On the advantages of using circularly polarized laser radiation in sensing crystal clouds

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It is shown that the using circularly polarized laser radiation and measuring the fourth Stokes parameter of scattered light offer certain advantages over the universally accepted measurements that use linearly polarized sounding. This eliminates the uncertainty in lidar depolarization, which may appear due to some spatial ordering of particles in a cloud. In addition, measurement of the fourth or second Stokes parameter of the scattered light at circular polarization of laser radiation has some promises in distinguishing between the plate and columnar crystals.

The backscattering phase matrix (BSPM) bears maximum information that can be obtained in the experiments on laser sensing of a scattering medium in order to determine its microphysical parameters. However, even in the case of single-frequency monostatic sensing, determination of the BSPM requires a large number of independent polarization measurements.[1] Therefore, it is worth analyzing the possibilities that may appear in polarization measurements that are simplified to a maximum degree.

Laser sensing of atmospheric aerosols often relies on the intensity measurement of two components of the backscatter that have orthogonal polarization. Laser radiation is usually linearly polarized, and measured parameters are the intensities of scattered light polarized parallel $I_\parallel$ and perpendicular $I_\perp$ to the polarization of sounding laser radiation. The presence of cross-polarized component, $I_\perp$, in the backscattered signal is indicative of particle nonsphericity. This statement is valid within the framework of the single-scattering approximation, that we use in this paper. As the parameter characterizing the particle nonsphericity in some way, we take the lidar depolarization ratio, which is sometimes defined as $I_\perp/I_\parallel$ or, for example, as $\Delta_L = I_\perp/(I_\parallel + I_\perp)$ (Ref. 2). It can be easily seen that the latter definition gives the value half as large as the one defined by a more acceptable definition[3]:

$$\Delta_L = 1 - |I_\parallel - I_\perp| / (I_\parallel + I_\perp) = 1 - |Q| / I. \quad (1)$$

The use of absolute values in Eq. (1) follows from the definitions

$$P = (Q^2 + U^2 + V^2)^{1/2}/I; \ \ \Delta = 1 - P, \quad (1')$$

where P is the degree of polarization; $I = (I_\parallel + I_\perp)$, Q, U, and V are the Stokes parameters; Δ is the true value of the radiation depolarization.

Equation (1) gives $\Delta_L = \Delta$ at zero values of the parameters U and V, and this is not always the case with sensing crystal clouds with a linearly polarized laser radiation. Deviations are possible if there is a preferred orientation of particles in a cloud that does not coincide with the direction of the wave vector of the incident radiation. At the random orientation of particles, all directions are equiprobable, and the backscattering phase matrix proves to be invariant relative to lidar rotation about its optical axis and takes especially simple form[3]:

$$\mathbf{M} = \begin{bmatrix} M_{11} & 0 & 0 & M_{14} \\ 0 & M_{22} & 0 & 0 \\ 0 & 0 & M_{33} & 0 \\ M_{41} & 0 & 0 & M_{44} \end{bmatrix}, \quad (2)$$

and the following identity is fulfilled

$$(M_{11} - M_{44})/2 \equiv - M_{33} \equiv M_{22}; \ M_{14} \equiv M_{41}. \quad (3)$$

Mathematically, the invariance relative to rotation about the wave vector of the incident radiation can be expressed as

$$\mathbf{M}' = \mathbf{R}(\Phi)\ \mathbf{M}\mathbf{R}(\Phi) = \mathbf{M}, \quad (4)$$

where $\mathbf{R}(\Phi)$ is the operator of turn of the coordinate system by the angle Φ.

It is seen from Eqs. (2) and (3) that the BSPM of an ensemble of randomly oriented particles is determined by three parameters M_{11}, M_{44}, and M_{14}, which are invariants of rotation for the BSPM of any ensemble of particles. From this it also follows that Eq. (1), even at the random particle orientation, gives true value of the radiation depolarization only if the incident radiation is linearly polarized. However, the available experimental results on the BSPM of crystal clouds[1,4] suggest that the values of M_{14} and M_{41} are close to zero in almost all cases. Therefore, we can state that the use of Eq. (1) in the case of random orientation is rather correct. Different situation occurs if some preferred orientation of cloud particles exists.

