# Separation of diffraction and geometric-optics components in asymptotically estimating lidar returns due to multiple scattering

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The structure of lidar equation is analyzed in the small-angle approximation taking into account multiple scattering of light. To describe lidar return signal in this approximation, the scattering phase function is modeled as a sum of diffraction and geometric-optics components. A method is proposed for separation of the diffraction and geometric-optics components of the return signals. It is shown that, at large receiver's field of view, this allows the information about the diffraction component of the scattering phase function to be replaced by its Hankel transform at zero point.

## Introduction

In laser sensing of optically dense media, the contribution of multiple scattering to lidar echo depends both on optical characteristics of the medium (extinction coefficient, single scattering albedo, scattering phase function) and on the parameters of the experiment geometry. Among the latter, the receiver's field of view and the distance from a lidar to a scattering volume are at the first place. In the general case, it is rather difficult to take into account the contribution coming to the lidar echo from multiple scattering. This problem involves solution of the radiative transfer equation (RTE), and difficulties of its solution increase especially at interpretation of lidar experiments. Various aspects of this problem have already been discussed for a long time at international conferences on laser sensing, including the International Workshop on Lidar Multiple Scattering Experiments (MUSCLE).

Among various approaches to solving this problem, I would note the methods based on the use of the small-angle approximation in solving the RTE.[1–4] These methods give a relatively simple analytical description for a lidar echo signal with the allowance for multiple scattering in the case of strongly forward-peaked scattering phase function. In Refs. 3 and 5 it was shown that in this case the lidar equation becomes significantly simpler at the large receiver's field of view. The small-angle scattering phase function in this case is replaced in the lidar equation by the derivative of its Hankel transform at the zero point, and the behavior of the lidar return acquires the asymptotic character. In atmospheric optics, the scattering phase function can often be presented as a sum of the diffraction (D) and geometric-optics (GO) components. However, the information about the GO component of the lidar signal is lost in the asymptotic approximation.[3,5]

In developing the approach considered in Refs. 3 and 5, we obtain, in this paper, a modified description of the lidar signal that overcomes the above disadvantage. The method proposed is based on solving the RTE for D and GO components separately with the following application of the asymptotic equation only to the D component. This makes it possible to obtain more accurate description of the lidar return behavior at large receiver's field of view and, consequently, more accurate solution of the inverse problems in lidar sensing of optically dense media.

## 1. Lidar equation with regard for multiple scattering in the small-angle approximation

Assume that the scattering medium is characterized by a strongly forward-peaked scattering phase function and its optical characteristics depend on only one spatial coordinate $z$. Assume also that both the source and receiver of laser radiation are in the plane $z = 0$; their optical axes coincide and sensing is conducted along the $Oz$ axis. As in Refs. 3 and 5, we are starting with a simplified model of the lidar return, in which scattering at large angles, including backscattering, is taken into account only in the single-scattering approximation, while multiple scattering is considered in description of sensing pulse propagation from the source to the scattering volume and from the scattering volume back to the receiver within the nonstationary RTE in the small-angle approximation.

With these assumptions, in the case of a point unidirectional (PU) source emitting δ-pulses with the unit energy at $t = 0$, we can obtain the following equation for the power of the lidar return coming at the time $t = 2z/c$ to the receiving system of a monostatic lidar:

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$$P(z)=\frac{c}{2}z^{-2}S_r\beta_\pi(z)(z\gamma_r)\int_0^\infty J_1(\nu z\gamma_r)F(\nu)d\nu, \quad (1)$$

where

$$F(\nu)=\exp[-2\tau(z)+g(\nu)],\ \ \tau(z)=\int_0^z\varepsilon(s)ds, \quad (2)$$

$$g(\nu)=2\int_0^z\sigma(z-s)\tilde{x}(\nu s)ds; \quad (3)$$

$S_r$ and $\gamma_r$ are the receiver's area and the field of view; $J_1(.)$ is the first-kind first-order Bessel function; $\beta_\pi(z)$ is the backscattering coefficient; $F(\nu)$ is the optical transfer function (OTF) for a stationary source in a fictitious medium, whose extinction and scattering coefficients are twice as large as their actual values $\varepsilon(z)$ and $\sigma(z)$. The OTF $F(\nu)$ also depends on the small-angle scattering phase function $x(\gamma)$, which enters into the equation for the function $g(\nu)$ (3) as a Hankel transform $\tilde{x}(.)$.

