

# Measurements of the emissivity of plane surfaces

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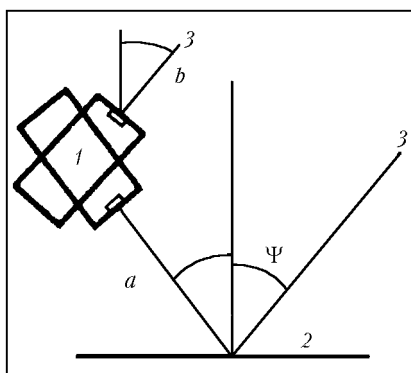
We present a comparison among three techniques for calculating the spectral emissivity  $\epsilon$  of specularly reflecting plane surfaces using narrowband IR radiometric data. Relations are given to calculate  $\epsilon$  as well as the instrumental errors and the errors due to the methodology used. The first method is suitable for field measurements of  $\epsilon$  of natural objects, including thin films on the water surface; and its accuracy is  $d\epsilon \leq 0.005$  when the radiometer noise level  $\delta t_n \approx 0.1^\circ$ . By selecting most appropriate method, the accuracy of  $\epsilon$  measurement can be made as small as  $d\epsilon < 0.0015$  ( $\epsilon \approx 0$ ) –  $0.004$  ( $\epsilon \approx 1$ ) at  $\delta t_n \approx 0.1^\circ$ .

## Introduction

Infrared remote sensing of surface temperature provides a good deal of information useful for weather prediction, ecological monitoring, and studies of different processes. To calculate the thermodynamic surface temperature, it is necessary to know the surface emissivity  $\epsilon$ . Actually, if the background radiation  $\Phi$  is incident on the surface with temperature  $t_s$  and emissivity  $\epsilon$ , the optical system of a radiometer located over this surface receives the radiation  $I_s$  which, in the absence of atmospheric effect, is given by (Fig. 1a):

$$I_s = \epsilon B(t_s) + (1 - \epsilon) \Phi. \quad (1)$$

Here and below,  $B(t)$  is the Planck's function of temperature  $t$  at the wavelength determined by the pass band of the entrance filter of the optical system. The validity of the approximation of the radiance by Planck's function is based on that the filter has narrow bandwidth:  $d\lambda/\lambda < 0.05$ .



**Fig. 1.** The schematics of radiometric measurements needed to calculate the emissivity  $\epsilon$ : radiometer measures either (a) surface emission or (b) background radiation  $\Phi$ : radiometer (1), surface with temperature  $t_s$  and either known ( $\epsilon_c$ ) or unknown ( $\epsilon$ ) emissivity (2), and background radiation  $\Phi$  (3).

From Eq. (1) we determine the Planck's function

$$B(t_s) = \Phi + 1/\epsilon (I_s - \Phi),$$

which is used to determine  $t_s$ . The error  $d\epsilon$  of  $\epsilon$  determination translates to  $t_s$  error given by

$$dt_s = -d\epsilon/\epsilon \lambda T_s^2/C_2 [1 - \Phi/B(t_s)]. \quad (2)$$

Here and below,  $t$  is in centigrade degrees,  $T$  is in Kelvin degrees;  $C_2 = 14388 \mu\text{m} \cdot \text{K}$ ; and  $\lambda$  is the center wavelength of the filter of optical system, taken here to be equal to  $11 \mu\text{m}$ , representing the center wavelength of atmospheric transparency window at  $10\text{--}12 \mu\text{m}$ . When  $d\epsilon/\epsilon = 1\%$ ,  $t_s = 20^\circ$ , and  $\Phi/B(t_s) = 0.2$  (typical for near-zenith observations under clear-sky conditions),  $dt_s \approx 0.4^\circ$ .

Clean and polluted water surfaces have different emissivities, and this can be used to solve ecological monitoring problems. If we let  $\epsilon_w$  and  $\epsilon_d$  denote the emissivities of clean and polluted waters, respectively, then, at the same thermodynamic temperature  $T_w$ , the brightness temperatures of upward radiation for the clean and polluted water surfaces,  $T_{wbr}$  and  $T_{dbr}$ , will differ by amount  $dT_{br} = T_{dbr} - T_{wbr} = \lambda T_w^2/C_2 (1 - \Phi_1/B(t_w)) (\epsilon_d - \epsilon_w)$ . When  $\Phi_1/B(t_w) = 0.2$  (clear sky in summer),  $t_w = 20^\circ$ , and  $\epsilon_d - \epsilon_w = 0.02$ , at  $\lambda = 11 \mu\text{m}$   $dT_{br} \approx -1^\circ$ . This is quite large  $dT_{br}$  value, suggesting that the difference between  $\epsilon_d$  and  $\epsilon_w$  can be thought as an indicator of pollution level in analysis of variations in the brightness temperature field; although in reality, this is not the only factor contributing to  $dT_s$ .<sup>1</sup> The knowledge of  $\epsilon_d$  is required for determination of thickness of thin films.<sup>2</sup>

Moreover, for reliable measurements, regular calibrations of the measuring instrumentation are required, and the plane calibrators are most suitable for this in many cases. In addition to its simplicity and technical feasibility, this calibration procedure provides for more adequate account of background illumination than more complex calibrators, ensuring higher calibrator emissivity  $\epsilon_c$ .