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It was found experimentally[1,4] that the preferred orientation of particles in some or other horizontal direction was observed roughly in 30% of cases of sensing crystal clouds. This fact manifests itself in significantly nonzero values of all or some off-diagonal elements of the BSPM. In Ref. 3, it was shown that experimental matrices could, as a rule, be reduced to a block-diagonal form. This is achieved by applying the transformation (4) to the measured matrix $\mathbf{M}$ at the turn angle Φ_0, which is determined from the same matrix. The experimental matrix $\mathbf{M}$ is thus reduced to the form $\mathbf{M}^0$, which it would have in the case of the preferred particle orientation parallel to the reference plane containing the axis x of the lidar polarization basis:

$$\mathbf{M}^0 = \begin{bmatrix} M_{11}^0 & M_{12}^0 & 0 & M_{14}^0 \\ M_{21}^0 & M_{22}^0 & 0 & 0 \\ 0 & 0 & M_{33}^0 & M_{34}^0 \\ M_{41}^0 & 0 & M_{43}^0 & M_{44}^0 \end{bmatrix}. \quad (5)$$

Having known $\mathbf{M}^0$ and Φ_0 we can write, using the transformation (4) and following the scheme proposed in Ref. 3, the matrix $\mathbf{M}(\Phi)$ in the coordinate system turned about the axis z or, what is the same, if the lidar polarization basis is turned about the sensing direction by an arbitrary angle Φ:

$$\mathbf{M}(\Phi) = \begin{pmatrix} A & B\cos 2\phi & B\sin 2\phi & H \\ B\cos 2\phi & E + F\cos 4\phi & F\sin 4\phi & D\sin 2\phi \\ -B\sin 2\phi & -F\sin 4\phi & -E + F\cos 4\phi & D\cos 2\phi \\ H & D\sin 2\phi & -D\cos 2\phi & C \end{pmatrix}, \quad (6)$$

where the following designations are introduced:

$$\phi = \Phi - \Phi_0;\ A = M_{11}^0;\ E = (M_{22}^0 - M_{33}^0)/2;$$
$$F = (M_{22}^0 + M_{33}^0)/2;\ \ C = M_{44}^0;\ \ B = M_{12}^0 = M_{21}^0;$$
$$D = M_{34}^0 = -M_{43}^0;\ \ H = M_{14}^0. \quad (7)$$

The measurements of lidar depolarization ratio in the case of linearly polarized sounding radiation can be represented as follows. One can assume, without loss of generality, that laser radiation propagates along the axis z, has unit intensity, and is polarized in the plane xOz of the chosen coordinate basis $\mathbf{e}_x \times \mathbf{e}_y = \mathbf{e}_z$. It can be described by the Stokes column-vector

$$\mathbf{S}_l = (1;\ 1;\ 0;\ 0)^{\mathrm{T}}, \quad (8)$$

where T denotes transposition. The receiving system of a lidar includes a device, for example, a Wollaston prism, which is used to split the scattered radiation into two components having mutually orthogonal linear polarization. After that their intensities are being measured

$$I_x = I_\parallel \ \text{ and } \ I_y = I_\perp.$$

The effect of an ideal splitter is described mathematically as the effect of two instrumental row vectors[5]:

$$\mathbf{G}_x = \frac{1}{2}(1;\ 1;\ 0;\ 0) \ \text{ and } \ \mathbf{G}_y = \frac{1}{2}(1;\ -1;\ 0;\ 0). \quad (9)$$

