

# Deposition of aerosol particles on surface: the model of gas-kinetic interaction and results for thermophoretic deposition

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The results of theoretical analysis of deposition of aerosol particles on a surface are presented. A gas-kinetic model of elementary particle–surface interaction processes allowing for different kinds of gas inhomogeneities is proposed. Classification schemes for deposition modes based on introduction of three Knudsen numbers are discussed. The problem of thermophoretic deposition normal to the surface is formulated, and the results for free-molecular and viscous slip-flow deposition modes are presented. A method is proposed to take into account the Brownian diffusion of submicron aerosol particles. The connection of the gas-kinetic model developed with the dynamic particle–surface interaction models is considered.

## Introduction

The processes of aerosol particle deposition on and detachment from surfaces play an important role in the evolution of air-disperse systems. The wide variety of deposition modes and conditions gives rise to numerous theoretical models and approaches to solution of the problem.<sup>1</sup>

Interaction of particles with a surface may be physical-chemical or electrostatic (including the effect of Van der Waals and Coulomb forces), as well as *gas-kinetic*, similar to the hydrodynamic interaction.<sup>2</sup> Unfortunately, characteristics of the gas-kinetic interaction of particles with the surface are not that thoroughly understood and studied as the characteristics of hydrodynamic interaction. A gas cannot be usually treated as continuum because of its rarefaction and molecular effects. Moreover, deposition processes usually proceed at widely varying gas pressure and temperature, and this leads to the particle–surface interaction different from that under continuum conditions.

Actual atmospheric or technological processes involve a wide variety of gas-kinetic deposition modes: from the continuum mode (coarse aerosol) to the mode of ultra rarefied gas (fine aerosol); various inhomogeneities of the gas medium caused by pressure, temperature and concentration gradients manifest themselves; particles may be under the exposure to external fields: gravitational, electrostatic, or electromagnetic. The corresponding phoretic processes in aerosols (thermo-, diffusion-, electro-, turbo-, and photophoresis) are studied largely within the model of aerosol particles in an unbounded gas volume. The presence of deposition surfaces gives rise to a wide variety of gas-kinetic effects and regularities that are studied rather fragmentarily and non-systematically yet.

In this study we have undertaken an attempt to apply logically the gas-kinetic model of elementary processes of the particle–surface interaction with allowance for various inhomogeneities of the gas medium. The model efficiency is demonstrated by the results on thermophoretic deposition for some gas-kinetic models. The model capabilities of accounting for other physical factors (for example, Brownian diffusion of particles) are briefly analyzed. It is shown that the model of gas-kinetic interaction applied is necessarily connected with the following dynamic models describing the adhesion interaction, collision, and detachment of particles from the surface.

## Statement of the problem and classification of deposition modes

Consider a spherical particle of radius  $R$  suspended in gas at a distance  $a$  from one of two parallel planes spaced by  $L$  (Fig. 1). The lower plane is assumed to be the deposition surface. The presence of the upper surface is convenient for classification of deposition modes and, in addition, it gives rise to a particular class of problems (deposition of particles from a bounded gas volume). The surfaces may have equal or different temperatures, and this is important for the problem of thermophoretic deposition. The particle may be affected by different forces caused by both inhomogeneities of the gas medium and inhomogeneity of the particle surface temperature (resistance force; thermo-, diffusion-, and photophoretic forces). The case of thermophoretic deposition of particles most known in the literature is considered below by default. The instantaneous directions of vectors of forces and particle motion are generally arbitrary with respect to the deposition surface.

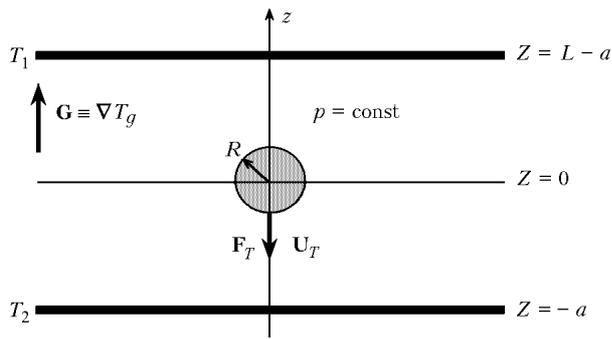


Fig. 1. Statement of the problem on thermophoretic deposition of a particle on a surface.

