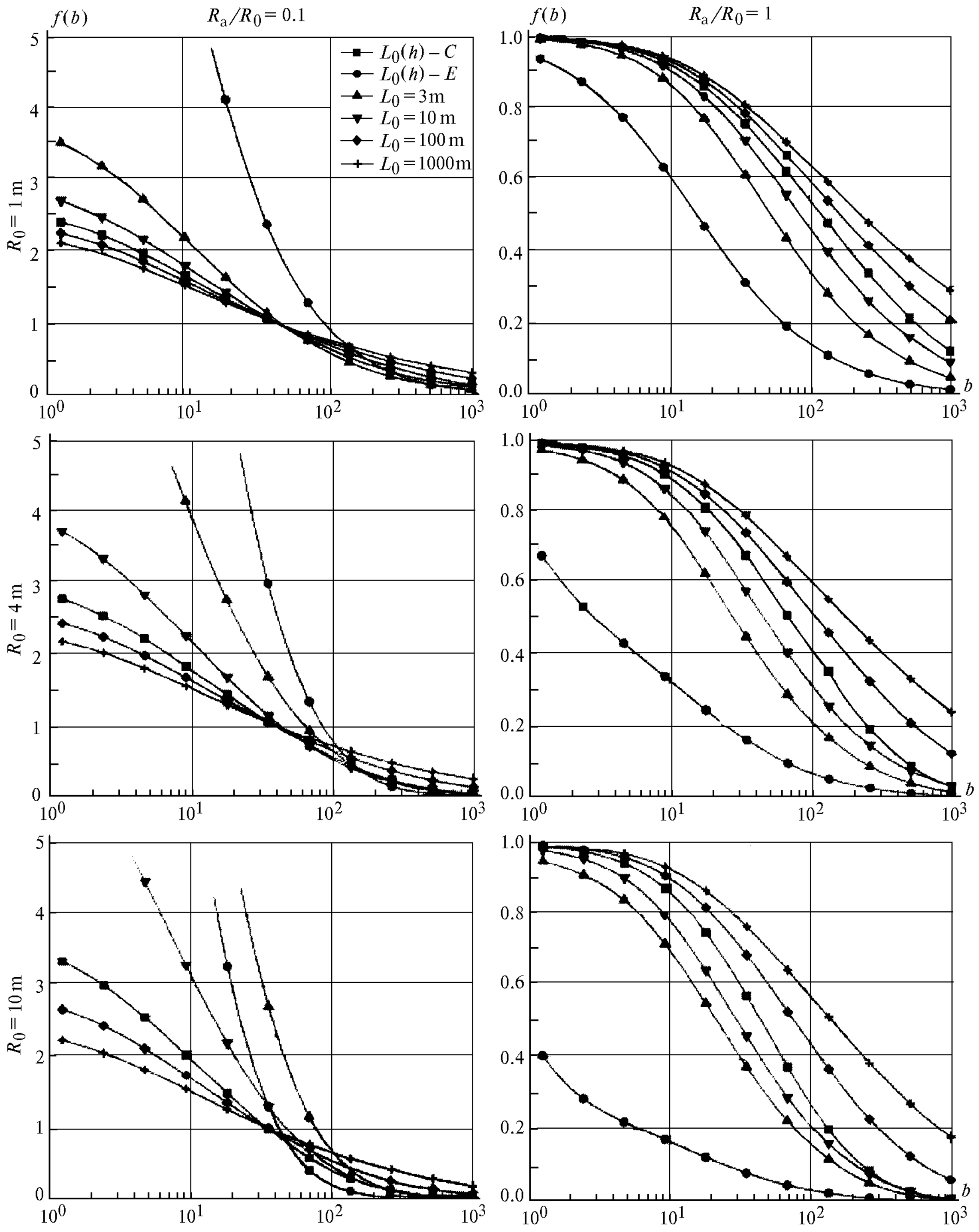

Fig. 3. The same as in Fig. 2, but for X = 100 km.