Omitting the factors accounting for instrumental parameters and attenuation along the sounding path in the lidar equation, we can represent the measurement process by the following equations:

$$I_\parallel(z) = \mathbf{G}_x\,\mathbf{M}(z,\phi)\,\mathbf{S}_l,\ \ I_\perp(z) = \mathbf{G}_y\,\mathbf{M}(z,\phi)\,\mathbf{S}_l,$$
$$I(z) = I_\parallel(z) + I_\perp(z),\ \ Q(z) = I_\parallel(z) - I_\perp(z). \quad (10)$$

Substituting the corresponding matrices into the right-hand sides and calculating the matrix products yield the following result:

$$I_\parallel = (A + B\cos 2\phi + B\cos 2\phi + E + F\cos 4\phi)/2,$$
$$I_\perp = (A + B\cos 2\phi - B\cos 2\phi - E - F\cos 4\phi)/2. \quad (11)$$

Here the arguments z of intensities and BSPM elements are omitted for simplicity, but they are implied. From Eqs. (10) and (11) we have

$$\frac{Q}{I} = \frac{I_\parallel - I_\perp}{I_\parallel + I_\perp} = \frac{E + B\cos 2\phi + F\cos 4\phi}{\mathrm{A} + B\cos 2\phi}. \quad (12)$$

It follows from Eq. (12) that in the presence of a preferred particle orientation, the Stokes parameters Q and I, and the lidar depolarization ratio defined by Eq. (1) depend on the angle ϕ. This angle should be considered as a random parameter, because the angle Φ_0 [see Eq. (7)] cannot be determined without measuring the complete BSPM. The problem of determining the lidar depolarization ration (1) is an ill-posed problem, dependent on the random direction of the preferred particle orientation with respect to the lidar polarization basis. It can be easily demonstrated that the true depolarization ratio also depends on ϕ. In Ref. 1 it was shown, with the experimentally measured BSPM taken as an example, that variations may be significant enough.

The situation is different, if we use circular polarization of sounding laser radiation and measure the first, I, and fourth, V, Stokes parameters of the scattered radiation, and define the lidar depolarization ration as

$$\Delta_c = 1 - |V|/I, \quad (13)$$

where the subscript c (circular) is used to distinguish it from the definitions of Δ_L and Δ.

The measurements of Δ_c are to be performed in the following way: the laser emits circularly polarized radiation, whose intensity can be thought equal to unity without any loss of generality and described by the Stokes column-vector

$$\mathbf{S}_c = (1;\ 0;\ 0;\ -1)^{\mathrm{T}}, \qquad (14)$$

where, for certainty, we took the clockwise polarization, as indicated by the minus sign of the fourth Stokes parameter; a $\lambda/4$ plate and a splitter, whose effect is similar to that described above, are placed in the optical system of the receiver and mutually arranged so that their common effect is described by the following instrumental vectors[5]:

$$\mathbf{G}_x^c = \frac{1}{2}(1;\ 0;\ 0;\ -1) \text{ and } \mathbf{G}_y^c = \frac{1}{2}(1;\ 0;\ 0;\ 1); \qquad (15)$$

similarly to Eq. (10), the measurement process is described by the equations

$$I_x^c(z) = \mathbf{G}_x^c\, \mathbf{M}(z,\phi)\, \mathbf{S}_c;\ \ I_y^c(z,\phi) = \mathbf{G}_y^c\, \mathbf{M}(z,\phi)\, \mathbf{S}_c; \qquad (16)$$
$$I = I_x^c + I_y^c;\ \ V = I_x^c - I_y^c.$$