The accuracy of  $\epsilon_c$  determination translates directly into that of  $t_s$  calculation: if  $\epsilon_c$  has the error  $d\epsilon_c/\epsilon_c$ , the calculated temperature,  $t_{s,c}$ , will diverge from the real one by amount

$$t_s - t_{s,c} = d\epsilon_c/\epsilon_c \lambda T_s^2/C_2 [B_m/B(t_w) - 1]. \quad (3)$$

Here  $B_m = B(t_{o,s})$ , and  $t_{o,s}$  is the temperature of the optical system. As written, this relation is valid only for the modulation-type radiometers<sup>3</sup>; and setting  $B_m = 0$  makes it applicable to the other types. When  $\lambda = 11 \mu\text{m}$ ,  $d\epsilon_c/\epsilon_c = 1\%$ , and  $t_{o,s} = 20^\circ$ ,  $t_s - t_{s,c} = 0.05^\circ$  for  $t_s = 15^\circ$  (sea surface emission) and  $t_s - t_{s,c} = 0.5^\circ$  for  $t_s = -42^\circ$  (clear-sky emission) in the case of a modulation-type radiometer, and  $t_s - t_{s,c} = 1^\circ$  in the other cases. Relations (2) and (3) provide for estimates  $\epsilon$  with the required accuracy.

In this paper we present radiometric methods of determining  $\epsilon$  for the plane, specularly-reflecting surfaces, together with the error estimates, the latter falling into two categories: irremovable inherent instrument errors and partially removable systematic errors due to bias between the model-specified and actual conditions of observations.

The output radiometer voltage will be assumed to be linearly related to the input voltage by:

$$U_x = a I_x + U_{\text{sys}}, \quad (4)$$

where  $U_{\text{sys}}$  is the voltage determined by the optical characteristics and temperature of elements of the optical system and by the bias voltage of the radiometer electronic system;  $a$  is the radiometer response, determined by the detector response, optical parameters of the elements of the optical system, and the gain of the electronic system. Relation (4) is applicable, e.g., to the widespread uncooled detectors of pyroelectric type.

### 1. The measurements of $\epsilon$ with the account for “cold” and “warm” backgrounds

In radiometric measurements of a horizontal surface, that has the temperature  $t_s$  and unknown emissivity  $\epsilon$ , and is oriented at an angle  $\psi$  with respect to nadir, the total radiation received by a radiometer is made up of two contributions: the surface emission  $\epsilon B_s$  ( $B_s = B(t_s)$ ) itself and the fraction  $r\Phi$  of the background radiation  $\Phi$  reflected from the surface that is traveling downward at an angle  $\psi$  with respect to nadir (see Fig. 1a); so that the radiometer output voltage can be written as

$$U = a (\epsilon B_s + r \Phi) + U_{\text{sys}}.$$

Here and below,  $r = 1 - \epsilon$  is the surface reflection coefficient.

For these two viewing geometries, incorporating (a) “cold” background  $\Phi_1$  taken to be the emission of a clear sky, and (b) “warm” background  $\Phi_2$  involving a

horizontal surface whose emission  $\Phi_2 \approx B_s$  exceeding  $\Phi_1$ , we obtain

$$U_1 = a (\epsilon B_s + r \Phi_1) + U_{\text{sys}}, \quad (5)$$

$$U_2 = a (\epsilon B_s + r \Phi_2) + U_{\text{sys}}. \quad (6)$$

We note that the clear sky is a unique natural absolutely black body with a stable emission  $\Phi$ : at night  $|d\Phi/B(T_{\text{air}})| < 0.3\%$  per hour; field measurements and calculations using LOWTRAN-7 model show that, for angles  $\varphi \leq 20^\circ$  off nadir, at  $\lambda = 11 \mu\text{m}$   $\Phi(\varphi)/B(T_{\text{air}})$  is 0.2–0.35 in summer and 0.07–0.15 in winter. Use of a clear-sky radiation as a source of “background” radiation in methods I and II allows one to reduce considerably the error in  $\epsilon$  computation.

When “cold”/“warm” background alone falls within the radiometer field of view (FOV) (Fig. 1b), we have:

$$U_3 = a \Phi_1 + U_{\text{sys}}, \quad (7)$$

$$U_4 = a \Phi_2 + U_{\text{sys}}. \quad (8)$$

In writing (6) and (8), it is assumed that the surface, from which the background radiation  $\Phi_2$  comes, is oriented so that the condition of no reflections between surface and optical system is satisfied; otherwise, a change in the form of equations (6) and (8) will be required and the solution of system of equations (5) through (8) for  $\epsilon$  will be impossible.