Let  $l$  be the mean free path of gas molecules at a fixed gas pressure  $p$  in the gap between the plates. Then we can introduce three dimensionless similarity parameters characterizing the degree of gas rarefaction:  $Kn_R = l/R$ ,  $Kn_L = l/L$ , and  $Kn_a = l/a$ . These three Knudsen numbers are not independent, because of the obvious condition that  $R < a < L$  and the practical restriction that  $R \ll L$  (particle size is usually much smaller than the characteristic size of the deposition area; the case  $R \approx L$  is principally possible, but it calls for a separate consideration). Therefore, usually  $Kn_R > Kn_a > Kn_L$ , and  $Kn_R \gg Kn_L$ . Thus, the following classification scheme of deposition modes (Fig. 2) can be considered.

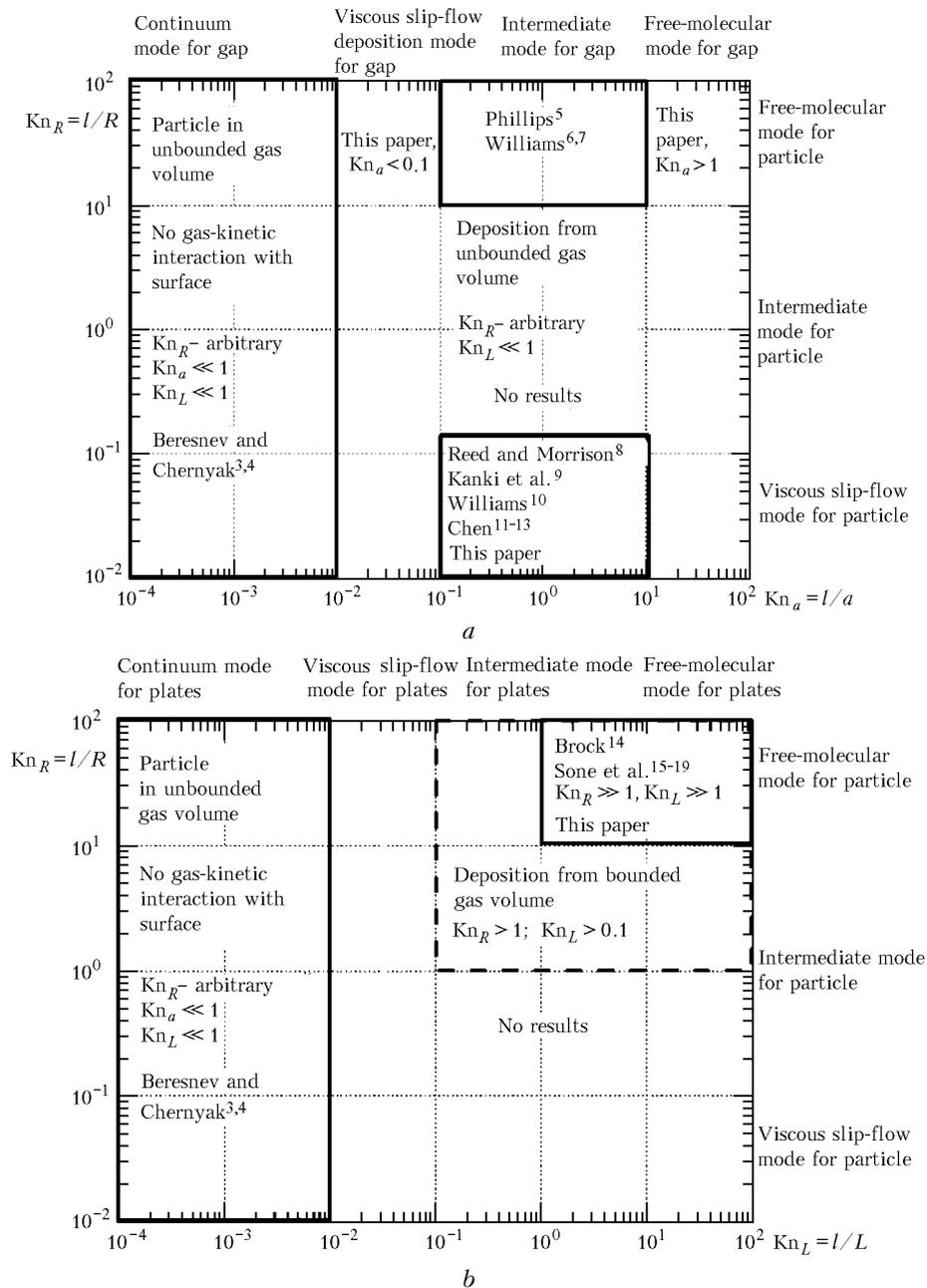


Fig. 2. Classification of thermophoretic deposition modes from unbounded (a) and bounded (b) gas volumes.

