

Stochastization of the laser beam intensity at nonlinear propagation in a droplet aerosol

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The paper presents some results of investigations into the dynamics of statistical characteristics of a laser beam with fluctuating power acting on a droplet aerodisperse medium with the determinate parameters. It is shown that nonlinear interaction processes result in swinging the initial fluctuations of the beam intensity. As a result, random variations of the beam intensity at the end of the path can far exceed the initial ones.

It is well known that propagation of electromagnetic wave in real media (e.g., in the turbulent atmosphere) is accompanied by the change of its statistical characteristics. The information on these changes is very important in many applications, in particular, at remote measurements and control of a random medium parameters. Thus, at radiation propagation in transparent random media the study of statistical intensity moments and wave phase enables one to determine both the value and the fluctuation spectrum of the medium refractive index. The account for random variations of the radiation parameters is also necessary when analyzing the variation of effective dimensions and shifts of center of gravity of wave beams.

In the general case real media are characterized by pulsations of both real (refractive index) and imaginary (absorption or amplification factor) components of the dielectric constant ϵ . The investigation of linear radiation propagation in such media has shown (see, e.g., Refs. 1–3) that the presence of fluctuations of an imaginary part of ϵ results not only in extra variations of traditional characteristics (variance and distribution function of the intensity fluctuations), when analyzing the wave transfer in transparent random media, but also in the occurrence of pulsations of such characteristics as the total radiation (power) flux. This case is important in some practical problems and it must be taken into account, in particular, in calculations of efficient laser beam parameters (such as broadening, shift of the energy gravity center, variance of the beam jitter), since the above characteristics are determined by spatial moments of radiation intensity normalized to the power in a separate realization. The role of power pulsations is essential in solving the problems of recording the optical fields. In this situation the irregularity of radiation flux incident on a receiving device, gives rise to photocurrent fluctuations, which are complementary to the shot noise.

The effect of power stochastization can be more significant at nonlinear wave propagation in dissipative (amplifying) media. In this case the thermal blooming of wave considerably changes conditions of its

propagation through the medium, since not only the medium affects radiation, but the radiation causes transformations of the medium parameters. Moreover, the medium with initially determined parameters can be chaotically disturbed by an intense incident radiation. This problem, namely, the evolution of stochastization of radiation power with a fluctuating intensity (at the exit from the radiation source) at nonlinear wave transfer in a dissipative medium with initially determined parameters, will be considered in this paper where high-power laser beam propagation in a droplet aerosol is taken as an example. As known^{4,5} the action of high-power laser radiation on the aerosol medium results in its clearing up accompanied by the appearance of fluctuations (due to different mechanisms of randomization) of the imaginary part ϵ_I of the dielectric constant that are compared in magnitude with the pulsations ϵ_R of its real part.^{4–9}

Let us next investigate the dynamics of fluctuations of the laser beam power P with the given initial pulsations of intensity at clearing up of a droplet-aerosol medium with the determined parameters. Note that such a study is not only of independent interest but it is necessary when solving the problems of the calculation of laser beam effective parameters in the media being cleared up.

Let us assume that a pulse beam of laser radiation is incident on the droplet aerodisperse medium with the determined parameters occupying the halfspace $z \geq 0$. The laser beam is well absorbed by droplets and the fluctuating intensity is \tilde{I}_0 . We consider the case when the pulse duration t_p is chosen to be much less than the time of wind blurring of the area of radiation interaction with the medium. In this case the process of radiation effect on the droplet aerosol can be described by a system of equations with an equation for a complex amplitude u of an electromagnetic wave

$$2ik \frac{\partial u}{\partial z} + \Delta_{\perp} u + k^2 [\epsilon_R(I) + i\epsilon_I(I)] u = 0, \quad (1)$$

an equation of heat transfer

$$\frac{\partial T}{\partial t} = (\rho_{\text{air}} c_p)^{-1} \int_0^{\infty} dR (1 - \beta_{\text{eff}}) \times \sigma_a(R) I(\mathbf{r}, R, t) f(\mathbf{r}, R, t), \quad (2)$$

an equation for droplet size distribution f , when taking the account of a possibility of regular and explosive destruction of the aerosol condensed phase,