From Eqs. (5)–(8) it follows that

$$\epsilon = 1 - (U_1 - U_2)/(U_3 - U_4), \quad (9)$$

according to which the instrumental error in  $\epsilon$  calculation is

$$D\epsilon_1 = \frac{1}{\Phi_2 - \Phi_1} \left( \frac{dU_1 - dU_2}{a} - r \frac{dU_3 - dU_4}{a} \right) \quad (10)$$

Since the summands in the right-hand side of Eq. (10) are statistically independent, assuming that the measurements according to (5) – (8) are made at the same time yields

$$D\epsilon_1 \approx C_2/\lambda \delta t_n/T_s^2 [2(1+r^2)]^{1/2}/(1-\Phi_1/\Phi_2), \quad (11)$$

where  $\delta t_n$  is the noise in the radiometer output voltage in units of temperature.

In measurements according to (5) through (8), the condition of no variations in  $t_s$ ,  $\Phi_1$ , and  $\Phi_2$  must be met; otherwise errors in  $\epsilon$  calculation will include:

1) measurement error  $t_s$

$$d\epsilon_1 [dt_s] = (\epsilon - \epsilon_{\text{calc}}) [dt_s] = \epsilon \frac{B_{s(6)} - B_{s(5)}}{\Phi_2 - \Phi_1} \approx \frac{C_2}{\lambda} \frac{dt_s}{T_s^2} \frac{\epsilon}{1 - \Phi_1/B_s}, \quad (12)$$

2) measurement error  $\Phi_1$

$$d\epsilon_1 [d\Phi_1] = (\epsilon - \epsilon_{\text{calc}}) [d\Phi_1] = (1 - \epsilon) \frac{\Phi_{1(7)} - \Phi_{1(5)}}{\Phi_2 - \Phi_1} \approx \frac{C_2}{\lambda} \frac{d\Phi_1}{T_{\Phi_1}^2} \frac{r}{1 - B_s/\Phi_1}, \quad (13)$$

3) measurement error  $\Phi_2$

$$d\epsilon_I [d\Phi_2] = (\epsilon - \epsilon_{calc}) [d\Phi_2] = (1 - \epsilon) \frac{\Phi_{2(8)} - \Phi_{2(6)}}{\Phi_2 - \Phi_1} \approx \frac{C_2}{\lambda} \frac{dt_{\Phi_2}}{T_{\Phi_2}^2} \frac{r}{1 - \Phi_1/B_s} \quad (14)$$

Here and below,  $\epsilon_{calc}$  is the calculated  $\epsilon$  value; the numbers in the parentheses in subscripts stand for equation numbers; and  $d\Phi_1$ ,  $d\Phi_2$ , and  $dB_s$  are instabilities in  $t_{\Phi_1}$ ,  $t_{\Phi_2}$ , and  $t_s$  (in intensity units) in measurements according to Eqs. (5)–(8).

For lower methodological errors, as defined by Eqs. (12) through (14), in the emissivity determination with  $\epsilon \approx 1$  special care must be given to ensure the stability of  $B_s$  and perform measurements according to Eqs. (5)–(8), in that order. When  $\epsilon \approx 0$ , special care should be given to ensure the stability of  $\Phi_1$  and  $\Phi_2$  and perform measurements according to Eqs. (5), (7), (8), and (6), in that order.

**Version I of the method at  $t_s = t_{o,s}$**

When the target surface has the same temperature as the optical system, for determining  $\epsilon$  it is sufficient to perform three measurements: first and third according to expressions (5) and (7), and second by pointing the radiometer at the right angle to the surface. At  $t_{o,s} = t_s$ , the surface-radiometer system constitutes blackbody, and the radiometer output voltage is given by

$$V_2 = a B_s + U_{sys} \quad (15)$$

From Eqs. (5), (7), and (15) it follows that

$$\epsilon = \frac{U_1 - U_3}{V_2 - U_3} \quad (16)$$

When  $dV_2 \rightarrow 0$ , the instrument error of  $\epsilon$  determination becomes

$$D\epsilon_I \approx D\epsilon_I / \sqrt{2} \quad (17)$$

If  $t_s \neq t_{o,s}$ , the methodological error in  $\epsilon$  determination will be

$$d\epsilon_I [dt_{o,s}] = (\epsilon - \epsilon_{calc}) [t_{o,s} \neq t_s] = \epsilon (1 - \epsilon / (1 - r r_0)) (B_{o,s} - B_s) / (B_s - \Phi), \quad (18)$$

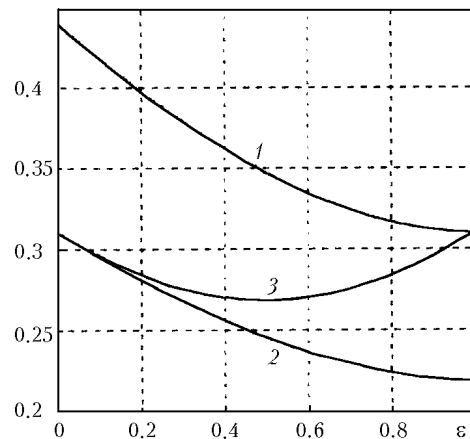
where  $r_0 \approx 0.15$  is the reflection coefficient of the entrance window of the optical system.