1)  $\text{Kn}_L \rightarrow 0$  (*deposition from unbounded gas volume*, Fig. 2a). The upper plate has almost no effect on the deposition process. If  $\text{Kn}_a \ll 1$  in this case, then we have the mode of particle motion in an unbounded gas volume, and  $\text{Kn}_R$  can have any arbitrary value. The gas-kinetic particle–surface interaction is absent. For this situation numerous theoretical results obtained by use of different methods (see, for example, Refs. 3 and 4) are known. The case of  $\text{Kn}_a \geq 0.1$  (the particle is separated from the lower plane by the distance of about the mean free path of gas molecules) already implies the interaction between the particle and the surface; in this case  $\text{Kn}_R$  can be arbitrary as well. There are only few theoretical results known for this case (for example, Refs. 5–7 for  $\text{Kn}_R \gg 1$ , Refs. 8–13 for  $\text{Kn}_R \ll 1$ ; the intermediate mode is unstudied yet). Finally, the case  $\text{Kn}_a \gg 1$  (the particle is located nearby the surface) is possible, for which both calculation of free-molecular gas-kinetic particle–surface interaction and accounting for adhesion forces between the particle and the surface are needed.

2)  $\text{Kn}_L \geq 0.1$  (*deposition from a bounded volume*, Fig. 2b). Here  $\text{Kn}_R \geq 1$  in the general case. A free-molecular deposition mode ( $\text{Kn}_R \gg 1$ ) usually occurs for the particle at any separation  $a$ . Some theoretical approaches and results are also known for this case, in particular, the results from Ref. 14 and the data of systematic analysis of free-molecular flows of a gas with particles.<sup>15–19</sup> It should be noted that in the literature there are no results on estimating the competition of gas-kinetic and adhesion interaction forces, which is important in principle for this deposition mode.

## Gas-kinetic particle–surface interaction

We know only one theoretical approach analyzing the gas-kinetic interaction between a particle and a plane surface.<sup>20</sup> According to it, the problem consists in determination how the linear and angular velocities of a spherical particle are related to the forces and angular momenta acting upon it near the surface. Let  $U$  and  $\Omega$  be the translational and angular velocities of the particle at some distance  $a$  from the plane, and  $F$  and  $T$  be the force and angular momentum acting on the particle from the gas medium. They are connected by the following equation:

$$\begin{pmatrix} F \\ T \end{pmatrix} = \eta_g \begin{pmatrix} A & \tilde{B} \\ B & C \end{pmatrix} \begin{pmatrix} U \\ \Omega \end{pmatrix}, \quad (1)$$

where  $\eta_g$  is the coefficient of gas viscosity. The square matrix in Eq. (1) is referred to as the resistance matrix.<sup>2,20</sup> The inverse matrix

$$\begin{pmatrix} U \\ \Omega \end{pmatrix} = \frac{1}{\eta_g} \begin{pmatrix} a & \tilde{b} \\ b & c \end{pmatrix} \begin{pmatrix} F \\ T \end{pmatrix} \quad (2)$$

is referred to as the mobility matrix. Using the methods of thermodynamics of irreversible processes and Onsager reciprocity relations, one can show that the coefficients of these matrices are symmetric.<sup>21</sup> Since no particular solutions of the gas flow equation are

invoked in this case, Eqs. (1)–(2) are valid at any gas-kinetic flow modes and conditions of gas – particle surface interaction.<sup>20</sup> If the coefficients of both matrices are represented in the form

$$\begin{aligned} A &= X^A \widehat{\mathbf{k}}\widehat{\mathbf{k}} + Y^A (\widehat{\mathbf{i}}\widehat{\mathbf{i}} + \widehat{\mathbf{j}}\widehat{\mathbf{j}}), & a &= x^a \widehat{\mathbf{k}}\widehat{\mathbf{k}} + y^a (\widehat{\mathbf{i}}\widehat{\mathbf{i}} + \widehat{\mathbf{j}}\widehat{\mathbf{j}}), \\ B &= Y^B (\widehat{\mathbf{i}}\widehat{\mathbf{j}} - \widehat{\mathbf{j}}\widehat{\mathbf{i}}), & b &= y^b (\widehat{\mathbf{i}}\widehat{\mathbf{j}} - \widehat{\mathbf{j}}\widehat{\mathbf{i}}), \\ C &= X^C \widehat{\mathbf{k}}\widehat{\mathbf{k}} + Y^C (\widehat{\mathbf{i}}\widehat{\mathbf{i}} + \widehat{\mathbf{j}}\widehat{\mathbf{j}}), & c &= x^c \widehat{\mathbf{k}}\widehat{\mathbf{k}} + y^c (\widehat{\mathbf{i}}\widehat{\mathbf{i}} + \widehat{\mathbf{j}}\widehat{\mathbf{j}}), \end{aligned} \quad (3)$$

where  $\widehat{\mathbf{i}}, \widehat{\mathbf{j}}, \widehat{\mathbf{k}}$  are unit vectors along the positive directions of the axes of the Cartesian coordinate system with the origin at the center of the particle,<sup>2</sup> then we obtain ten scalar functions,  $X^A, Y^A, Y^B, X^C, Y^C$  and  $x^a, y^a, y^b, x^c, y^c$ , which should be calculated by the gas-kinetic method for complete determination of the resistance and mobility matrices.<sup>20</sup>