The known lidar equation in the single scattering approximation follows from Eq. (1), if it is assumed that $g(\nu) = 0$ in it:

$$P_1(z)=\frac{c}{2}z^{-2}S_r\beta_\pi(z)e^{-2\tau(z)}. \quad (4)$$

Equation (1) generalizes the ordinary lidar equation (4) with the allowance for the contribution from multiple scattering in the small-angle approximation of the radiative transfer theory. The initial lidar return $P(z)$ given by Eq. (1) can be expressed through the single-scattered signal $P_1(z)$ by including an extra term $m(z)$ in the form

$$P(z)=P_1(z)[1+m(z)]. \quad (5)$$

The function $m(z)$ describing the ratio between the multiple- and single-scattering components is determined as follows:

$$m(z,\gamma_r)=z\gamma_r\int_0^\infty J_1(\nu z\gamma_r)\left(e^{g(\nu)}-1\right)d\nu. \quad (6)$$

The ratio $m(z, \gamma_r)$ increases monotonically as a function of the receiver's field of view $\gamma_r$ and tends to the limit $m_\infty = \exp(2\Lambda\tau) - 1$ at $\gamma_r \to \infty$, where $\Lambda = \sigma/\varepsilon$ is the single scattering albedo. It is seen from the latter equation that at some optical depth and large receiver's field of view $\gamma_r$ the contribution from multiply-scattered radiation to the lidar return may become dominant, tens times exceeding the single-scattering signal.

As is shown in Refs. 3 and 5, at a rather large receiver's field of view, the behavior of the function $m(z)$ given by Eq. (6) acquires the asymptotic character and can be described by the following equation:

$$\hat{m}(z) = \left(e^{2\tau(z)\Lambda} - 1\right) - e^{2\tau(z)\Lambda}\,\Delta(z), \quad (7)$$

where

$$\Delta(z)=-\frac{2\Lambda\tilde{x}'(0)}{z\gamma_r}\int_0^z s\varepsilon(z-s)ds. \quad (8)$$

It can be seen from Eqs. (7) and (8) that at large $\gamma_r$ the lidar return $P(z)$ [Eq. (5)] depends only on the Hankel transform of the small-angle scattering phase function at the zero point, rather than on this function itself.

Because of the particular role of the derivative $\tilde{x}'(0)$ in the further reasoning, let us briefly analyze the model of the scattering phase function in the problem considered. In problems of atmospheric optics related to scattering by large particles, for which $kr|m - 1| \gg 1$, where $r$, $m$ are the particle radius and refractive index, $k = 2\pi/\lambda$, and $\lambda$ is the radiation wavelength, the scattering phase function can satisfactorily be described as a superposition of the two main components:

$$x(\gamma)=a^{(D)}x^{(D)}(\gamma)+a^{(GO)}x^{(GO)}(\gamma),$$
$$a^{(D)}=\sigma^{(D)}/\sigma,\quad a^{(GO)}=\sigma^{(GO)}/\sigma. \quad (9)$$

The one is caused by light diffraction on particles, and the other one obeys the geometric optics law. In the case of spherical particles, the diffraction component $x^{(D)}(\gamma)$ is described by the known Airy equation,[6] while the geometric-optics component $x^{(GO)}(\gamma)$ in the region of small-scattering angles can be approximated by the linear combination of the exponential and Gaussian functions[7]:

$$x^{(GO)}(\gamma)=c_1e^{-\alpha\gamma}+c_2e^{-\beta\gamma^2}, \quad (10)$$

in which the parameters $c_1$, $c_2$, $\alpha$, and $\beta$ depend on the refractive index of the particulate matter. For the considered model of small-angle scattering phase function, the product

$$2\Lambda\tilde{x}'(0)=\left[\tilde{x}^{(D)}(0)\right]'=-2/(\pi k R_{eff}) \quad (11)$$

is determined only by the diffraction component of the scattering phase function being independent of the geometric-optics component. In Eq. (11) $R_{eff}$ is the effective particle radius.