$$\frac{\partial f}{\partial t} + \frac{\partial(fR)}{\partial R} = \int_{R_{\text{th}}}^{\infty} dR' \phi(R', R) f(\mathbf{r}, R', t) \delta(t - t_w(R', I)) - f(\mathbf{r}, R, t) \delta(t - t_w(R, I)) \theta(R - R_{\text{th}}(I)), \quad (3)$$

an equation of evaporation of an isolated droplet of the radius R

$$\dot{R} = \frac{dR}{dt} = -\beta_{\text{eff}} K_a(R) I / 4\pi \rho_d L \quad (4)$$

with the initial and boundary conditions

$$\begin{aligned} u(\mathbf{p}, z, t) \Big|_{z=0} &= (I_0(\mathbf{p}))^{1/2} e^{i\tilde{s}_0(\mathbf{p})} \theta(t_p - t); \\ T \Big|_{t=0} &= T_0; \\ f(\mathbf{r}, R, t) \Big|_{t=0} &= f_0(R); \\ R \Big|_{t=0} &= R_0. \end{aligned} \quad (5)$$

In Eqs. (1)–(5) the following designations are used: $\varepsilon_R = \frac{\partial \varepsilon}{\partial T} (T - T_0)$ is the variation of the real part of the dielectric constant of the medium due to the radiation heating of the medium; $\varepsilon_I = \frac{1}{k} \alpha(I)$ is the imaginary part of ε ; k is the wave number; $\alpha(I) = \int_0^{\infty} dR \sigma_e(R) f(\mathbf{r}, R, t)$; σ_e is the cross section of radiation attenuation by a droplet with the radius R ; $\rho_{\text{air}} c_p$ is the specific heat of the air unit volume; $I = \frac{c |u|^2}{8\pi}$ is the radiation intensity; c is the speed of light; $\mathbf{r} = (\mathbf{p}, z)$; $\mathbf{p} = (x, y)$ is the radius-vector of the point in a plane perpendicular to the z axis; t is time; $\sigma_a(R)$, $K_a(R)$ is the cross section and the efficiency factor of radiation absorption by a droplet of radius R ; β_{eff} is the efficiency factor of droplet evaporation depending, in the general case, on the radiation intensity and the droplet size and determined by solving the system of equations of conservation of energy and mass of a droplet in the radiation flux⁴; $\phi(R, R')$ is the distribution function of the fragments of the exploded droplet of the radius R over the size R' ; t_w is the time of droplet warming up to the temperature of explosive fragmentation; $R_{\text{th}}(I)$ is the threshold value of the radius of exploding droplets at a given value of the radiation intensity I ; ρ_d , L denote the

density and specific heat of droplet evaporation; T_0 , f_0 , R_0 are the initial values (before the effect) of the medium temperature, the distribution function of droplets, and droplet radius; \tilde{s}_0 is the initial wave phase distribution; $\delta(x)$, $\theta(x)$ are the delta and theta functions. The quantities $t_w(I)$, $R_{\text{th}}(I)$ determining the conditions of realization of explosion of an isolated droplet in the radiation field with the intensity I , can be calculated and found from the experimental data (see, e.g., Ref. 4).

From the above system it follows that the irregularity of variations of the intensity of radiation incident on the medium results in the irregularity of the process of destruction of droplet fraction of the medium (immediately from the inlet layer) and in the occurrence of fluctuations of the imaginary and real parts of ε that were determinate before the action. The latter components, in turn, affect the statistical characteristics of the complex wave amplitude (see Eq. (1)).

Solution of the above problem can be achieved either numerically, based on the above-stated system of equations, or analytically when performing the appropriate simplifications.

First we consider the analytical version. It should be noted that for the case of a linear wave propagation through a turbulent absorbing medium² as well as for the nonlinear problem in the approximation of a preset field from Eq. (1) the following equation can be derived for the beam power $P = \int d^2\rho I(\mathbf{p}, z, t)$:

$$\frac{\partial P(z, t)}{\partial z} = -k \int d^2\rho \varepsilon_I(I) I(\mathbf{p}, z, t). \quad (6)$$

As would be expected, the variation of the beam radiation power on the path depends on whether the medium has imaginary part of ε . At $\varepsilon_I = 0$ the beam power is retained along the path and equals the initial value $P = P_0$. If ε_I varies randomly, then from Eq. (6) it follows that the radiation power fluctuates as well. In this case the dynamics of power pulsations depends on the dynamics of destruction of aerosol droplet fraction on the path of the wave propagation.