Thus, if we know the vectors of the angular and translational velocity of a particle at some point near the deposition surface, then the force and the angular momentum can be determined by Eq. (1), provided that the coefficients of the resistance matrix are known. On the contrary, if we know the force and the angular momentum, then the angular and translational velocities can be determined by Eq. (2) at the given coefficients of the mobility matrix. This formalism likely exhausts all possible versions of the dynamic behavior of the particle near the deposition surface with the allowance for their gas-kinetic interaction.

The statement of the necessity of calculating two determining parameters: either  $F$  and  $T$  or  $U$  and  $\Omega$ , is of principal importance. Results available from literature not always meet this requirement, and the problem of determination of the angular momentum  $T$  or the angular velocity  $\Omega$  is traditionally ignored. The classical example of the need in taking these characteristics into account is the Saffman problem,<sup>22</sup> namely, appearance of the lifting force acting on the particle in the shearing gas flow near the surface. Besides, within this approach, it is possible to consider particle deposition at an arbitrary angle to the surface (such results are usually obtained as a superposition of the normal and tangential particle motion, see, e.g., Ref. 13).

The coefficients of the resistance and mobility matrices can be determined only through solution of particular elementary problems with the allowance made for features noticed at classification of the deposition modes (the formalism itself<sup>20</sup> does not include these peculiarities). Such elementary problems are, in our opinion, the problems of gravitational deposition of particles, deposition of particles in temperature, concentration, and electromagnetic radiation fields (thermo-, diffusion-, and photophoretic deposition). This approach may probably be useful in solving the problem on turbulent deposition of aerosols as well.<sup>1</sup> Some results on the coefficients of the matrices are given in Ref. 20 (the results of calculation of the resistance force of the particle at its motion normal to the deposition surface are mostly analyzed). Rather a

detailed review of the results and methods for description of *hydrodynamic* interaction between the particle and the deposition surface can be found there as well. In this paper we present some results of solution of the elementary problem about thermophoretic deposition of the particle normally to the deposition surface in the free-molecular and viscous slip-flow deposition modes.

## Thermophoretic particle deposition

*Statement of the problem.* Consider a problem that a spherical particle with the thermal conductivity  $\lambda_p$  deposits normally to the surface in a plane-parallel gap with the width  $L$  and the wall temperature  $T_1 > T_2$  at a constant gas pressure  $p$  (see Fig. 1). This formulation of the problem corresponds to the deposition modes from the bounded and unbounded gas volumes according to the above classification. Unless otherwise specified, the angular momentum  $T$  does not arise at the normal deposition, and the angular velocity of the particle  $\Omega$  is equal to zero.

Since the velocity of the thermophoretic particle motion  $U_T$  is assumed low as compared with the characteristic velocities of thermal motion  $\bar{v}_g$ , the quasi-stationary formulation of the problem is valid.<sup>23</sup> It is obvious that the presence of surfaces makes the particle motion non-stationary, because the particle moves along one of the directions: the distance  $a$  between the particle and the plane is the function of time  $a = a(t)$ , therefore the thermophoretic force is  $F_T = F_T(a)$ , the resistance force is  $F_D = F_D(a)$ , other possible forces and the particle velocity  $U_T = U_T(a)$  are also the functions of time  $t$ . However, if the particle velocity  $U_T = |da/dt| \ll \bar{v}_g$ , then for the period of relaxation of the temperature field around the particle  $t_T \approx l/\bar{v}_g$  it moves at a distance  $\sim |da/dt| \bar{v}_g^{-1} \ll l$ , that is, it can be thought almost fixed. Therefore, all the considered macro parameters establishing for the time  $t_T$  are believed dependent only on the gap  $a$ , and the value of  $a$  is assumed given at every time  $t$ .

In any deposition mode, we first solve the problem of determination the temperature fields in the gas  $T_g$  and on the particle surface  $T_s$  and then find the thermophoretic force  $F_T(a)$  and the resistance force  $F_D(a)$  that are functions of the separation between the particle and the deposition surface  $a$ . After that we calculate the instantaneous velocity of thermophoretic motion of the particle  $U_T(a)$ .