One of the advantages of using the extra term $m(z)$ in the form (7) in describing the lidar return is the possibility of solving both direct and inverse problems of laser sensing without invoking complete information on the scattering phase function at its substitute by a single parameter determined by the effective particle radius. It is a disadvantage of such an approach that we have a loss of information about the geometric-optics component of the scattering phase function. In Section 3, the effect of these factors is studied numerically in model calculations. Further analysis is aimed at a more correct account for the geometric-optics component in the lidar equation while meeting all the requirements to the *a priori* information on the scattering phase function.

## 2. Separating the diffraction and geometric-optics components of the lidar equation

As has already been noted, the use of the extra term $m(z)$ in the asymptotic approximation (7) in describing the lidar return at multiple scattering is efficient with the allowance for only the diffraction component of the scattering phase function. This gives grounds for separating the component $P^{(D)}(z)$ from the lidar return $P(z)$ [Eq. (1)]. This component describes the lidar return in the case that the scattering phase function is considered in purely diffraction approximation. The equation for the component $P^{(D)}(z)$ can be presented in the form similar to Eq. (5), if the function $m(z)$ is replaced with the function $m^{(D)}(z)$, which is determined in the diffraction approximation from the general equations (3), (6), and (7) at substitution of $\sigma^{(D)}(z)$, $\tilde{x}^{(D)}(.)$, and $\Lambda^{(D)} = 1/2$ in place of $\sigma(z)$, $\tilde{x}(.)$, and $\Lambda$. In this case, the equation for an asymptotic approximation of $\hat{m}^{(D)}(z)$ takes the following form:

$$\hat{m}^{(D)}(z) = (e^{\tau(z)} - 1) - e^{\tau(z)}\,\Delta(z). \qquad (12)$$

Thus, we have obtained all equations needed to find the diffraction component $P^{(D)}(z)$ of the lidar return.

Let us turn to analysis of the residual term $\delta = P(z) - P^{(D)}(z)$. By analogy with Eq. (5), it can be written as

$$\delta P = P_1(z)\left[m(z) - m^{(D)}(z)\right], \qquad (13)$$

where the difference $\delta m(z) = m(z) - m^{(D)}(z)$ is determined as

$$\delta m(z) = z\gamma_r \int_0^{\infty} J_1(\nu z\gamma_r)\exp\left[g^{(D)}(\nu)\right]\left\{\exp\left[g^{(GO)}(\nu)\right] - 1\right\}d\nu. \qquad (14)$$

In the integrand in Eq. (14), the functions under the exponent sign have the form similar to Eq. (3):

$$g^{(p)}(\nu) = 2\int_0^{z} \sigma^{(p)}(z-s)\,\tilde{x}^{(p)}(\nu s)\,ds, \quad p = \{D, GO\}. \qquad (15)$$

At a relatively small receiver's field of view $\gamma_r$, the decisive contribution to the formation of multiple-scattering signal is due to the component $m^{(D)}(z, \gamma_r)$ achieving saturation with the increasing $\gamma_r$. Therefore, it should be expected that the leading role will then be played by the extra term $\delta m(z, \gamma_r)$ [Eq. (14)]. As can be seen from Eqs. (14) and (15), the extra term $\delta m(z, \gamma_r)$ depends on both components of the scattering phase function: the diffraction and the geometric-optics ones. However, it is clear from the physical reasoning that in the periphery of the angular reception pattern, where $\delta m(z, \gamma_r)$ is most significant, the role of the component $x^{(GO)}(\gamma)$ increases, while that of the component $x^{(D)}(\gamma)$ decreases.