In the general case of a nonlinear problem the real component of ε can affect the beam power. But this effect can be observed only along very extended paths, such that their length is sufficient for wave phase inhomogeneities due to gradients ε_R to transform into the amplitude inhomogeneities due to the refraction. The amplitude ones, in their turn, should convert into the inhomogeneity of an imaginary part of ε due to the variation of conditions of droplet destruction.

In the present paper attention must be given to the basic primary effect of excitation of fluctuations of ε_I caused by the immediate effect of the radiation with the irregular initial intensity on droplet aerosol. Therefore instead of Eqs. (1), and (2), neglecting the effects of deformation of the phase wave front, we can write the transfer equation for the radiation beam intensity:

$$\frac{\partial I}{\partial z} + k\varepsilon_1(I) I = 0 \tag{7}$$

with the boundary condition

$$I(\mathbf{p}, z, t)|_{z=0} = \tilde{I}_0(\mathbf{p}) \theta(t_p - t).$$

The analytical solution will be derived for the case of the regime of regular droplet destruction with the use of frequently used in the problems of clearing up of droplet aerosol the approximation of water content and constancy of β_{eff} (see, e.g., Ref. 4). Taking into account the above assumptions, Eqs. (2) and (3) can be written as follows:

$$\frac{\partial W}{\partial t} + \mu WI = 0, \tag{8}$$

where $\mu = \beta_{\text{eff}} C_1/L$; $W = k\varepsilon_1/C_0$ is the aerosol water content; C_1, C_0 are the constants of the water content approximation (for the CO₂ laser radiation and cloudy medium $C_1 = 0.5C_0$ and is equal to $0.75 \cdot 10^{-3} \text{ cm}^2/\text{g}$).

The equations (7) and (8) give, for realization of the random intensity field, the following expression:

$$I(\mathbf{p}, z, t) = \tilde{I}_0(\mathbf{p}) [1 + (e^\tau - 1) e^{-\mu \tilde{I}_0 t}]^{-1}, \tag{9}$$

where $\tau = k \int_0^z dz' \varepsilon_{I_0}(z')$ is the optical depth of aerosol medium unperturbed by the radiation; $\varepsilon_{I_0} = \varepsilon_1(I)|_{z=0}$.

Using the above ratio for the radiation intensity we calculate statistical characteristics of the beam power. We analyze the mean value $\langle P \rangle$, the variance D_P^2 and the relative variance σ_P^2 of the power fluctuations. By the definition we have

$$\begin{aligned} \langle P(z, t) \rangle &= \int d^2\mathbf{p} \langle I(\mathbf{p}, z, t) \rangle = \\ &= \int d^2\mathbf{p} \left\langle \frac{\tilde{I}_0(\mathbf{p})}{[1 + (e^\tau - 1) e^{-\mu \tilde{I}_0 t}]} \right\rangle, \end{aligned} \tag{10}$$

$$D_P^2 = \langle P^2 \rangle - \langle P \rangle^2, \quad \sigma_P^2 = \frac{\langle P^2 \rangle - \langle P \rangle^2}{\langle P \rangle^2} = \frac{D_P^2}{\langle P \rangle^2}, \tag{11}$$

$$\begin{aligned} \langle P^2 \rangle &= \iint d^2\mathbf{p}_1 d^2\mathbf{p}_2 \langle I(\mathbf{p}_1, z, t) I(\mathbf{p}_2, z, t) \rangle = \\ &= \iint d^2\mathbf{p}_1 d^2\mathbf{p}_2 \langle \tilde{I}_0(\mathbf{p}_1) [1 + (e^\tau - 1) e^{-\mu \tilde{I}_0(\mathbf{p}_1)t}]^{-1} \times \\ &\quad \times \tilde{I}_0(\mathbf{p}_2) [1 + (e^\tau - 1) e^{-\mu \tilde{I}_0(\mathbf{p}_2)t}]^{-1} \rangle = \\ &= \iint d^2\mathbf{p}_1 d^2\mathbf{p}_2 B_1(\mathbf{p}_1, \mathbf{p}_2, z, t) + \langle P \rangle^2, \end{aligned} \tag{12}$$

where $B_1(\mathbf{p}_1, \mathbf{p}_2, z, t)$ is the correlation function of the wave intensity fluctuations; averaging over the ensemble of realizations is denoted by angle bracket.