## Free-molecular deposition mode

### 1. Analysis of the case $\text{Kn}_R \gg 1$ , $\text{Kn}_a \ll 1$

If the particle is small, that is,  $\text{Kn}_R = l/R \gg 1$ , then it does not disturb the gas state, and for its

description we can use volume Chapman–Enskog velocity distribution functions for gas molecules.<sup>24</sup> Consider the situation that a particle is rather far from both plates ( $\text{Kn}_a \leq 0.1$ ). In this case, the thermophoretic force should differ only slightly from the corresponding value for the unbounded gas volume. The temperature profile between the plates can be determined through solution of the problem of heat transfer between two parallel planes<sup>25</sup>:

$$\begin{aligned} [T_1 - T(z)] / (T_1 - T_2) &= \delta / 2 + \\ &+ (1 - \delta) (L - a - z) / L, \\ \delta &= [1 + 4 / (15 \text{Kn}_L)]^{-1}. \end{aligned} \quad (4)$$

Then

$$T(z) = T_0 (1 + \tau),$$

where

$$\tau(z) = (1/T_0) (1 - \delta) Gz,$$

and the temperature

$$\begin{aligned} T_0 &= T(z = 0) = T_1 - \\ &- (T_1 - T_2) [1 - \delta / 2 - (1 - \delta) a / L]. \end{aligned}$$

For further solution, we should specify the distribution functions of gas molecules falling on and reflected from the particle surface. The first of them can be constructed using the first approximation of the Chapman–Enskog theory,<sup>24</sup> and it keeps unchanged until collision with the particle surface. For the distribution function of reflected molecules, we can use the specular-diffusion scheme of the boundary conditions.<sup>25</sup> Estimating the particle temperature as  $T_s(\theta_0) = T_0 [1 + \tau_s(\theta_0)]$  ( $\theta_0$  is the polar angle taken on the particle surface), in the process of solution of the thermal problem we obtain

$$\begin{aligned} \tau_s(\theta_0) &= [15Gl / (8T_0) - \pi^{1/2} u_\infty / 4] \times \\ &\times \varepsilon \cos \theta_0 \{ \lambda_p T_0 / [Rp(2kT_0 / (\pi m))^{1/2}] + \varepsilon \}^{-1}, \end{aligned} \quad (5)$$

where  $u_\infty = U_\infty [m / (2kT_0)]^{1/2}$  is the dimensionless velocity of the gas center of gravity with respect to a fixed particle;  $m$  is the molecular mass;  $k$  is the Boltzmann constant;  $\varepsilon$  is the coefficient of diffuse reflection of molecules from the particle surface.

The total force acting on the particle from the gas can be calculated through surface integration of the total momentum flux brought by falling and reflected molecules:

$$\begin{aligned} F &= F_D + F_T = \int_S (P^- + P^+) dS = \\ &= 2\pi R^2 \int_0^\pi (P^- + P^+) \sin \theta_0 d\theta_0, \\ P^\mp &= m \int v_r v_z f^\mp(r=R) dv. \end{aligned} \quad (6)$$

The resistance force has the form

$$F_D(a) = - (16/3) \pi^{1/2} pR^2 [m/(2kT_1)]^{1/2} \times \{1 - (1 - T_2/T_1) [1 - \delta/2 - (1 - \delta) a/L]\}^{-1/2} \times [1 + (\pi/8) \varepsilon] U_\infty. \quad (7)$$

In the case of small temperature differences ( $T_1 - T_2$ )/ $T_1 \ll 1$  we have

$$F_D(a) = F_D^0 [1 + (1/2) \Gamma(a)], \quad \Gamma(a) = (1 - T_2/T_1) [1 - \delta/2 - (1 - \delta) a/L], \quad (8)$$

where  $F_D^0$  is the free-molecular resistance force in the unbounded gas volume.<sup>4</sup>

The thermophoretic force has the form

$$F_T(a) = - 4pR^2 Gl/T_1 \{1 - (1 - T_2/T_1) [1 - \delta/2 - (1 - \delta) a/L]\}^{-1}. \quad (9)$$

For small temperature differences we have

$$F_T(a) = F_T^0 [1 + \Gamma(a)], \quad (10)$$

where  $F_T^0$  is the thermophoretic force in the unbounded gas volume.<sup>3</sup> The rate of the thermophoretic particle deposition can be presented as

$$U_T = - (3\pi^{-1/2}/4) [2kT_1/m]^{1/2} (Gl/T_1) \times [1 + (\pi/8) \varepsilon] \{1 - (1 - T_2/T_1) [1 - \delta/2 - (1 - \delta) a/L]\}^{-1/2}. \quad (11)$$

For small temperature differences this equation can be reduced to the following form:

$$U_T = - [5 + (\pi/8) \varepsilon]^{-1} (\lambda_g G/p) [1 + \Gamma(a)], \quad (12)$$

where  $\lambda_g$  is the gas thermal conductivity.