Let us transform equation (14) for $\delta m(z, \gamma_r)$ to avoid the necessity of using the information on the diffraction component of the scattering phase function to the greater extent than it is presented in the asymptotic description of $\hat{m}^{(D)}(z, \gamma_r)$ [Eq. (12)]. Recall that the asymptotic dependence of $\hat{m}^{(D)}(z, \gamma_r)$ on the scattering phase function in the diffraction approximation $x^{(D)}(\gamma)$ and, consequently, on the disperse composition of the medium reduces to the dependence on the effective particle radius $R_{eff}$, and the scattering phase function in the geometric-optics approximation $x^{(GO)}(\gamma)$ is fully independent on the disperse composition of the medium.

The possibility of making this transformation becomes apparent, if we take into account that the monotonically decreasing functions $\exp[g^{(D)}(\nu)]$ and $\exp[g^{(GO)}(\nu)]$ have significantly different widths[3] because of the different widths of the diffraction peak of the scattering phase function and its geometric-optics part. It is just the width of the function $\exp[g^{(D)}(\nu)]$ that far exceeds the width of the function $(\exp[g^{(GO)}(\nu)] - 1)$. In the frequency range, where the second factor $(\exp[g^{(GO)}(\nu)] - 1)$ in Eq. (14) changes most significantly, the first factor $\exp[g^{(D)}(\nu)]$ changes insignificantly. Therefore, we can expand the factor $\exp[g^{(D)}(\nu)]$ into the Taylor series and restrict our consideration to the first approximation

$$\exp\left[g^{(D)}(\nu)\right] \approx e^{\tau}\left\{1 + \nu\left[\tilde{x}^{(D)}(0)\right]' \int_0^{z} s\,\varepsilon(z-s)\,ds\right\}. \qquad (16)$$

As a result of substitution of Eq. (16) into Eq. (14) based on Eq. (11), the function $\delta m(z, \gamma_r)$, as well as the function $\hat{m}^{(D)}(z, \gamma_r)$, are already dependent only on the derivative $[\tilde{x}^{(D)}(0)]'$ and, consequently, on the effective radius $R_{eff}$, rather than on the diffraction component of the scattering phase function $x^{(D)}(\gamma)$.

The results of model calculations presented in Section 3 show that the linear term in Eq. (16), along with the diffraction component of the scattering phase function $x^{(D)}(\gamma)$, play so insignificant role in the formation of the function $\delta m(z, \gamma_r)$ that we can ignore them without any essential loss in the accuracy and assume that $\tilde{x}^{(D)}(\nu s) \approx \tilde{x}^{(D)}(0) = 1$ in calculations of the function $\delta m(z, \gamma_r)$. This is equivalent to replacement of the diffraction component of the scattering phase function $x^{(D)}(\gamma)$ by the $\delta$-function. This approximation is used rather widely in solving the RTE (see, for example, the transport approximation[8]). At such a replacement, the extra term $\delta m(z, \gamma_r)$ becomes completely independent on the disperse composition of the medium and takes very simple form

$$\hat{\delta} m(z, \gamma_r) = e^{\tau}\, m^{(GO)}(z, \gamma_r), \qquad (17)$$

where the function $m^{(GO)}(z, \gamma_r)$ can be determined by analogy with the function $m^{(D)}(z, \gamma_r)$. As a result, the extra term $\hat{\delta} m(z, \gamma_r)$ [Eq. (17)] is the function $m^{(GO)}(z, \gamma_r)$ "extended" by $e^{\tau}$ times.

With the allowance for the above-said, the final form of the lidar equation can be presented in the form (5) with the replacement of the function $m(z, \gamma_r)$ by the asymptotic equation of the following form:

$$\hat{m}(z, \gamma_r) = \hat{m}^{(D)}(z, \gamma_r) + e^{\tau}\, m^{(GO)}(z, \gamma_r). \quad (18)$$