Considering the expression for $\langle P^2 \rangle$, Eq. (11) for D_P^2 can be written in the following form:

$$D_P^2 = \iint d^2\mathbf{p}_1 d^2\mathbf{p}_2 B_1(\mathbf{p}_1, \mathbf{p}_2, z, t),$$

from this expression it follows that the variance of power fluctuations depends not only on the value of random intensity variations but on their correlation.

Analysis of expression (9) indicates that in the time interval of interaction of radiation with the medium two typical regions can be separated out. In the first one the condition $(e^\tau - 1) e^{-\mu \tilde{I}_0 t} \gg 1$ is fulfilled, and the state with low degree of the medium clearing up is determined. The second region is characterized by a significant clearing up effect, and within its limits the inequality $(e^\tau - 1) e^{-\mu \tilde{I}_0 t} < 1$ is fulfilled. The second time interval is less interesting from the point of view of nonlinear excitation of random variations of intensity and beam power since in the limiting case of a complete clearing up the statistics of I and P in the beginning and the end of the path became identical. Therefore, we focus on the study of the development of the process of stochastization of the medium and laser beam within the first of the above time intervals, within the limits of which the following representation for the radiation intensity

$$I(\mathbf{p}, z, t) \cong \tilde{I}_0 e^{-\tau} e^{\mu \tilde{I}_0(\mathbf{p})t} \quad (\tau - \mu \tilde{I}_0 t > 1) \tag{13}$$

is valid.

Let us consider the case when the intensity fluctuations of radiation, incident on the medium are small, i.e., $D_0 \ll \langle \tilde{I}_0 \rangle$ (here $D_0^2 = \langle \tilde{I}_0^2 \rangle - \langle \tilde{I}_0 \rangle^2$ is the dispersion of the beam intensity fluctuations at the input to medium). Let us assume that \tilde{I}_0 has quasi-Gaussian statistics, for which the single-point and double-point functions of pulsation distribution I_0 (these functions are necessary to calculate $\langle P \rangle$, D_P^2 and σ_P^2) are of the form:

$$\begin{aligned} f_1(\tilde{I}_0) &= \frac{1}{(2\pi D_0^2)^{1/2}} \exp \left\{ -\frac{1}{2 D_0^2} (\tilde{I}_0 - \langle \tilde{I}_0 \rangle)^2 \right\}, \tag{14} \\ f_2(\tilde{I}_0(\mathbf{p}_1); \tilde{I}_0(\mathbf{p}_2)) &= f_2(\tilde{I}_{01}; \tilde{I}_{02}) = \\ &= \{4\pi^2 D_0^4 [1 - \beta^2(\mathbf{p})]\}^{-1/2} \exp \{-[2D_0^2 (1 - \beta^2(\mathbf{p}))]^{-1} \times \\ &\quad \times [(\tilde{I}_{01} - \langle \tilde{I}_{01} \rangle)^2 + (\tilde{I}_{02} - \langle \tilde{I}_{02} \rangle)^2 - 2\beta(\mathbf{p}) \times \\ &\quad \times (\tilde{I}_{01} - \langle \tilde{I}_{01} \rangle) (\tilde{I}_{02} - \langle \tilde{I}_{02} \rangle)]\}, \end{aligned} \tag{15}$$

where $\beta^2(\mathbf{p}) = \langle (\tilde{I}_{01} - \langle \tilde{I}_{01} \rangle) (\tilde{I}_{02} - \langle \tilde{I}_{02} \rangle) \rangle / D_0^2$ is the normalized correlation function of intensity fluctuations at the input to medium. Note that for the given distributions the condition $D_0 \ll \langle \tilde{I}_0 \rangle$ provides only small contribution to the averaged values of a physically unrealistic region $\tilde{I}_0 < 0$.