In the limit  $Kn_L \rightarrow 0$  the equations obtained transform into the well known equations for the model of a particle in an unbounded gas volume.<sup>3,4</sup>

### 2. Analysis of the case with $Kn_R \gg 1, Kn_L \rightarrow 0$

Consider the situation that at  $Kn_R \gg 1$  the particle is in the immediate proximity to the lower surface with the temperature  $T_2$ , so that the gas molecules move from the plate to the particle without collisions ( $Kn_a \geq 1$  or even  $Kn_a \gg 1$ ). The upper plate is very far from the particle or absent, therefore  $Kn_L \rightarrow 0$ . The distribution function of the gas molecules fallen on the particle and the surface from above is again taken in the form of the first approximation of the Chapman–Enskog theory.<sup>24</sup> The distribution function of molecules falling on the particle from the plate is chosen in the form of the Maxwell function with the allowance for the temperature difference.<sup>25</sup> After solving the thermal problem, we can determine the thermophoretic force

$$F_T = (\pi/4) R^2 p (Ga/T_1) (1 - \delta) [1 - \Gamma(a)]^{-1}. \quad (13)$$

The rate of thermophoretic deposition  $U_T$  can be determined similarly to the previous case.

### 3. Analysis of the case with $Kn_R \gg 1, Kn_L \gg 1$

Let us try to generalize the result of Ref. 14 for the case of a collisionless ultra-rarefied gas between the plates (see Fig. 2b). Since the particle is small and it does not disturb the gas state, we can again make use of the solution of the problem on heat transfer between the plates<sup>25</sup> to find the temperature distribution and the number density. At completely diffuse reflection of gas molecules from the plates, these are as follows:

$$n_0 = (n_1 + n_2)/2, \quad T_0 = (T_1 T_2)^{1/2}, \quad n_{1,2} = 2n_0 T_{2,1}^{1/2}/(T_1^{1/2} + T_2^{1/2}).$$

The distribution functions  $f_1$  and  $f_2$  of molecules reflected from the surfaces are taken to be the Maxwell ones. Since there are no molecular collisions between the plates, the total distribution function of gas molecules falling on the particle surface is

$$f^- = f_1 H_1(v_z) + f_2 H_2(v_z), \quad H_1(v_z) = \begin{cases} 0, & v_z \geq 0 \\ 1, & v_z < 0 \end{cases}, \quad H_2(v_z) = \begin{cases} 1, & v_z \geq 0 \\ 0, & v_z < 0 \end{cases}. \quad (14)$$

Calculation of the thermophoretic force according to Eq. (6) gives

$$F_T = - \pi R^2 p (T_1^{1/2} - T_2^{1/2})/(T_1^{1/2} + T_2^{1/2}). \quad (15)$$

At small temperature differences, the equation obtained can be linearized as follows<sup>14</sup>:

$$F_T = - (\pi/4) R^2 p (1 - T_2/T_1). \quad (16)$$

Equations (15) and (16) can easily be generalized for the case of arbitrary accommodation of the energy of gas molecules on the plate surfaces. It can be shown that the value of energy accommodation on the particle surface does not affect the thermophoretic force in the absence of molecular collisions. Let  $\alpha_{E1}$  and  $\alpha_{E2}$  be the coefficients of energy accommodation on the plates 1 and 2, respectively. Then, according to Ref. 25, gas molecules are reflected from the plates with the temperatures  $T_{r1}$  and  $T_{r2}$ , rather than  $T_1$  and  $T_2$ , with allowance for possible arbitrary energy accommodation. In this case for the thermophoretic force we have

$$F_T = -\pi R^2 p \left\{ \sqrt{\alpha_{E2}(1-\alpha_{E1})T_2 + \alpha_{E1}T_1} - \sqrt{\alpha_{E1}(1-\alpha_{E2})T_1 + \alpha_{E2}T_2} \right\} / \left\{ \sqrt{\alpha_{E2}(1-\alpha_{E1})T_2 + \alpha_{E1}T_1} + \sqrt{\alpha_{E1}(1-\alpha_{E2})T_1 + \alpha_{E2}T_2} \right\}. \quad (17)$$