If  $t_s$  and  $\Phi_1$  vary during the measurements then, according to Eqs. (5)–(7), the methodological error of  $\epsilon$  determination will be

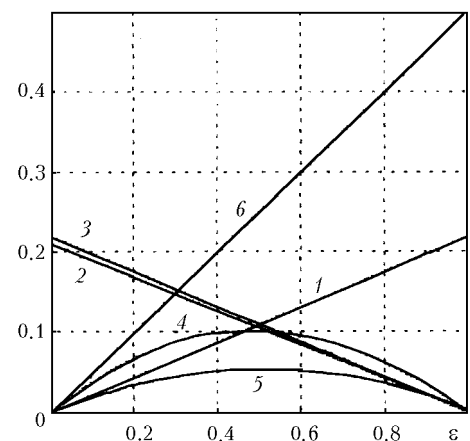
$$d\epsilon_I [dt_s] = (\epsilon - \epsilon_{calc}) [dt_s] = \epsilon \frac{B_{s(6)} - B_{s(5)}}{B_s - \Phi_1} \approx \frac{C_2}{\lambda} \frac{dt_s}{T_s^2} \frac{\epsilon}{1 - \Phi_1/B_s}, \quad (19)$$

$$d\epsilon_{I'} [d\Phi_1] = (\epsilon - \epsilon_{calc}) [d\Phi_1] = (1 - \epsilon) \frac{\Phi_{1(17)} - \Phi_{1(16)}}{B_s - \Phi_1} \approx \frac{C_2}{\lambda} \frac{dt_{\Phi_1}}{T_{\Phi_1}^2} \frac{r}{1 - B_s/\Phi_1} \quad (20)$$

Evidently, this method is most accurate and is based on three (rather than four) measurements; so, it should be chosen whenever possible (i.e., at  $t_s = t_{o,s}$ ). Figures 2 and 3 show the instrumental and methodological errors of the method I for  $t_{\Phi_1} = -42^\circ$ ,  $dt_{\Phi_1} = 0.2^\circ$ ,  $t_s = t_{\Phi_2} = 20^\circ$ , and  $dt_s = dt_{\Phi_2} = 0.1^\circ$ .



**Fig. 2.** Instrumental errors of methods I and II:  $D\epsilon_I$  (11) (curve 1),  $D\epsilon_{I'}$  (17) (curve 2);  $D\epsilon_{II}$  (25) (curve 3). In the parentheses there are the equation numbers. Calculations have been made for  $\delta t_n = 0.1^\circ$ ;  $t_{\Phi_1} = -42^\circ$ ,  $t_{\Phi_2} = 20^\circ$ ,  $dt_s = 0.1^\circ$ ,  $dt_{o,s} = 0.1^\circ$ ,  $dt_{\Phi_1} = 0.2^\circ$ ,  $\epsilon_c = 0.993$ ,  $d\epsilon_c/\epsilon_c = 0.5\%$ .



**Fig. 3.** Components of methodological errors of methods I and II:  $d\epsilon_I [dt_s]$  (12),  $d\epsilon_{I'} [dt_s]$  (19),  $d\epsilon_{II} [dt_s]$  (26) (curve 1);  $d\epsilon_I [d\Phi_1]$  (13),  $d\epsilon_{I'} [d\Phi_1]$  (20) (curve 2);  $d\epsilon_I [d\Phi_2]$  (14) (curve 3);  $d\epsilon_I [dt_{o,s}]$  (18) (curve 4);  $d\epsilon_{II} [d\Phi]$  (27) (curve 5); and  $d\epsilon_{II} [d\epsilon_c]$  (28) (curve 6). In the parentheses there are the equation numbers. Calculations have been made for  $t_{\Phi_1} = -42^\circ$ ,  $t_{\Phi_2} = 20^\circ$ ,  $t_s = 20^\circ$ ,  $t_{o,s} = t_s + 0.2^\circ$ ,  $dt_s = 0.1^\circ$ ,  $dt_{\Phi_1} = 0.2^\circ$ ,  $dt_{\Phi_2} = 0.1^\circ$ ,  $\epsilon_c = 0.993$ ,  $d\epsilon_c/\epsilon_c = 0.5\%$ ;  $d\Phi = 0.2^\circ$  in the units of temperature.