Substituting the corresponding matrices to the right-hand sides and calculating the matrix products yield the following equations:

$$I = A - H;\ V = -(C + H);$$
$$\Delta_c = 1 - |-C - H| / (A - H). \qquad (17)$$

Both of the Stokes parameters are expressed through the BSPM rotation invariants. If we take into account the fact that the element $M_{14} = H$ is close to zero, as was described above, then we can see from Eqs. (16) and (17) that

$$(I_x^c - I_y^c)/(I_x^c + I_y^c) = -C/A = -M_{44}/M_{11}. \qquad (18)$$

This means that the ratio between the difference and the sum of intensities measured by the method described above is determined by the BSPM element m_{44} normalized to M_{11}. The significance of this fact will become clear in the following consideration. Here we only note that just the normalized BSPM was determined in the experiments. According to Eq. (13), the lidar depolarization ratio is described by the equation

$$\Delta_c = |I_x^c - I_y^c| / (I_x^c + I_y^c) = 1 - |m_{44}|. \qquad (19)$$

It can easily be demonstrated that the degree of polarization of the radiation scattered from a medium with the BSPM (6) irradiated by a circularly polarized radiation is described by the equation

$$P = \sqrt{B^2 + D^2 + V^2 + 2CH + H^2} / (A - H) \qquad (20)$$

with obvious simplification at $H = 0$.

It is seen from the definitions (1′) and (13) and Eq. (20) that the lidar depolarization ratio Δ_c is also equal to the true depolarization Δ only if the BSPM parameters B, D, and H are equal to zero, but in any case neither Δ_c nor Δ depend on ϕ, as, on the contrary, Δ_L does.

Manifestation of the ambiguity in the depolarization as a characteristic of scattering by an ensemble of crystal particles is not necessarily connected with the presence of a preferred orientation in some horizontal direction. It should always be expected in sensing along a slant path. The cause is that rather large particles, when falling, are oriented so that their larger diameters are directed horizontally. If there are no other orienting factors, the vertical is the symmetry axis of infinite order for such an ensemble of particles, and the BSPM is diagonal at sensing into zenith or nadir. If the sensing path deviates from these directions, the BSPM takes the block-diagonal form with respect to the reference plane containing the vertical and the sensing direction.[3] The process of transformation of the BSPM from the diagonal to the block-diagonal form depending on the zenith angle of the path is illustrated in Fig. 1, which shows the results we have calculated on the BSPM for layers of horizontally arranged hexagonal ice plates and columns. The elements of the normalized BSPM $m_{12} = M_{12}/M_{11}$ and $m_{34} = M_{34}/M_{11}$ becomes nonzero as the zenith angle Θ increases.

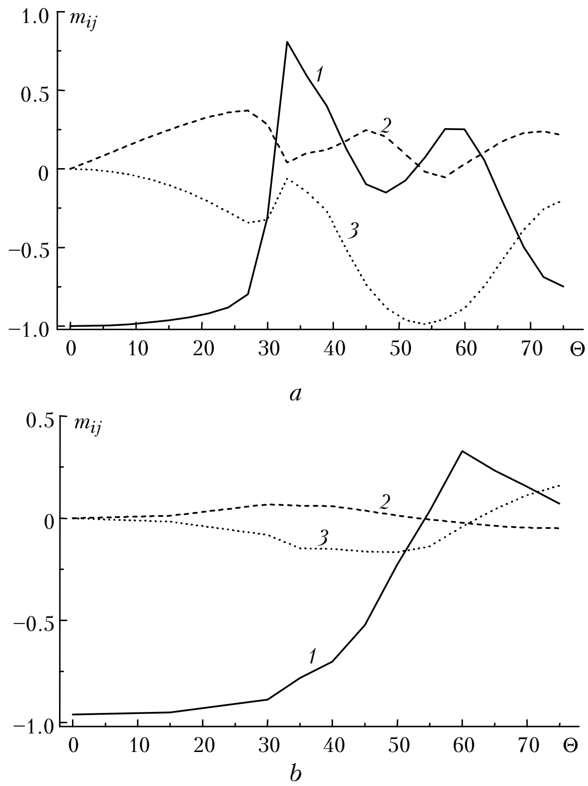

Fig. 1. Dependence of the normalized BSPM elements $m_{ij} = M_{ij}/M_{11}$ on the angle of deviation of the sensing radiation wave vector from the vertical Θ for an ensemble of hexagonal plates oriented so that the edges of the largest side are directed horizontally (*a*) and the ensemble of hexagonal columns whose main axes are oriented horizontally and distributed uniformly over the azimuth direction (*b*): M_{44}/M_{11} (curve *1*), M_{34}/M_{11} (*2*), M_{12}/M_{11} (*3*).

