

# Combined algorithm for spatiotemporal extrapolation of mesometeorological fields in problems of atmospheric-ecological monitoring of local territories

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An original technique and an algorithm for spatiotemporal extrapolation of mesometeorological fields based on the procedure of optimal combination of the modified Group Method of Data Handling and the Kalman filtering method are considered. The results of experimental study of the combined algorithm performance are discussed for the case of its application to the problem of spatial interpolation of the temperature and wind fields to a territory not covered by observations.

## Introduction

In recent years, growing requirements to the data of atmospheric and ecological monitoring of local territories (e.g., a big city or an industrial zone) necessitate development of new, more reliable methods and algorithms for spatial extrapolation of meteorological fields on the mesoscale. The temperature and wind play a key role in atmospheric transport of pollutants, therefore determination of the fields of just these weather parameters is needed to evaluate spread of pollutants to small (up to 100–200 km) distances.

For solution of such ecological problems, it is necessary that the mesoscale temperature and wind fields be specified with a sufficient vertical and horizontal resolution. Thus, according to Refs. 1 and 2, extrapolation of meteorological fields in the atmospheric boundary layer, just where the transport of pollutants mostly occurs, should be performed with the height step from 200 to 400 m and from 5 to 20 km in the horizontal plane.

In practice such strict requirements are difficult to satisfy, since

– the scale of extrapolation of mesometeorological fields is much smaller than the separation between stations of the global aerological network (this separation ranges from 300 to 400 km [Ref. 3] even in densely populated regions) and, consequently, the data of this network do not allow reliable evaluation of the structure of these fields with high spatial resolution;

– methods of spatial interpolation and extrapolation used in current schemes of objective analysis of meteorological fields (e.g., methods of optimal interpolation, polynomial and spline approximation<sup>3,4</sup>) do not yield reliable results in

regions with wide spacing between stations, as well as in predicting these fields to a territory not covered by observations.

Taking into account all the above-said, specialists from the Institute of Atmospheric Optics have developed in mid-1990s an original dynamic-stochastic algorithm for solving the problem on spatial extrapolation of mesometeorological fields based on the combined procedure of the method of optimal extrapolation, which is widely used in practice of objective analysis of meteorological fields, with the Modified Group Method of Data Handling (MGMDH). This algorithm is described in detail elsewhere in the literature.<sup>5,6</sup>

Regardless of its remarkable advantages over the traditional methods of regression analysis, this algorithm has some shortcomings. In particular, some sample of previous spatiotemporal observations is needed, for its implementation, and calculation of the weighting coefficients in the equation of optimal extrapolation<sup>3</sup> requires also the spatial correlation functions obtained from the data of many-year observations.

In this connection, this paper proposes an original dynamic-stochastic algorithm for solution of the problem on spatial extrapolation of meteorological fields on the mesoscale. It is based on the procedure of combining the MGMDH with the Kalman filtering method and is free of shortcomings inherent in the combination of MGMDH with the method of optimal extrapolation.

An important peculiarity of the proposed combined algorithm is that it allows spatial extrapolation of mesometeorological fields to be obtained not only during measurements, but also in the

gaps between standard (synoptic) measurements. Another peculiarity of the algorithm is that the scheme of its realization differs by the number of observations. With the number of observations  $K < h + 1$  (where  $h$  is the number of altitude levels, including the surface one), the procedure of spatial extrapolation is based on the single method of Kalman filtering, which, in its turn, is based on the simplified dynamic-stochastic model. With the number of observations  $K \geq h + 1$ , extrapolation is performed by the following algorithm:

- at the first stage, the Kalman filtering method is used to extrapolate the surface values of the weather parameter at issue to a given point based on measurements at neighboring stations;

- at the second stage, this weather parameter at the prediction point and the given time is reconstructed at the needed atmospheric levels based on the MGMDH and aerological observations at the nearest (to the prediction point) station with the total number  $K = h$ .

Below we describe the proposed algorithm and consider the results of evaluation of its performance based on the data of many-year aerological observations at the local network.