## 2. The measurements of $\epsilon$ with the account for surface with known $\epsilon_c$ and “cold” background

The idea of the second method consists of (a) sensing two surfaces with the same temperature  $t_s$  and emissivities  $\epsilon_c$ , assumed to be known (e.g., taken to be the emissivity of water), and  $\epsilon$ , which is to be determined (see Fig. 1a); and (b) measuring “cold” background radiation  $\Phi_1$  which is considered as illumination source for these surfaces in the measurements (a) (see Fig. 1b). Mathematically, these measurement conditions can be expressed as:

$$U_c = a (\epsilon_c B_s + r_c \Phi_1) + U_{sys}, \quad (21)$$

$$U_x = a (\epsilon B_s + r \Phi_1) + U_{sys}, \quad (22)$$

$$U_\Phi = a \Phi_1 + U_{sys}. \quad (23)$$

From Eqs. (21)–(23),

$$\epsilon = \epsilon_c (U_x - U_\Phi) / (U_c - U_\Phi). \quad (24)$$

The instrument error in  $\epsilon$  determination is:

$$D\epsilon_{II} \approx \frac{C_2 dt_s}{\lambda T_s^2} \frac{(2(\epsilon^2 + \epsilon_c^2 - \epsilon \epsilon_c))^{1/2}}{(1 - \Phi_1/B_s) \epsilon_c}. \quad (25)$$

The methodological errors arise here due to (a) poor knowledge of  $\epsilon_c$ , (b) different  $t_s$  used in measurements according to Eqs. (21) and (22), and (c) drift of  $\Phi_1$  in the course of measurements according to Eqs. (21)–(23).

Factor (b) causes the error:

$$\begin{aligned} d\epsilon_{II} [dt_s] &= (\epsilon - \epsilon_{calc}) [dt_s] = \\ &= \epsilon \frac{B_{s(22)} - B_{s(21)}}{B_s - \Phi_1} \approx \frac{C_2 dt_s}{\lambda T_s^2} \frac{\epsilon}{1 - \Phi_1/B_s}. \end{aligned} \quad (26)$$

When  $dt_s = t_{s(22)} - t_{s(21)} = 0.1^\circ$ ,  $d\epsilon_{II} (dt_s) / \epsilon < 0.2\%$ .

If in the course of measurements according to Eqs.(21) – (23) the background  $\Phi_1$  varies linearly in the manner:

$$\left. \begin{aligned} U_c &= a (\epsilon_c B_s + r_c \Phi_1) + U_{sys} \\ U_x &= a (\epsilon B_s + r (\Phi_1 + d\Phi)) + U_{sys} \\ U_\Phi &= a (\Phi_1 + 2d\Phi) + U_{sys} \end{aligned} \right\},$$

then the calculated  $\epsilon$  value will be biased relative to the actual one by the amount

$$d\epsilon_{II} [d\Phi] = (\epsilon - \epsilon_{calc}) [d\Phi] = \epsilon \left( 1 - \frac{2}{\epsilon_c} + \epsilon \right) \frac{d\Phi}{B_s - \Phi_1}. \quad (27)$$

Where  $\epsilon_c$  is with an error

$$d\epsilon_{II} [d\epsilon_c] = (\epsilon - \epsilon_{calc}) [d\epsilon_c] = \epsilon d\epsilon_c / \epsilon_c. \quad (28)$$

The instrumental and methodological errors for method II are presented in Figs. 2 and 3.

When the distance  $L$  between the radiometer and the target surface is fixed, the range of permissible angles for methods I and II of  $\epsilon$  determination has lower limit given by some quantity  $\Psi$ . If the radiometer is shaped as a cylinder, and if  $D$  denotes the diameter of the optical system and  $d$  is the diameter of coaxial entrance window, then  $L$  is related to  $D$ ,  $d$ , and radiometer FOV  $\alpha$  via

$$L = \frac{1}{2} \left( D \tan(\Psi) + \frac{(d + D) \cos(2\Psi - \alpha/2)}{2\sin(\Psi - \alpha/2) \cos(\Psi)} \right).$$

For  $\psi < \Psi$ , in measurements according to Eqs. (5) and (7), the target surface falling within the radiometer FOV, instead of  $\Phi_1$  and  $\Phi_2$  components, will receive radiation coming from the optical system, and it increases as  $\psi$  vanishes. When  $\Psi = 10^\circ$ ,  $D = 12$  cm,  $d = 3$  cm, and  $\alpha = 3^\circ$ ,  $L = 24.7$  cm. For  $\Psi = 3^\circ$ ,  $L$  is larger than 1.4 m. Thus, the methods I and II have a restriction in angle. On the other hand, for  $\epsilon(0^\circ) > 0.8$  this is a less stringent constraint since, as  $\epsilon$  calculations from Fresnel formulas show,  $\epsilon$  varies by less than 0.01% over the angular range 0–10° off the vertical direction.

For laboratory implementation of methods I and II, melting ice and water at room temperature can be used as sources of  $\Phi_1$  and  $\Phi_2$  components, with the appropriately pointed measurement device. However, the errors in this case will be 2–2.5 times larger than when “cold” downward clear-sky radiation is used as a source of  $\Phi_1$ . The method I was used to calculate the reflection coefficient of the polished mirror; at  $\lambda = 11$  and 12  $\mu\text{m}$  it was found to be  $r = 0.92 \pm 0.01$ , which is quite close to the value  $r = 0.87$ –0.92 reported previously.<sup>4</sup>