*Note.* Equation (17) for the extra term $\hat{\delta} m(z, \gamma_r)$ can be also obtained in other way through application of the method of component-wise expansion of RTE.[9] Consider the stationary RTE in the medium without sources

$$DI = LI \quad (19)$$

for the intensity $I$ with the differential transfer operator $D = \mathbf{n}\nabla + \varepsilon$ and the collision integral

$$LI = \sigma \int_{4\pi} I\, x(\gamma)\, d\mathbf{n}'. \quad (20)$$

According to the representation of the scattering phase function $x(\gamma)$ in the form of the sum (9), the operator $L$ can be presented as $L = L^{(D)} + L^{(GO)}$, where the components $L^{(D)}$ and $L^{(GO)}$ are determined by analogy with Eq. (20). Let then $I_1$ be standing for the RTE solution

$$DI_1 = L^{(D)} I_1 \quad (21)$$

in the medium with the scattering phase function $x^{(D)}(\gamma)$ and the scattering coefficient $\sigma^{(D)}$. Then the intensity difference $\delta I = I - I_1$ satisfies the equation

$$D(\delta I) = L(\delta I) + B, \quad (22)$$

which differs from Eq. (19) by the source function $B = L^{(GO)} I_1$ in the right-hand side.

Considering a fictitious medium with the doubled scattering $2\sigma$ and extinction $2\varepsilon$ coefficients, from solution of Eq. (21) in the small-angle approximation we can obtain the equation for OTF $F_1(\nu)$ similar to $F(\nu)$ (2):

$$F_1(\nu) = \exp\left[-2\tau(z) + g^{(D)}(\nu)\right]. \quad (23)$$

The intensity difference $\delta I$ determined from solution of Eq. (22) corresponds to the function difference

$$\delta F(\nu) = F(\nu) - F_1(\nu). \quad (24)$$

The width of the diffraction part of the scattering phase function $x^{(D)}(\gamma)$ is far less than that of the function $\delta I$ depending on the full scattering phase function. Therefore, the scattering phase function $x^{(D)}(\gamma)$ can be approximated by $\delta$-function without significant loss in accuracy when solving Eq. (22). This leads to the following approximate equation for the function difference:

$$\hat{\delta} F(\nu) = e^{-\tau(z)}\, [e^{g^{(GO)}(\nu)} - 1]. \quad (25)$$

Taking into account the general equation (6), we can easily notice that the function $\hat{\delta} F(\nu)$ [Eq. (25)] corresponds to the term $\hat{\delta} m(z, \gamma_r)$ in the form (17).

## 3. Results of model calculations

The efficiency of the proposed description of the lidar returns in the case of sensing dense coarse disperse media is illustrated by the calculated data on the characteristic $m(\gamma_r)$ presented below. As a model of the medium, we took a plane homogeneous layer, whose microstructure was specified by the generalized gamma-distribution with the effective particle radius $R_{eff} = 10$ μm. The depth of penetration into the layer spaced by 1 km was also 1 km. In the results presented, the scattering phase function was chosen according to the model (9) and (10) at the wavelength $\lambda = 0.55$ μm. Figures 1–3 illustrate the situation with the layer having optical depth $\tau = 1$, and Figs. 4 and 5 are for a denser layer with $\tau = 3$.

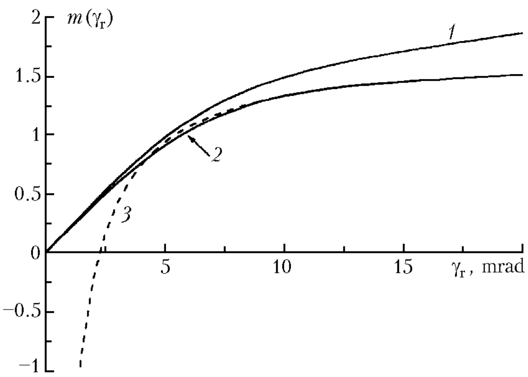


**Fig. 1.** Angular dependence of $m(\gamma_r)$ taking into account (*1*) and neglecting (*2*) the geometric-optics part of the scattering phase function at the depth of 1 km inside a homogeneous layer with the optical depth $\tau = 1$ spaced by 1 km from the lidar. Asymptotic approximation of $\hat{m}^{(D)}(\gamma_r)$ (curve *3*).