Scanning schemes of the LGS formation

A rather different technique of realizing the formation of a guide source in the form of two intersecting lines is possible. One can even say about the laser guide star in the form of *a guide cross*.

To do this, very fast angular scanning with two narrow laser beams is used. In this case, two narrow focused beams are emitted from a point nearby the aperture of the primary telescope (but beyond it). The beams are being moved along two mutually orthogonal directions. The frequency of such an angular modulation is far higher than the characteristic frequencies of turbulent jitter of focused beams. Sufficiently fast angular scanning with these two beams in their focusing plane gives rise to a luminous object: LGS in the form of two intersecting lines, i.e., the laser guide cross.

The signal is received almost monostatically. The wave front sensor of the primary telescope uses two identical CCD arrays (CCD array Y and CCD array Z). Note that the image of the guide cross in two channels can be divided into two guide bands by using the initially orthogonal polarizations of the laser beams used. The optical arrangement of the wave front sensor is designed so that the image of only one line is formed in each of the two CCD arrays. The signal in each array is processed in the following way: two signals are measured simultaneously, namely, the signal recorded by the entire array φ^y for the CCD array Y (and φ^z for the CCD array Z) and the signal recorded only in the central part φ_c^y for the CCD array Y (and φ_c^z for the CCD array Z), and then the signal differences are calculated. As a result, we have

$\Delta^y = \varphi^y - \varphi_c^y$ for the CCD array Y,

$\Delta^z = \varphi^z - \varphi_c^z$ for the CCD array Z.

It is easy to show that the components of these differences, in their turn, can be expressed as

$$\varphi^y = \varphi_{\rm lb}^y + \varphi_{\rm ss}^y, \quad \varphi_c^y = \varphi_{\rm lb}^y + \varphi_{\rm sp}^y,$$

$$\varphi^z = \varphi_{\rm lb}^z + \varphi_{\rm ss}^z, \quad \varphi_c^z = \varphi_{\rm lb}^z + \varphi_{\rm sp}^z.$$

Their vector of the difference signal is $\boldsymbol{\Delta} = \boldsymbol{\varphi}_{\rm ss} - \boldsymbol{\varphi}_{\rm sp}$. In using the signal $\boldsymbol{\Delta}$ for making correction for the wave front tilt of a natural star $\boldsymbol{\varphi}_{\rm pl}(R_0)$, the level of residual fluctuations of the wave front tilts is the following:

$$<[\boldsymbol{\varphi}_{\rm pl} - \boldsymbol{\Delta}]^2> \approx <[\boldsymbol{\varphi}_{\rm pl} - \boldsymbol{\varphi}_{\rm sp}]^2> + <\boldsymbol{\varphi}_{\rm ss}^2>,$$

and the relative variance is

$$\beta^2 = \frac{<[\boldsymbol{\varphi}_{\rm pl} - \boldsymbol{\Delta}]^2>}{<(\boldsymbol{\varphi}_{\rm pl})^2>} \approx \frac{<[\boldsymbol{\varphi}_{\rm pl} - \boldsymbol{\varphi}_{\rm sp}]^2>}{<(\boldsymbol{\varphi}_{\rm pl})^2>} + f(b,\ 1).$$

The first term here is connected with the cone non-isoplanatism, and the second one is due to the finite length of the LGS. It should be noted that the optimal correction algorithm gives in this case somewhat lower level of residual distortions, namely,

$$\beta^2 = \frac{<[\boldsymbol{\varphi}_{\rm pl} - \boldsymbol{\Delta}]^2>}{<(\boldsymbol{\varphi}_{\rm pl})^2>} = 1 - \frac{K^2(X)}{1 + f(b,\ 1)}\ .$$

Since the angle of scanning with the laser beams is rather large, the linear size of the guide line can be much longer than the correlation length of fluctuations of the wave front tilts, and then

$$<(\boldsymbol{\varphi}_{\rm ss})^2>/<(\boldsymbol{\varphi}_{\rm sp})^2> = f(b,\ 1) \Rightarrow 0.$$