It is worth considering the results shown in Fig. 1 from the viewpoint of using lidar polarization measurements along slant paths to evaluate the shape of particles. It is noteworthy that a behavior of the normalized BSPM elements m_{44} and m_{12} for plates and columns is strongly different (see Fig. 1). For the ensemble of plates, the element m_{44} rapidly increases near the zenith angle of 30° with the following four alternations of the sign of m_{44} as the zenith angle increases. The ensemble of columns is characterized by the slower and more monotonic, up to $\Theta = 60°$, increase of m_{44} with alternation of the sign at $\Theta = 54°$. These peculiarities may prove useful from the viewpoint of evaluation of the particle shape, because the above scheme of sensing with the use of circular polarization just assumes determination of only the element m_{44} (Eq. (18)).

The element m_{12} experiences strong variations in the case of plates and much more weak variations in the case of columns. However, in the experiment with the linear polarization this element is not measured directly, but it enters, in combination with other parameters, into the definition of the normalized Stokes parameter $q = Q/I$ according to Eq. (12). It should be noted here that, as the lidar optical axis deviates from the symmetry axis of the ensemble of particles, i.e., from the vertical, the plane containing both these directions is the plane of mirror symmetry of the ensemble, and the reference plane, as well as the axis x of the lidar polarization basis, can be made coincident with it. Then the BSPM proves to be reduced, i.e., $\phi = 0$, and Eq. (12), with the allowance made for the definitions (7), can be written in the following form:

$$Q/I = (m_{12} + m_{22})/(1 + m_{12}). \quad (21)$$

It is implied here that the Stokes vector of laser radiation is defined by the expression (8), i.e., the radiation is polarized in the reference plane xOz. In this case, as in the case with the laser radiation polarized normally to the reference plane, the definition (1) proves to be equivalent to the definition (1′).

Figure 2 shows the depolarization (1′) as a function of the angle Θ. It is noteworthy that the value of the depolarization light scattered from a layer of plates irradiated by radiation polarized linearly in the plane containing the vertical and the sensing direction is high at $\Theta = 45°$. On the contrary, if the layer is illuminated with the light polarized circularly or linearly and normally to the plane mentioned above, the scattered radiation keeps a rather high degree of polarization, but depolarization peaks are observed at $\Theta = 38°$ and, besides, for all the three types of polarization there exists a peak at $\Theta = 30°$. The ensemble of columns gives an even growth of depolarization within the range of Θ angles from 0 to 55° for all types of polarization of the incident radiation.

The material presented in this paper allows us to conclude the following. When sensing along slant paths, to evaluate the shape of particles in an ensemble with the axial symmetry about the vertical direction, a rapid increase of depolarization, as the zenith angle exceeds 25°, is a characteristic that distinguishes ensemble of plates from ensembles of columns. In this case, the depolarization ratio has large values at the zenith angles from 30 to 50° and then decreases.