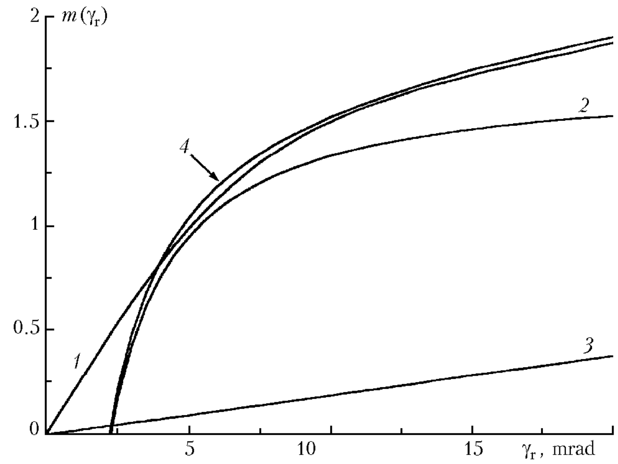


**Fig. 2.** Asymptotic approximation of $m(\gamma_r)$ with regard for the geometric-optics part of the scattering phase function: exact dependence $m(\gamma_r)$ (curve *1*), asymptotic approximation $\hat{m}^{(D)}(\gamma_r)$ (*2*), $\hat{\delta} m(\gamma_r)$ (*3*), approximation of $\hat{m}(\gamma_r)$ by Eq. (18) (*4*). Observation conditions are the same as in Fig. 1.

Curves *1* and *2* in Figs. 1 and 3 describe the behavior of $m(\gamma_r)$ for the full scattering phase function $x(\gamma_r)$

[Eq. (9)] and its diffraction part, respectively. Within $\gamma_r < 15$ mrad, the difference between $m(\gamma_r)$ and $m^{(D)}(\gamma_r)$ does not exceed 15%. As the receiver's field of view $\gamma_r$ increases these curves differ more widely, and at $\gamma_r \to \infty$ they asymptotically tend to their limits: $m_\infty = 5.55$ and $m_\infty^{(D)} = 1.72$. Curve *3* in Fig. 1 shows the behavior of the asymptotic function $\hat{m}^{(D)}(\gamma_r)$. The error in approximation of the diffraction component $m^{(D)}(\gamma_r)$ with the function $\hat{m}^{(D)}(\gamma_r)$ decreases monotonically with the increase of $\gamma_r$, and already at $\gamma_r > 3.4$ mrad it does not exceed 15%, while the error in approximation of $m(\gamma_r)$ has a minimum of 4% in the vicinity of $\gamma_r = 5.5$ mrad.

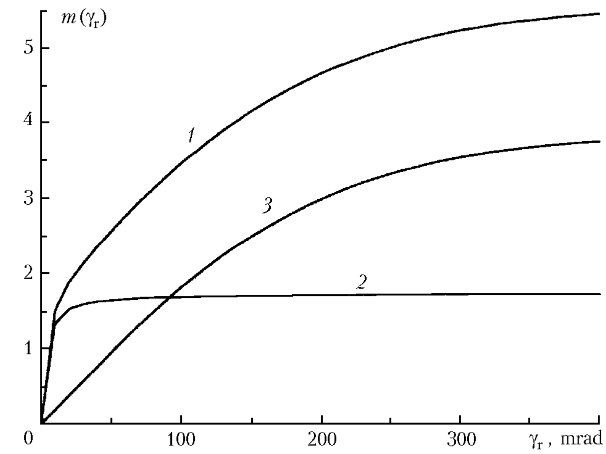


**Fig. 3.** Ratio $m(\gamma_r)$ (*1*) and its components $m^{(D)}(\gamma_r)$ (*2*), $\hat{\delta m}(\gamma_r)$ (*3*) at the layer optical depth $\tau = 1$ for large $\gamma_r$.

Figure 2 illustrates the efficiency of the proposed method of component separation in description of the ratio $m(\gamma_r)$. The exact dependence is $m(\gamma_r)$ shown by curve *1*, and curve *4* describes the approximate dependence $\hat{m}(\gamma_r)$ [Eq. (18)] formed by the sum of the diffraction component $\hat{m}^{(D)}(\gamma_r)$ in the asymptotic form (curve *2*) and $\hat{\delta m}(\gamma_r)$ in the form (17) (curve *3*). As can be seen from the comparison of the curves in Figs. 1 and 2, the domain of applicability of the approximation $\hat{m}(\gamma_r)$ [Eq. (18)] for $m(\gamma_r)$ is at least no smaller than the region, in which the asymptotic function $\hat{m}^{(D)}(\gamma_r)$ satisfactorily describes the diffraction component $m^{(D)}(\gamma_r)$.