## 3. The $\epsilon$ measurement by the method of two calibrations

This method uses results of two calibrations: (a) against a surface with known  $\epsilon_c$ , and (b) against a surface whose  $\epsilon$  value is to be determined. The calibration (a) (such as against water surface for which  $\epsilon_c = 0.993$  at  $\lambda = 11 \mu\text{m}^5$ ), under the assumption of no multiple reflections between surface and optical system, can be expressed as

$$U_{ci} = a (\epsilon_c B_i + r_c \Phi) + U_{sys}.$$

Here,  $B_i = B(t_i)$ , while  $\{t_i\}$  is the set of calibration temperatures. Using the dataset  $\{B(t_i), U_{ci}\}$  and applying the least squares method,  $a_c = a \epsilon_c$  is determined (with  $a$  not known *a priori*). Similarly, using the dataset  $\{B(t_j), U_{xj}\}$  of calibration against the characteristics of the surface whose  $\epsilon$  is to be

determined,  $a_x = a\epsilon$  is evaluated. Then the desired quantity is as follows

$$\epsilon = \epsilon_c a_x / a_c;$$

and, hence,

$$\Delta\epsilon_{III} = \epsilon (da_x^2/a_x^2 + da_c^2/a_c^2 + d\epsilon_c^2/\epsilon_c^2)^{1/2} \quad (29)$$

is the total error in calculating  $\epsilon$  assuming the absence of multiple reflections. In Eq. (29), first two terms represent the instrumental error, and the last term is the methodological error. If both of the calibrations are accurate to 0.5%, and  $\epsilon_c$  is known with the same accuracy, then  $\Delta\epsilon_{III}/\epsilon \approx 1\%$ . For  $\epsilon \leq 0.2$ , the error  $\Delta\epsilon_{III} < \Delta\epsilon_I, \Delta\epsilon_I', \Delta\epsilon_{II}$ .

If the radiometer is aimed at the target surface approximately along the normal, multiple reflections occur between the radiometer and the surface, and the calibration ratios can now be written as

$$U_{xi} = \frac{a}{1 - r r_0} (\epsilon B_i + r B_{o,s}) + U_{sys},$$

$$U_{cj} = \frac{a}{1 - r_c r_0} (\epsilon_c B_j + r_c B_{o,s}) + U_{sys}.$$

Here,  $B_{o,s} = B(t_{o,s})$ . Thus, from the above described calibrations we determine  $a_x = a\epsilon/(1 - r r_0)$  and  $a_c = a\epsilon_c/(1 - r_c r_0)$ , and their ratio is  $A = a_x/a_c = \epsilon/\epsilon_c (1 - r_c r_0)/(1 - r r_0)$ . Hence,

$$\epsilon = A \epsilon_c / [1 + \epsilon_c (1 - A) r_0 / (1 - r_0)]. \quad (30)$$

If we neglect multiple reflections (MR), by taking that  $\epsilon_{calc} = \epsilon_c A$ , then  $\epsilon_{calc}$  will be biased by the amount

$$d\epsilon_{III} [MR] = (\epsilon - \epsilon_{calc}) [\text{MR}] = \frac{(\epsilon - \epsilon_{calc}) \epsilon}{1 - r r_0} r_0. \quad (31)$$

If  $\epsilon_c = 0.993$  (water at  $\lambda = 11 \mu\text{m}$ ),  $\epsilon = 0.8$ , and  $r_0 = 0.15$ , then  $d\epsilon_{III} [MR] = -0.024$ . Hence, if  $\epsilon$  is to be determined quite accurately in the presence of multiple reflections, those ought to be taken accurately into account; and for this,  $r_0$  must be known.

The  $r_0$  value can be determined using four measurements. The first two, involving sensing along nadir of a horizontal "reference" surface with known emissivity  $E = 1 - R$  at temperatures  $t_{1,2}$ , are expressed as

$$U_{1,2} = a (E B_{1,2} + R B_{o,s}) / (1 - R r_0) + U_{sys1}, \quad (32)$$

while the second two involve sensing of this surface (with the same  $E$ ) with the same temperatures along the directions at the angles  $\leq 10^\circ$  provided that the radiometer-surface distance is such that the background radiation  $\Phi$ , rather than radiation from the optical system, is incident on the surface. In this case, the

multiple reflections are absent, and so the corresponding expression takes the form:

$$\begin{aligned} V_{1,2} &= a (E B_{1,2} + R \Phi) + U_{sys2}, \\ B_{1,2} &= B(t_{1,2}). \end{aligned} \quad (33)$$

From Eqs. (32) and (33) it follows that

$$r_0 = [1 - (V_2 - V_1)/(U_2 - U_1)]/R. \quad (34)$$

Using expressions (34) in (30), total error in calculating  $\epsilon$  can be estimated with the account for the correction made for the multiple reflection effects:

$$\begin{aligned} \Delta\epsilon_{III} [MR_c] \approx \epsilon &\left\{ \left( 1 - r_0 \frac{r - r_c}{1 - r_c r_0} \right)^2 \frac{d\epsilon_c^2}{\epsilon_c^2} + \left( \frac{1 - r r_0}{1 - r_0} \right)^2 \times \right. \\ &\times \left( \frac{da_x^2}{a_x^2} + \frac{da_c^2}{a_c^2} \right) + \left( \frac{r - r_c}{(1 - r_c r_0)(1 - r_0)} \right) \left[ \left( r_0 \frac{dE}{R} \right)^2 + \right. \\ &\left. \left. + 2(1 + (1 - R r_0)^2) \left( \frac{1 - R r_0}{ER} \frac{C_2/\lambda}{1 - B_1/B_2} \frac{\delta t}{T_2^2} \right)^2 \right] \right\}^{1/2}. \quad (35) \end{aligned}$$