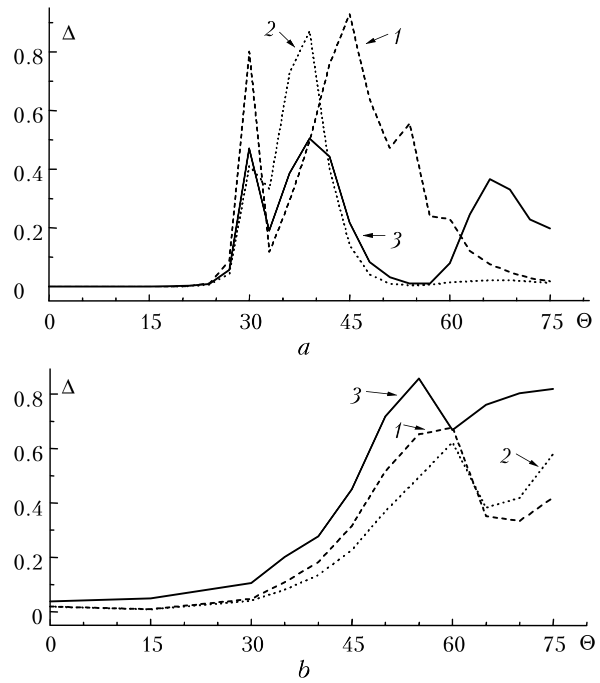

Fig. 2. Dependence of depolarization of the backscattered radiation Δ on the zenith angle of a sounding path Θ for ensembles of hexagonal plates (*a*) and columns (*b*) oriented as in Fig. 1 when irradiated by radiation polarized linearly in the tilt plane (*1*), linearly while normally to the tilt plane (*2*), and circularly polarized (*3*).

Ensembles of columns are characterized by the even increase of the depolarization with the peak at the zenith angle of 55–60°. The highest contrast occurs at $\Theta = 30°$, where the depolarization generated by plates is 5 to 10 times higher than that generated by columns. The maximum effect is achieved under exposure of the layer to the radiation polarized linearly in the tilt plane. The above applies to the true depolarization Δ, which in this case is equal to the lidar depolarization ratio Δ_L [see the definition (1)]. However, the lidar depolarization ratio Δ_c defined by Eq. (13) has higher values and achieves unity at $\Theta = 30°$, since the element m_{44}, along with the fourth Stokes parameter, changes its sign (see Fig. 1*a*). This alternation of sign or, at least, nearly zero values of the fourth Stokes parameter at the zenith angles about 30° seems to be most significant distinction between the ensembles of plates, and measurements by the scheme described by Eqs. (14)–(18) seem to be more promising than measurements of lidar depolarization from a layer irradiated with a linearly polarized radiation.

A slight change of the experiment with the circularly polarized laser radiation allows us to find another one distinguishing characteristic – large negative values of the element $m_{12} = m_{21}$ of the normalized BSPM of an ensemble of plates at the zenith angles of 50–60°. Determination of m_{21} is reduced to measurement of the normalized Stokes parameter q_c when the medium is irradiated by a circularly polarized radiation. This can be easily shown by multiplying matrices (5) and (14). It was already mentioned above that the BSPM of the medium with the axial symmetry must have the form (5) at sensing along the direction not coinciding with the symmetry axis. As to the changes in the experiment, they are reduced to turning the $\lambda/4$ plate from the position when its fast axis makes the angle of 45° with the axis x of the lidar polarization basis[5] to the position when this angle is zero.

It can be concluded from the above-said that circular polarization of laser radiation is preferable over a linear one for sensing crystal clouds. Here it is assumed that one state of polarization of laser radiation is used, rather than a set of such states, as, for example, for measurements of the complete BSPM, i.e., we consider the minimum version of polarization measurements. An advantage of using circularly polarized sounding radiation consists, first, in the fact that in this case the depolarization ratio and backscattering coefficient are independent of the azimuth orientation of the lidar, what may be the case with the use of linear polarization of sounding radiation. Second, in sensing along a slant path of a layer axially symmetric about the vertical direction, use of circularly polarized radiation allows direct measurement of the elements m_{21} and m_{44} of the normalized BSPM. As was shown above, these elements have a significantly different dependence on the zenith angle for hexagonal plates and columns. This gives us some criteria for distinguishing between the particle shapes, and we guess that these criteria are more efficient than those, which could be obtained with the use of linearly polarized sounding radiation.

It can be expected that in actual clouds consisting of particles having quite various shapes the above criteria will manifest themselves as tendencies, whose degree of manifestation is still to be studied both experimentally and through mathematical simulation of the BSPM of more complicated ensembles of particles.

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