The result of correction for the wave front tilts in this case proves to be better than for any known bistatic arrangement. The proposed version of the optical arrangement has some advantages: (1) there is no need for use of additional telescopes, (2) two laser illuminating sources can be placed on the telescope mounting, what removes the problem on alignment of the telescope and the illuminating sources, (3) laser beams are formed beyond the aperture of the primary telescope, therefore there is no phosphorescence of the optical system, (4) there is no dependence of the signal $\boldsymbol{\Delta} = \boldsymbol{\varphi}_{\rm ss} - \boldsymbol{\varphi}_{\rm sp}$, which provides for the tilt correction, on the parameters of laser beams, (5) the size of the laser guide cross can be changed by changing only the control voltage of optical deflectors of the laser beams.

It should be noted that some restrictions are imposed here on the value of the telescope focal length in the optical system of a wave front sensor. Let us consider a telescope with 8-m focal length as an example. The suppression of image jitter in the telescope is sufficient if $b = a_{\rm b}/R_0 > 10^3$. Here $a_{\rm b}$ is the apparent size of the guide band. We should provide such a field of view FOV for the wave front sensor that the whole guide band is inside it, i.e., we should have

$${\rm FOV} = \frac{2R_0}{f} > \frac{bR_0}{X}\ .$$

The focal length in the wave front sensor system should be (at $X = 100$ km, $2R_0 = 8$ m, $b = a_{\rm b}/R_0 > 10^3$) less than 200 m.

Thus, it is sufficient to provide high-frequency (about 10 to 20 kHz) angular scanning with the laser beams within the angle about 2–3°, and this is equivalent to the 40-km separation of the axis of an additional telescope.

Attempt to decrease the effect of angular non-isoplanatism

Let us consider one approach to minimizing the effect of angular non-isoplanatism.[21,22] It is well known that this effect is connected with the angular difference between the direction toward the guide star and that toward the star whose image is to be corrected. Let the guide star be formed in zenith, and the star under study at the angular distance θ from the zenith. As is known, any function, for example, the function of fluctuation of the optical wave phase, can be presented in the form of a finite-dimensional series expansion over orthogonal polynomials within a circle of the radius R:

$$S(y,\ z,\ 0) = \sum_1^N a_j\ F_j(y/R,\ z/R). \qquad (14)$$

Let us use the fact that we can pre-determine the normalized angular correlation functions $b_j(\theta)$ of the mode components a_j $(j = 1,\ \dots\ ,\ N)$ of phase fluctuations $S(y,\ z)$. Then the measurement data (14) obtained using the guide star can be corrected for the direction at the angle θ by the following scheme:

$$S(y,\ z,\ \theta) = \sum_1^N b_j(\theta)\ a_j\ F_j(y/R,\ z/R). \qquad (15)$$

It is easy to show that the residual error due to angular non-isoplanatism for any mode component a_j ($j = 1, \ldots, N$) of the phase series expansion (14) is expressed as follows:

$$e_1^2 = \langle[a_j(0) - a_j(\theta)]^2\rangle = D_j(\theta) = 2\langle a_j^2\rangle\, [1 - b_j(\theta)].$$

Using the estimate (15), we can obtain the residual error:

$$e_2^2 = \langle[a_j(\theta) - b_j(\theta)\, a_j(0)]^2\rangle = \langle a_j^2\rangle\, [1 - b_j^2(\theta)].$$

It is seen that the second estimate of the residual distortions due to angular non-isoplanatism is less than the first one. Let us determine the ratio of the two variances

$$e_2^2/e_1^2 = \langle[a_j(\theta) - b_j(\theta)\, a_j(0)]^2\rangle/\langle[a_j(0) - a_j(\theta)]^2\rangle =$$

$$= [1 + b_j(\theta)]/2.$$

As known, the correlation length (or angle) of a mode decreases with the increase in its number, and therefore at a fixed angular difference θ we can obtain some gain just due to more correct account for the high modes. It is obvious that the residual error is halved at the angle θ exceeding the angular correlation of this mode. Thus, the proposed algorithm almost halves the effect of the focal length non-isoplanatism.

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