Here  $dE$  is the error of  $E$  determination; and  $\delta t$  is the noise in radiometer voltage in units of temperature when measurements based on the algorithm (32) – (33) are used. The errors of method III are presented in Fig. 4.

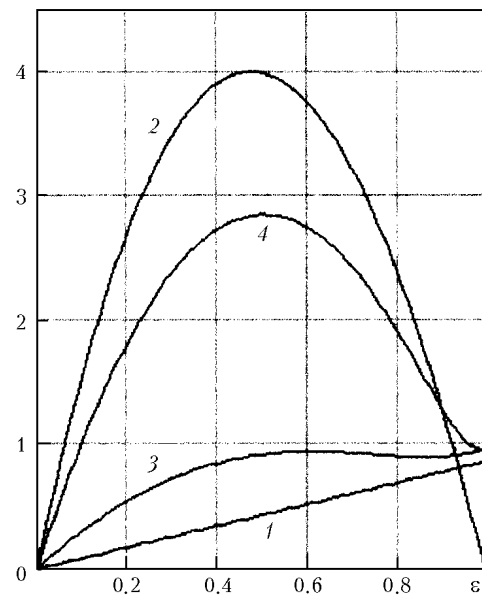


Fig. 4. Errors of method III:  $\Delta\epsilon_{III}$  (29) (curve 1);  $d\epsilon_{III} [MR]$  (31) (curve 2);  $\Delta\epsilon_{III} [MR_c]$  (35) (curves 3 and 4). Given in parentheses are the equation numbers. Calculations were made for  $\delta t_n = 0.1^\circ$ ;  $t_1 = 20^\circ$ ,  $t_2 = 40^\circ$ ,  $dt_1 = dt_2 = 0.1^\circ$ ;  $\epsilon_c = 0.993$ ,  $d\epsilon_c/\epsilon_c = da_x/a_x = da_c/a_c = dE/E = 0.5\%$ ;  $\delta t = 0.05^\circ$ ,  $r_0 = 0.15$ ; curve 3 is calculated with  $E = 0.8$ , and curve 4 with  $E = 0.95$ .

As  $E \rightarrow 1$ ,  $\Delta\epsilon_{\text{III}} [\text{MR}_c]$  rapidly increases (Fig. 4, curve 4). Thus, water surface cannot be used as a "reference" surface in determination of  $r_0$ , since the inequality  $E \leq 0.8$  does not hold in this case.

#### 4. Application

The first method is useful in the field study of  $\epsilon$  of plane objects such as thin films of natural or anthropogenic origin on the water surfaces. For instance, the oceanographic platform in Katsiveli had been used to study the influence of most frequently observed natural surface films with the thickness of about  $0.1 \mu\text{m}$  on the sea surface emissivity. Using special probe, we could collect water samples on the slice fragments of water surface without breaking surface films on them. Then,  $\epsilon$  measurements at the angles of  $30^\circ$ ,  $40^\circ$ ,  $50^\circ$ , and  $60^\circ$  had been conducted for these samples. As a result, it was found that, within the uncertainty  $d\epsilon \approx 0.005$ ,  $\epsilon$  does not depend on the presence of a film; whereas the brightness temperatures of clean and slice fragments of sea surface *in situ* differed by  $0.5\text{--}0.8^\circ$ . These measurement results, as well as the model calculations, have shown that fluctuations of the sea surface brightness temperature in the presence of natural slices are caused by variations in the thermodynamics temperature of the emitting surface layer; whereas surface emissivity variations due to the presence of thin films are negligibly small.<sup>6</sup>

We have calculated the emissivity of a clean sea surface as viewed by a radiometer at an angle of  $30^\circ$  using a filter with the passband of  $8\text{--}13 \mu\text{m}$ , and found that  $\epsilon = 0.987$ , which coincides, accurate to the measurement error, with the value determined from the data on complex refractive index of water.<sup>5</sup> The calculated  $\epsilon$  value had then been used to determine the sea surface natural emission and temperature difference in the skin layer.<sup>7</sup>

#### Conclusion

The methods I, II, and III presented here can be used to calculate the emissivity of plane surfaces over entire range of possible  $\epsilon$  values,  $\epsilon = 0\text{--}1$ . The question on which method suits best depends on the specific technological and methodological conditions of their implementation.

Neither calibration nor measurements over surface with a known  $\epsilon_c$  are required to use the first method. It suits quite well for the determination of the emissivity of polluted water surface under field conditions.

