Record and reconstruction of Gabor hologram for formation of interference patterns in diffusely scattered light

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It is shown that an interferometer using diffusely scattered light is sensitive to lens astigmatism.

In Ref. 1, it was shown that single-exposure records of the hologram of the focused image of an amplitude scatterer by use of Gabor optical arrangement gives the interference pattern in equally wide fringes that characterizes spherical aberration of a lens (objective) used for reconstruction of the actual image of the scatterer. On the one hand, formation of the interference pattern occurs due to correlation of the subjective speckle fields in the (−1) and (+1) orders of diffraction in the far zone. On the other hand, the superposing of speckle fields doubles the interferometer sensitivity to spherical aberration as compared with that of the classical double-slit interferometers. The results of research in Ref. 2 showed that the use of a binary optical system (Kepler telescope) for formation of the actual image of an amplitude scatterer when recording the hologram following Gabor optical arrangement also provides for the controlling spherical aberration of an optical system.

In this paper, we consider the method for recording the hologram of the focused image of an amplitude scatterer. This method involves recording of the interference pattern characterizing the astigmatism of the lens used for construction of the scatterer image.

As shown in Fig. 1, the hologram is recorded by use of Gabor optical arrangement with a coherent radiation of a quasi-plane wave. This radiation is used to illuminate the amplitude scatterer *1*, i.e., in the general case, it is assumed, as in Refs. 1 and 2, that there are phase distortions of the wave due to, for example, aberrations of the optical system forming the wave front. The scattered radiation with a coherent background passes through the beam splitter *2* and a controlled positive lens *3* and then reflects from the mirror *4*. The reflected beam passes through the lens *3*, reflects from the beam splitter *2*, and then it is recorded on a photographic plate *5* lying in the plane of the actual image *1*.

After developing the plate, the hologram is reconstructed, as in Refs. 1 and 2, using a source of coherent light that was used at the stage of its recording as an interference pattern in the far diffraction zone at spatial filtering of the diffraction field using, for example, a round aperture in an opaque screen on the optical axis in the hologram plane.

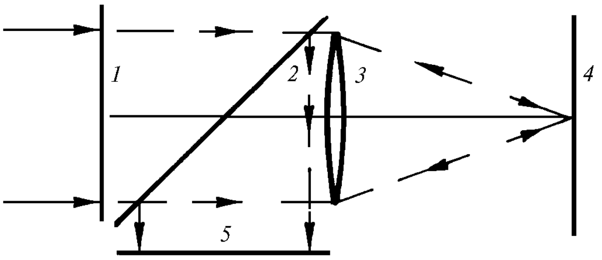

Fig. 1. Geometry of recording the Gabor hologram: amplitude scatterer *1*, beam splitter *2*, controlled lens *3*, mirror *4*, and photographic plate *5*.

Since the scatterer image constructed in Fig. 1 is the result of two consecutive Fourier transformations, the complex transmission amplitude of the hologram can be determined based on Ref. 2. Then, at the stage of hologram reconstruction, let us assume for brevity that the distribution of illumination in the back focal plane is recorded with a lens having the focal length f equal to the focal length of the controlled lens. Then in the observation plane (x, y) (ignoring beam turn by the beam splitter), the distribution of illumination takes the form

$$I(x, y) \sim [1 + \cos 4\varphi(x, y)]\,|F(x, y) \otimes P(x, y)|^2, \quad (1)$$

where $\varphi(x, y)$ is the phase function characterizing aberrations of the controlled lens;

$$F(x, y) = \iint\limits_{-\infty}^{\infty} t(-x_0,-y_0) \exp\left[-\frac{ik}{f}(x_0x + y_0y)\right] \mathrm{d}x_0\,\mathrm{d}y_0$$

is the Fourier transform of the absorption amplitude of the scatterer that is a random real function of coordinates; x_0 and y_0 are the coordinates in the hologram (photographic plate) plane; k is the wave number;

$$P(x, y) = \iint\limits_{-\infty}^{\infty} p(x_0, y_0) \exp\left[-\frac{ik}{f}(x_0x + y_0y)\right] \mathrm{d}x_0\,\mathrm{d}y_0$$

is the Fourier transform of the transmission function $p(x_0, y_0)$ of the opaque screen with a round aperture.[3]

From Eq. (1) it follows that the subjective speckle structure is modulated by interference fringes of equal width due to aberrations of the controlled lens.