Now suppose that the mean intensity distribution at the medium input is taken to be homogeneous and substituting Eqs. (14) and (15) in (10) and (11) with the allowance for Eq. (13) we obtain for the mean value and variance of the power fluctuations

$$\langle P \rangle = \pi a_0^2 \langle \tilde{I}_0 \rangle (1 + \bar{q}_0 \sigma_0^2) \exp(\bar{q}_0 - \tau + \bar{q}_0^2 \sigma_0^2 / 2); \quad (16)$$

$$\begin{aligned} D_P^2 &= \langle \tilde{I}_0 \rangle^2 \exp(2\bar{q}_0 - 2\tau + \bar{q}_0^2 \sigma_0^2) \times \\ &\times \iint d^2 \rho_1 d^2 \rho_2 \theta(a_0 - |\rho_1|) \theta(a_0 - |\rho_2|) \times \\ &\times \{ \exp[\bar{q}_0^2 \sigma_0^2 \beta(|\rho_1 - \rho_2|)] \{1 + 2\bar{q}_0 \sigma_0^2 [1 + \beta(\rho)] + \\ &+ \bar{q}_0^2 \sigma_0^4 [1 + \beta(\rho)] [1 - \beta^2(\rho)] + \\ &+ \beta(\rho) \sigma_0^2 [1 + \bar{q}_0^2 \sigma_0^2 (1 + \beta)^2] - (1 + \bar{q}_0 \sigma_0^2)^2 \}, \quad (17) \end{aligned}$$

where $\bar{q}_0 = \mu \langle \tilde{I}_0 \rangle t$ is the mean value of thermal effect function at the input to medium; $\sigma_0^2 = D_0^2 / \langle \tilde{I}_0 \rangle^2$; $\rho = \rho_1 - \rho_2$.

From Eq. (16) it follows that the presence of intensity fluctuations in the beam affecting the aerosol results in an increase of its mean power transmitted through the medium, as compared with the situation, when the input intensity is determinate ($\sigma_0^2 = 0$). In this case, at the considered time interval, the value $\langle P \rangle$ grows fast with the increase of time of action and, hence, of the thermal effect function \bar{q}_0 ($\langle P \rangle \sim \exp(\bar{q}_0^2 \sigma_0^2 / 2)$).

Such a growth of the mean power within this time interval ($\mu \tilde{I}_0 t < \tau$) is caused by the growth with time of fluctuations of the optical depth $\tau_{cl} = \tau - \mu \tilde{I}_0 t$ of the medium cleared up (see Eq. (13)) as well as by known effect of the mean intensity enhancement in an absorbing random medium as compared with the determinate optical depth.

As to the variance of the power fluctuations, it grows fast with the increasing time of action on the medium, but the degree of its growth depends on the correlation of intensity pulsations at different points of the wave front at the medium input. Really, in the case when the degree of correlation of random variations I_0 is significant over the entire beam cross section ($\rho_{c_0} \geq a_0$, $\beta \sim 1$, where ρ_{c_0} is the initial value of the correlation radius of pulsations I_0), for D_P^2 we have

$$\begin{aligned} D_{P1}^2 &\cong \langle P_0 \rangle^2 \exp(2\bar{q}_0 - 2\tau + \bar{q}_0^2 \sigma_0^2) \times \\ &\times [\exp(\bar{q}_0^2 \sigma_0^2) (1 + \sigma_0^2 + 4\bar{q}_0 \sigma_0^2 + 4\bar{q}_0^2 \sigma_0^4) - \\ &- (1 + \bar{q}_0 \sigma_0^2)^2]. \quad (18) \end{aligned}$$

In the opposite limiting case ($\rho_{c_0} \ll a_0$) for the variance of power fluctuations we can write

$$\begin{aligned} D_{P2}^2 &\cong \langle P_0 \rangle^2 (\rho_{c_0} / a_0)^2 \exp(2\bar{q}_0 - 2\tau + \bar{q}_0^2 \sigma_0^2) \times \\ &\times [\exp(\bar{q}_0^2 \sigma_0^2) (1 + \sigma_0^2 + 4\bar{q}_0 \sigma_0^2 + 4\bar{q}_0^2 \sigma_0^4) - \\ &- (1 + \bar{q}_0 \sigma_0^2)^2] = (\rho_{c_0} / a_0)^2 D_{P1}^2 \ll D_{P1}^2. \quad (19) \end{aligned}$$

Comparative analysis of expressions for D_P^2 and $\langle P \rangle^2$ indicates that the value D_P^2 depends stronger on \bar{q}_0 than on $\langle P \rangle^2$. Consequently, in the cleared up medium at the first stage of the irradiation, as time increases, the relative variance σ_P^2 of power fluctuations also increases by the end of the path; true enough, the increase will be slower than for the value D_P^2 .