At small temperature difference between the plates, we obtain the generalized result from Ref. 14:

$$F_T = - (\pi/4) [\alpha_{E1} \alpha_{E2} / (\alpha_{E1} + \alpha_{E2} - \alpha_{E1} \alpha_{E2})] \times R^2 p (1 - T_2/T_1). \quad (18)$$

## Viscous slip-flow deposition mode

### Analysis of the case with $\text{Kn}_R \ll 1$ , $\text{Kn}_L \rightarrow 0$ .

Consider the situation that a spherical particle of radius  $R$  is at the distance  $a$  from the plane surface with the constant temperature  $T_0$ . The temperature gradient  $G$  along the  $OZ$  axis far from the particle and the deposition surface is constant. To solve problems of such a kind, it is convenient to use bipolar coordinates  $(\xi, \eta, \theta)$  described in detail in Ref. 10. The thermal problem on determining the temperature fields of the gas  $T_g$  and the particle  $T_p$  is reduced to solution of Laplace equations in this coordinate system with the boundary conditions of equal temperatures of the gas, particle, and deposition surface (neglecting temperature differences) and continuous heat fluxes. To find the gas flow fields, let us use the method of flow functions.<sup>2</sup> The boundary conditions on the particle surface accounts for possible thermal gas slip flow. From analysis of the external (gas) and internal (particle) thermal problem, it follows<sup>26</sup> that the analytical solution is possible only for rather a large gap  $a$  (the particle is not too close to the deposition surface). Using this approximation, we obtain an analytical solution of the boundary-value problem for the flow functions and then calculate the total force  $F = F_D + F_T$  acting on the particle. As a result, we have

$$F_D = -8\pi\eta_g R U \sinh\alpha \sum_{n=1}^{\infty} \frac{n(n+1)}{(2n+3)(2n-1)} \times \left\{ \frac{(2n+1)\sinh 2\alpha + 2\sinh(2n+1)\alpha}{4\sinh^2[(2n+1)\alpha/2] - (2n+1)^2 \sinh^2\alpha} - 1 \right\}, \quad (19)$$

$$F_T = -16\pi\eta_g \sigma G R \sinh^2\alpha \sum_{n=1}^{\infty} n(n+1) \sinh[(2n+1)\alpha/2] \times \frac{2\sinh\alpha \exp[-(2n+1)\alpha/2] + (1-1/\Lambda)\phi_n}{4\sinh^2[(2n+1)\alpha/2] - (2n+1)^2 \sinh^2\alpha}, \quad (20)$$

where the angle  $\alpha$  in the bipolar coordinate system is determined from the condition  $\cosh\alpha = a/R$ ,  $\sigma = \eta_g K_{ts}/(\rho_g T_0)$ ,  $K_{ts}$  is the coefficient of thermal slip flow of the gas,<sup>24</sup> all other designations are standard.<sup>10</sup> Determination of the rate of thermophoretic deposition  $U_T$  from the condition  $F = 0$  completes solution of the problem.

### Account for Brownian diffusion at thermophoretic deposition of particles

In calculations of thermophoretic forces for different deposition modes, the effect of the Brownian motion (both translational and rotational) of particles was neglected. The answer to the question about the

effect of the Brownian motion was given in Refs. 23 and 27: for not extremely small particles the characteristic time of rotation of the sphere through some marked angle is much longer than the characteristic time of temperature distribution on the particle surface. However, correct allowance for the translational Brownian motion of particles is needed, because it considerably modifies the process of deposition of submicron aerosols.<sup>27</sup> In Ref. 28 an attempt was undertaken to introduce the generalized coefficient of Brownian diffusion for problems of aerosol deposition onto a surface. It accounts for the presence of the deposition surface and allows the significance of the Brownian motion to be evaluated for different deposition modes depending on the set of determining parameters. Thus, the model of gas-kinetic interaction should include the mechanism of Brownian diffusion as well.

## Dynamic models of particle–surface interactions

The final result of the model of gas-kinetic interaction under development is instantaneous values of the vectors of forces and velocities depending on the distance to the deposition surface. This information is necessary and sufficient for realization of different versions of dynamic models accounting for adhesion interaction, collisions, and attachment and detachment of particles from the deposition surface. Among models of this type, the results of Refs. 29–32 are worth noting. Thus, it becomes clear what are the role and the place of the model of gas-kinetic interaction in complex and multi-stage deposition processes: it gives the primary information about the forces, angular momenta, and velocities of particles caused by various inhomogeneities of the gas medium and features of the deposition mode. It is undoubted that further dynamic behavior of the particle near the surfaces is determined, to a great extent, by their “gas-kinetic” pre-history.