Figure 3 illustrates the behavior of the functions $m(\gamma_r)$, $m^{(D)}(\gamma_r)$, and the extra term $\delta m(\gamma_r)$ at a very large receiver's field of view $\gamma_r$. We can distinguish two sections with different slope on the $m(\gamma_r)$ curve. The initial section with the steep slope at $\gamma_r <$ 11–13 mrad is largely formed by the diffraction component $m^{(D)}(\gamma_r)$ (curve *2*), which first sharply increases in this region and then saturates. Starting from this point, the dependence $m(\gamma_r)$ becomes more gently sloping and its slope is determined by the second term $\hat{\delta m}(\gamma_r)$ (curve *3*). Nevertheless, the diffraction component $m^{(D)}(\gamma_r)$ continues to contribute largely to the total dependence $m(\gamma_r)$ at $\gamma_r < 92$ mrad. The extra term $\hat{\delta m}(\gamma_r)$ in this region increases almost linearly and at $\gamma_r > 92$ mrad it becomes dominant, smoothly approaching saturation.

Let us then discuss the main tendencies observed in the behavior of $m(\gamma_r)$ with the increasing optical depth of the layer. Figures 4 and 5 show the dependence $m(\gamma_r)$ for the optical depth $\tau = 3$.

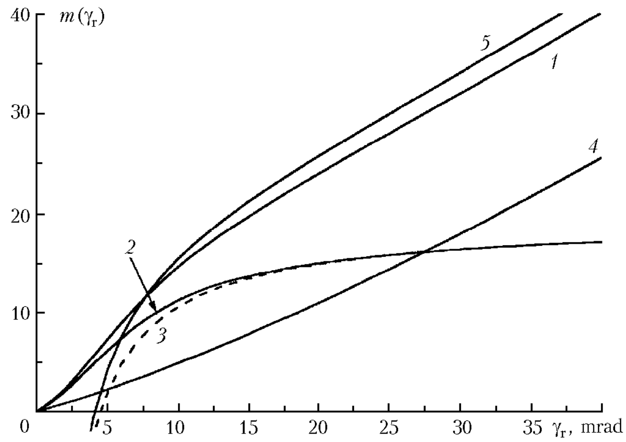


**Fig. 4.** Asymptotic approximation of $m(\gamma_r)$ with regard for the geometric-optics part of the scattering phase function for the layer with the optical depth $\tau = 3$: exact dependence $m(\gamma_r)$ (*1*), diffraction component $m^{(D)}(\gamma_r)$ (*2*), asymptotic approximation $\hat{m}^{(D)}(\gamma_r)$ (*3*), extra term $\hat{\delta m}(\gamma_r)$ (*4*), approximation of $\hat{m}(\gamma_r)$ by Eq. (18) (*5*).

The fact is to be noted first that the transition from $\tau = 1$ to $\tau = 3$ is accompanied by a significant increase in the level of $m(\gamma_r)$ saturation. The diffraction component increases more than tenfold and tends to the limit $m_\infty^{(D)} = 19.09$ (see Figs. 4 and 5, curve *2*). Additional allowance for the geometric-optics component in the scattering phase function leads to an even more significant growth of the asymptotic limit of $m(\gamma_r)$, which increases from $m_\infty = 5.55$ at $\tau = 1$ to $m_\infty = 280.5$ at $\tau = 3$ (Fig. 5, curve *1*). That significant increase of $m(\gamma_r)$ at large $\gamma_r$ is mostly due to the extra term $\hat{\delta m}(\gamma_r)$ (Fig. 5, curve *3*).