To investigate the behavior of power fluctuations over the entire time interval of the radiation effect on the droplet aerosol, the problem must be solved numerically based on the system of Eqs. (1)–(5) and without the use of approximations of water content and constancy of β_{eff} , limiting the applicability of the results obtained. We consider also a possibility of not only regular, but the explosive destruction of a condensed phase of aerosol medium.

The calculations of statistical power characteristics were made for a CO₂ laser beam with homogeneous distribution of the mean intensity $\langle \tilde{I}_0 \rangle = I_0 \theta(a_0 - \rho)$ over the beam cross section for different values of the initial optical thickness τ of droplet aerosol, the correlation radius ρ_{c_0} and the intensity I_0 .

As an illustration Figure 1 shows the calculated results on the dynamics of the relative variance σ_P^2 of the beam power fluctuations with the intensities $I_0 = 10^3$ W/cm² (that is below the threshold of explosive destruction of droplets), $I_0 = 10^5$ W/cm² (that is higher than the explosion threshold) while having same pulse energy in the course of thermal blooming of a pulse in droplet aerosol with the initial thickness $\tau = 5, 10, 20$. The initial distributions of droplets and explosion fragments, over their radii, are taken as gamma-distributions with the parameters $R_m = 5$ μm , $s = 3$ and $r_m = 0.5$ μm , $s' = 3$, respectively. The values σ_P^2 and ρ_{c_0} are taken to be equal to 0.1 and $0.2a_0$.

Figure 1 shows that as the time of action on the medium increases, the beam power fluctuations at the end of the path first grow fast and then, after reaching maximum, the fluctuations decrease. The maximum of σ_P^2 far exceeds the initial value of $\sigma_{P0}^2 = \sigma_P^2|_{t=0}$ and depends strongly on the optical thickness τ . Thus, for $\tau = 20$ at a maximum $\sigma_P^2 \cong 60 \sigma_{P0}^2$. Such a significant increase of power fluctuations is due to the effect of a “swinging” of random intensity variations in the course of nonlinear transfer of laser radiation when the initial pulsations of radiation intensity induce irregular fields of physical characteristics of a cleared-up medium,

which, in turn, increases the fluctuations of beam intensity and power as a result of synchronous random effect on a light flux.

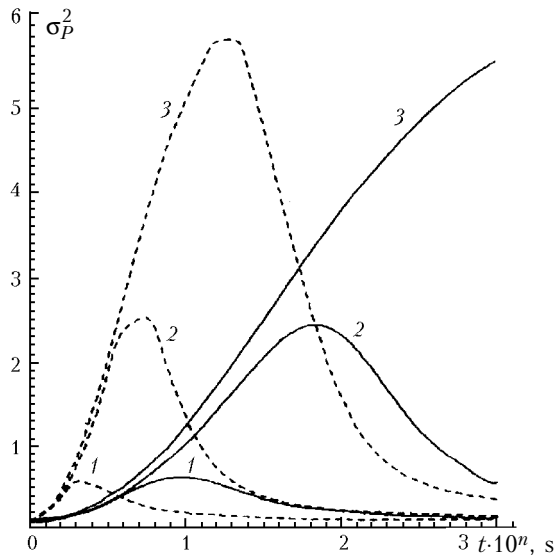


Fig. 1. Variation in time of the variance of power fluctuations for $I_0 = 10^3$ W/cm², $n = 2$ (solid curves) and $I_0 = 10^5$ W/cm², $n = 4$ (dashed lines); $\tau = 5$ (1), 10 (2), 20 (3).

The value of σ_p^2 decreases only at the final stage of thermal blooming when the degree of clearing up of the medium is high, that is, the degree of droplet destruction is high and the cause of random dissipation of radiation power disappears.

Analysis of the calculated results (see also the figure) indicates that the character of variation of σ_p^2 in time is similar for the cases of both of regular and

explosive regimes of destruction of droplet aerosol fraction. The relative time of reaching the maxima of σ_p^2 is different. At droplet explosion the maxima of σ_p^2 are attained at smaller values of \bar{q}_0 since the medium clearing up occurs more efficiently.

In conclusion it should be noted that the possibility of initiating swinging effect of such an averaged over the beam cross section characteristic as power, occurs only in the dissipative (amplifying) media, especially in the case when the thermal blooming of intense radiation takes place under conditions of the forced destruction of the medium structure.

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