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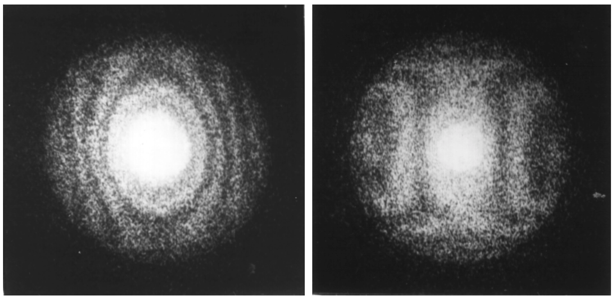
In this case, as in Refs. 1 and 2, the diffracting quasi-plane waves for the (−1) and (+1) diffraction orders in the Fourier plane that form the interference pattern are turned around the optical axis by 180° relative to each other. This allows us to exclude distortions of the wave front due to asymmetric aberrations.

As in the case of the classic Twyman – Green interferometer,[4] the use of the beam passages forth and back through the controlled lens excludes its spherical aberration. Then, as the hologram is recorded in the plane of best focusing, the phase function $\varphi(x, y)$ in Eq. (1) corresponds to astigmatism of the controlled lens.



a *b*

Fig. 2. Interference patterns recorded when the photographic plate is set in the plane of best focusing (*a*) and off this plane (*b*).

In the experiment, we used the radiation of a He–Ne laser at 0.63 μm wavelength for recording and reconstructing the holograms. Holograms were recorded onto Mikrat–VRL photographic plates. As an example, Figure 2*a* shows the interference pattern characterizing the astigmatism of the lens *3* (see Fig. 1) with the coefficient $C = 0.5\lambda$, where λ is the wavelength of coherent radiation used for recording and reconstruction of the hologram (the additional component of spherical aberration is caused by hologram aberrations). The focal length of the controlled lens 18 mm in diameter was 160 mm. The interference pattern was recorded in the focal plane of the objective with the focal length of 80 mm at spatial filtering of the diffraction field in the hologram plane by reconstructing it with a small-aperture (≈ 2 mm) laser beam.

The interference pattern localized in the pupil plane of the controlled lens characterizes this type of aberration provided that the hologram has been recorded in the plane of best focusing. If this condition is not fulfilled, the interference pattern shown in Fig. 2*a* changes due to defocusing. Thus, Fig. 2*b* corresponds to the case of hologram recorded at the distance of 0.9 mm from the plane of best focusing, and the interference pattern characterizes the combination of astigmatism and defocusing.

In conclusion, it should be noted that the considered interferometer provides for formation, in diffusely scattered fields, of the interference pattern in equally wide fringes. This pattern characterizes the astigmatism of the controlled lens. In this interferometer, there is no need in obtaining an aberration-free wave front for making a comparison. In contrast to the classic double-slit interferometer, the proposed interferometer has twice as high sensitivity, because the formation of the interference pattern occurs due to superposition of quasi-plane reversed waves diffracting in the (−1) and (+1) orders of diffraction.

References

1. V.G. Gusev, Opt. Zh. **65**, No. 2, 36–40 (1998).
2. V.G. Gusev, Atmos. Oceanic Opt. **9**, No. 7, 569–574 (1996).
3. M. Born and E. Wolf, *Principles of Optics* (Pergamon, New York, 1959).
4. P. Hariharan and D. Sen, Proc. Phys. Soc. **77**, 328–334 (1961).