The second method requires measurements over surface with known  $\epsilon_c$ .

With the first and second methods, the  $\epsilon$  measurements can be performed at the angles not very close to the normal. They must include sensing of background with stable radiative properties differing considerably from those of the target surface. The emission measured under clear-sky conditions is very useful for this purpose.

The third method, in fact, compares the calibration results obtained for two surfaces: with the known  $\epsilon_c$  and with  $\epsilon$  to be determined.

At viewing angles close to the normal, multiple reflections between the radiometer and the target surface become important; so if one neglects this difference, the systematic error of method III can increase by  $(0.02\text{--}3) \cdot 10^{-2}$ , depending on the value of  $\epsilon$ . In the paper, we present a correction procedure, based on calculation of the reflection coefficient  $r_0$  of entrance window with the known emissivity  $E$ . The total error removal is impossible because of the presence of the radiometer noise  $\delta t$  and nonzero  $E$  determination error, especially at  $E \geq 0.8$ . At the viewing angles such that multiple reflections are important,  $\epsilon$  has very weak angular dependence; therefore, the measurements in this angular range ( $0\text{--}10^\circ$  off the normal) are of a very little concern here.

The third method requires two, and even four (when  $r_0$  must be determined to correct for the multiple reflection effects) calibrations; so it is more expensive than the methods I and II. Even though two measurements are sufficient to make a single calibration, their number is usually increased to 15 (or the duration of the measurements is increased when two stable temperatures,  $t_1$  and  $t_2$ , are used for the calibration) to make them accurate to 0.5%. If compared with the methods I and II, the third method is more accurate when  $\epsilon \leq 0.2$ ; while at  $\epsilon \approx 1$ , though not as accurate, it is more universal in that it can be used for the  $\epsilon$  determination at viewing angles close to  $90^\circ$ , by accounting for multiple reflections.

When a steering mirror with the reflection coefficient  $r_2$  is used in the measurement system to allow  $\epsilon$  measurements at different angles to the surface for a fixed radiometer orientation, the instrument response becomes lower, and the instrument error increases by a factor of  $1/r_2$ . According to this modification, the calculation formulas of the method III will change somewhat to include an additional factor  $r_2^2$  in the products of the form  $r r_0$ .

The Table gives the results of comparing the methods as well as estimates of calculation errors in  $\epsilon$  for  $\delta t_n = 0.1^\circ$ ,  $t_{\Phi 1} = -42^\circ$ ,  $t_{\Phi 2} = 20^\circ$ ,  $t_s = 20^\circ$ ,  $\epsilon_c = 0.993$ ,  $d\epsilon_c/\epsilon_c = 0.5\%$ ,  $t_1 = 20^\circ$ ,  $t_2 = 40^\circ$ ,  $t_{o,s} = 20.2^\circ$ ,  $dt_{1,2} = 0.1^\circ$ ,  $dt_s = 0.1^\circ$ ,  $dt_{o,s} = 0.1^\circ$ ,  $dt_{\Phi 1,2} = 0.2^\circ$ ,  $r_0 = 0.15$ ,  $E = 0.8$ ,  $dE/E = 0.5\%$ ,  $\delta t = 0.05^\circ$ .

**Table. Comparison of methods I, II, and III**

Parameter		I, I'	II	III
Number of calculations required		4; I' : 3	3	$\geq 4$ or $\geq 8$ <sup>1)</sup>
Are the measurements over surface with known $\epsilon_c$ required?		No	Yes	Yes
Can $\epsilon$ be measured at angles close to 90°?		No	No	Yes <sup>2)</sup>
Instrument error $D\epsilon, \cdot 0.01$	$\epsilon = 0-0.2$	0.44-0.4 I' : 0.31 $\rightarrow$ 0.28	0.31 $\rightarrow$ 0.29	0-0.14 <sup>3)</sup> 0-0.15 <sup>4)</sup>
	$\epsilon = 0.8-1$	0.32 $\rightarrow$ 0.31 I' : 0.23 $\rightarrow$ 0.22	0.29 $\rightarrow$ 0.31	0.57-0.71 <sup>3)</sup> 0.65-0.83 <sup>4)</sup>
Methodological error $d\epsilon, \cdot 0.01$	$\epsilon = 0-0.2$	0.31 $\rightarrow$ 0.25 I' : 0.21 $\rightarrow$ 0.19	0 $\rightarrow$ 0.11	0-0.1 <sup>3)</sup> 0-0.61 <sup>4)</sup>
	$\epsilon = 0.8-1$	0.19 $\rightarrow$ 0.22 I' : 0.19 $\rightarrow$ 0.22	0.44 $\rightarrow$ 0.55	0.4-0.5 <sup>3)</sup> 0.71-0.5 <sup>4)</sup>

<sup>1)</sup> In presence of multiple reflections.

<sup>2)</sup> A correction for multiple reflection effects is required.

<sup>3)</sup> In absence of multiple reflections.

<sup>4)</sup> Corrected for existing multiple reflections.

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