## 1. Formulation of the problem and algorithms for its solution

The problem of spatial extrapolation of a centered meteorological field  $\xi$  consists in estimation of its values at the spatial point with the coordinates  $(x_n, y_n, z_n)$ , measurements for which are lacking, based on observations  $\xi_i$  at the points with the coordinates  $(x_i, y_i, z_i)$  ( $i = 1, 2, \dots, n - 1$ ) and some mathematical model describing variations of the field  $\xi$  in space and time. The combined algorithm proposed assumes simultaneous use of the MGMDH and the Kalman filtering method that differ by spatiotemporal models and the computational procedure. Let us consider briefly the main peculiarities of the algorithms used.

### (a) Modified group method of data handling

According to Refs. 5 and 6, the modified group method of data handling is a method of structural-parametric identification, which allows synthesizing prognostic models based on limited *a priori* information under the conditions of partial or complete uncertainty of knowledge about the structure of the process modeled and the properties of noise in the initial data. The MGMDH algorithms employs, as basis functions, difference dynamic-stochastic models:

$$\xi_{h,K+1} = \sum_{\tau=1}^{K^*} A_{h,\tau} \xi_{h,K+1-\tau} + \sum_{j=0}^{h-1} B_{h,j} \xi_{j,K+1} + \varepsilon_{h,K+1}, \quad (1)$$

$$h = h^* + 1, h^* + 2, \dots, h_{\max}$$

(here  $K^*$  is the order of time delay ( $K^* < [K - h - 1]/2$ );  $A_{h,1}, \dots, A_{h,K^*}$  and  $B_{h,0}, \dots, B_{h,h-1}$  are unknown model parameters;  $\varepsilon_{h,K+1}$  is the model discrepancy) and, as the initial data, spatiotemporal observations of the centered field  $\xi$  at the points  $i$  in the form

$$\begin{aligned} &\{\xi_{h,k}, h = 0, 1, \dots, h_{\max}, k = 1, \dots, K\}, \\ &\{\xi_{h,k}, h = 0, 1, \dots, h^* \leq h_{\max}, k = K+1\}. \end{aligned} \quad (2)$$

In Eqs. (1) and (2),  $h$  is the altitude;  $h^*$  is the altitude of the top measurement level at the time  $K + 1$ ;  $h_{\max}$  is the maximum observation top before the time  $K$ ;  $k$  is the current time of observations;  $K$  is the sample size.

It should be noted that the first term in the right-hand side of Eq. (1) reflects the relation between observations at the altitude  $h$  and at the time point  $K + 1$  and those at the previous time points that arises due to nonzero correlation length of the temporal correlation function for the mesoscale atmospheric processes. The second term reflects the correlation between the altitude levels at the time point  $K + 1$  that arises because of turbulent mixing and regular vertical motions.

To find the best prognostic model of the form (1) and predict (reconstruct) successfully the vertical structure of the field  $\xi$  at the point  $i$ , all initial spatiotemporal observations (2) are used. These observations are divided into two samples: the learning sample  $a_1$  (it includes all observations before the time point  $k = K - 1$  inclusive) and the control sample  $a_2$  including observations only at the time point  $k = K$ . Besides, two specialized methods are used for this purpose, i.e., to find the best prognostic model.

1) Directional group trial-and-error method for optimization of the model structure with the two-stage model selection performed with such a selection criteria as:

- final prediction error (H. Akaike<sup>7</sup>)

$$FRE = \frac{(K - K^* - 1) + s}{(K - K^* - 1) - s} RSS(s), \quad (3)$$

where  $RSS(s) = \sum_{j=1}^{K-K^*-1} [\xi_{h,K-j} - \hat{\xi}_{h,K-j}(s)]^2$  is the rest

square sum for the current model  $\hat{\xi}_{h,K-j}(s)$  including  $s$  nonzero estimates of parameters;

- root-mean-square prediction error for the control sample, that is, the sample  $a_2$ :

$$|\xi_{h,K} - \hat{\xi}_{h,K}(s)|^2 \rightarrow \min, \quad (4)$$

where the minimum is sought over all  $K^*+h$  structures, each described by its own model  $\hat{\xi}_{h,K}(s)$ .