As the optical depth increases, the region of $\gamma_r$, in which the diffraction component $m^{(D)}(\gamma_r)$ prevails, becomes narrower. The boundary of this region is determined in Fig. 4 by the point of intersection of the curves *2* and *4*, at which $\gamma_r = 27.5$ mrad. The upper boundary of the $\gamma_r$ region, in which the diffraction approximation is applicable to description of the function $m(\gamma_r)$, shifts to the left in a similar way. The acceptable error of this approximation in the considered case is less than 15% for $\gamma_r < 4.6$ mrad.

To the contrary, the upper boundary of the $\gamma_r$ region, in which the asymptotic approximation $\hat{m}^{(D)}(\gamma_r)$ is applicable to description of the diffraction component $m^{(D)}(\gamma_r)$ (see Fig. 4, curve *3*) with the error within 15%, shifts to the right up to $\gamma_r = 7.7$ mrad. It follows from the above estimates that at $\tau = 3$, unlike the case of $\tau = 1$, the asymptotic approximation $\hat{m}^{(D)}(\gamma_r)$ does not provide for the 15% error in approximation of the total ratio $m(\gamma_r)$ at any receiver's field of view $\gamma_r$. The estimates show that the minimum possible error in this case is 28%.

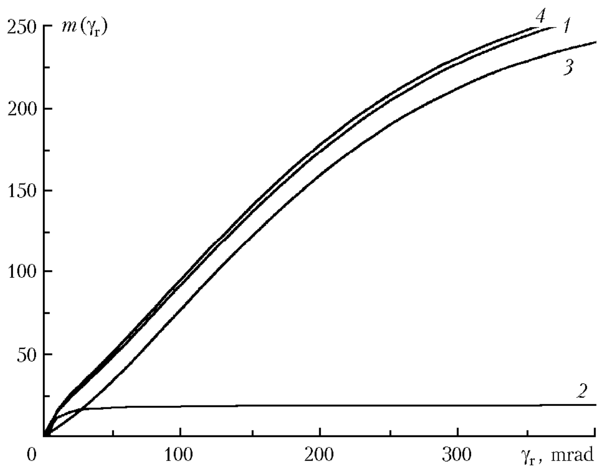


**Fig. 5.** Ratio $m(\gamma_r)$ (*1*) and its components $m^{(D)}(\gamma_r)$ (*2*) and $\hat{\delta m}(\gamma_r)$ (*3*) at the layer optical depth $\tau = 3$ for large $\gamma_r$; asymptotic approximation of $\hat{m}(\gamma_r)$ (*4*).

The situation changes in the case that the term $\hat{\delta m}(\gamma_r)$ is taken into account in addition to the diffraction component (see Fig. 4, curve *4*). Curve *5* in Fig. 4 describes the behavior of the total estimate $\hat{m}(\gamma_r)$. As can be seen from Fig. 4, the estimate $\hat{m}(\gamma_r)$ gives good results even at the receiver's field of view somewhat smaller than that, at which the approximation $\hat{m}^{(D)}(\gamma_r)$ is applicable to description of the diffraction component $m^{(D)}(\gamma_r)$. In the region of the receiver's field of view, at which the diffraction component $m^{(D)}(\gamma_r)$ achieves saturation ($\gamma_r > 20$ mrad), the behavior of $\hat{m}(\gamma_r)$ is determined by the term $\hat{\delta m}(\gamma_r)$ shifted by the constant $m_\infty^{(D)}$ (see Fig. 5).

## Conclusion

The structure of the lidar equation has been considered taking into account multiple scattering in the small-angle approximation. The role of the diffraction and the geometric-optics components of the scattering phase function has been studied depending on the receiver's field of view. The method has been proposed for separation of the diffraction and geometric-optics components in the lidar return. The advantage of this approach is the possibility of making analysis of the lidar return components separately. In particular, it has been shown that at a large receiver's field of view the asymptotic approximation can be used for description of the diffraction component of the lidar return. This allows the information about the diffraction component of the scattering phase function to be replaced by the value of the effective particle radius, keeping the information about the geometric-optics part unchanged. The description proposed may be useful in the development of methods and algorithms for solution of inverse problems in laser sensing of optically dense media.

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