## Conclusion

This paper (along with Ref. 28) is only the first stage in the program of extensive studies of the gas-kinetic interaction between a particle and a deposition surface. The limited paper does not allow us to present detailed analysis of the results obtained within the theory of thermophoretic deposition of particles and the complicated mathematical apparatus. In the following papers, it is planned to present the results given by the gas-kinetic model for diffusion- and photophoretic deposition of particles. The main difficulties in calculations arise in analysis of the deposition mode at  $\text{Kn}_R \approx 1$  and  $\text{Kn}_L \approx 1$  (the results of such a kind are now absent in the literature).

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### References

1. M.M.R. Williams and S.K. Loyalka, *Aerosol Science: Theory and Practice* (Pergamon Press, Oxford, 1991), 446 pp.
2. J. Happel and H. Brenner, *Low Reynolds Number Hydrodynamics* (Kluwer, Dordrecht, 1991).
3. S.A. Beresnev and V.G. Chernyak, *Phys. Fluids* **7**, No. 7, 1743–1756 (1995).
4. S.A. Beresnev, V.G. Chernyak, and G.A. Fomyagin, *J. Fluid Mech.* **219**, 405–421 (1990).
5. W.F. Phillips, *Phys. Fluids* **15**, No. 6, 999–1003 (1972).
6. M.M.R. Williams, *J. Colloid and Interface Sci.* **117**, No. 1, 193–199 (1987).
7. M.M.R. Williams, *Phys. Fluids* **31**, No. 5, 1051–1057 (1988).
8. L.D. Reed and F.A. Morrison, *J. Aerosol Sci.* **6**, 349–365 (1975).
9. T. Kanki, S. Iuchi, T. Miyazaki, and H. Ueda, *J. Colloid and Interface Sci.* **107**, No. 2, 418–428 (1985).
10. M.M.R. Williams, *J. Colloid and Interface Sci.* **122**, No. 1, 110–119 (1988).
11. S.H. Chen, *Aerosol Sci. Technol.* **30**, No. 4, 364–382 (1999).
12. S.H. Chen, *J. Colloid and Interface Sci.* **224**, No. 1, 63–75 (2000).
13. S.H. Chen, *AIChE Journal* **46**, No. 12, 2352–2368 (2000).
14. J.R. Brock, *J. Colloid and Interface Sci.* **25**, 564–567 (1967).
15. Y. Sone, *J. Mecanique Theor. Appliquee* **3**, No. 2, 315–328 (1984).
16. Y. Sone, *J. Mecanique Theor. Appliquee* **4**, No. 1, 1–14 (1985).
17. Y. Sone, *Phys. Fluids* **28**, No. 1, 419–420 (1985).
18. K. Aoki, C. Bardos, F. Golse, M.N. Kogan, and Y. Sone, *Eur. J. Mech. B/Fluids* **12**, No. 5, 565–577 (1993).
19. C. Cercignani and A. Frezzotti, *Phys. Fluids* **5A**, No. 10, 2551–2556 (1993).
20. R. Ying and M.H. Peters, *Aerosol Sci. Technol.* **14**, No. 4, 418–433 (1991).
21. L.D. Landau and E.M. Lifshitz, *Theoretical Physics*. Vol. 5. Part. 1. *Statistical Physics* (Nauka, Moscow, 1995), 608 pp.
22. P.G. Saffman, *J. Fluid Mech.* **22**, Part 2, 385–400 (1965).
23. N.A. Fuks, *Mechanics of Aerosols* (USSR Academy of Sciences Press, Moscow, 1955), 352 pp.
24. J.H. Ferziger and H.G. Kaper, *Mathematical Theory of Transport Processes in Gases* (North Holland, Amsterdam, 1972).
25. Yu.A. Koshmarov and Yu.A. Ryzhov, *Applied Dynamics of Rarefied Gas* (Mashinostroenie, Moscow, 1977), 184 pp.
26. P.M. Morse and H. Feshbach, *Methods of Theoretical Physics* (McGraw Hill, 1953).
27. P.C. Reist, *Introduction to Aerosol Science* (Macmillan, New York, 1984).
28. D.D. Ryvkin and S.A. Beresnev, *Atmos. Oceanic Opt.* **13**, Nos. 6–7, 638–639 (2000).
29. R.M. Brach and P.F. Dunn, *Aerosol Sci. Technol.* **23**, No. 1, 51–71 (1995).
30. B. Dahneke, *Aerosol Sci. Technol.* **23**, No. 1, 25–39 (1995).
31. R.P. Andres, *Aerosol Sci. Technol.* **23**, No. 1, 40–50 (1995).
32. U.K.T. Paw and D.A. Braaten, *Aerosol Sci. Technol.* **23**, No. 1, 40–50 (1995).