2) Minimax estimation method (MEM)<sup>8</sup> used to obtain estimates of parameters of the prognostic model  $\hat{\Theta}$ , which ensures the quality of the corresponding prediction estimated by the equation

$$\mathbf{M}|\mathbf{M}(\xi_{h,K+1}) - \hat{\xi}_{h,K+1}|^2 \leq \delta_{h,K+1}$$

$$(h = h^* + 1, \dots, h_{\max}), \tag{5}$$

where  $\mathbf{M}(\bullet)$  is the operator of mathematical expectation that performs averaging over all possible realizations of observation errors;  $\hat{\xi}_{h,K+1}$  and  $\delta_{h,K+1}$  are the minimax estimates depending on the variance of observation errors and *a priori* information about maximum acceptable prediction errors.

**(b) Kalman filtering method**

This method falls in the group of methods of Markov optimal filtration theory<sup>10</sup> and provides for estimation of the sought parameters with the minimum rms error at every step. To formulate the problem of spatial prediction by terms of the Kalman filtering method, the weather parameters variable in space and time were represented as the following dynamic system<sup>9</sup>:

$$\begin{cases} X_1(k+1) = X_n(k)(1 - \beta\Delta r_{n1})(1 - \alpha\Delta t) + \omega_1(k); \\ X_2(k+1) = X_n(k)(1 - \beta\Delta r_{n2})(1 - \alpha\Delta t) + \omega_2(k); \\ \dots \\ X_{n-1}(k+1) = X_n(k)(1 - \beta\Delta r_{n,n-1})(1 - \alpha\Delta t) + \omega_{n-1}(k); \\ X_n(k+1) = X_n(k)(1 - \alpha\Delta t) + \omega_n(k), \end{cases} \tag{6}$$

where

$$|X_1(k+1), X_2(k+1), X_3(k+1), \dots, X_n(k+1)|^T = \mathbf{X}(k+1)$$

is the vector of state, whose elements are the values of the homogeneous centered field  $\xi$  at the points with the coordinates  $x_i$  and  $y_i$  ( $i = 1, 2, \dots, n$ ) at the time point  $k+1$  ( $X_n(k+1)$  is the value of the weather parameter at the point  $(x_n, y_n)$ , for which measurements are lacking);  $\Delta r_{ni}$  is the distance between the predicted point  $n$  with the coordinates  $(x_n, y_n)$  and the measurement points  $i = 1, 2, \dots, n-1$ ;  $\Delta t$  is the time discretization interval;  $k = 0, 1, 2, \dots, K$  is the current time with the discretization interval  $\Delta t$  ( $t_k = k\Delta t$ );

$$\mathbf{\Omega}(k) = |\omega_1(k), \omega_2(k), \omega_3(k), \dots, \omega_n(k)|^T$$

is the column vector of state noise.

The system of difference equations (6) is specified on the assumption that the temporal and spatial correlation functions of the sought weather parameter  $\xi$  on the mesoscale can be approximated (with a small error) by the exponential equations:

$$\mu_\xi(\tau) = \exp(-\alpha\tau); \tag{7}$$

$$\mu_\xi(\rho) = \exp(-\beta\rho), \tag{8}$$

here  $\alpha = 1/\tau_0$  is the coefficient inversely proportional to the length of temporal correlation  $\tau_0$ ;  $\beta = 1/\rho_0$  is the coefficient inversely proportional to length of spatial correlation  $\rho_0$ .

Besides, the specific property of the mesoscale allows us to apply the splitting method and to estimate

(predict) the weather parameters at a fixed altitude level ignoring relations between neighboring levels. Thus, for each altitude range, its own Kalman filter should be used, and each filter processes measurements obtained for the given altitude level at all aerological stations of a specified mesoscale territory. The set of these measurements obtained synchronously at the time point  $k$  is described by the following system of equations of observations :

$$\begin{cases} Y_1(k) = X_1(k) + \varepsilon_1(k); \\ Y_2(k) = X_2(k) + \varepsilon_2(k); \\ \dots \\ Y_{n-1}(k) = X_{n-1}(k) + \varepsilon_{n-1}(k), \end{cases} \tag{9}$$

where  $Y_i(k) = \xi_i(k) - \bar{\xi}(k)$  is the centered value of the field at the  $i$ th point (here  $\xi_i(k)$  and  $\bar{\xi}(k)$  are respectively the value of the weather parameter measured at the  $i$ th point and that averaged over the studied territory at the level  $h$  at the time  $k$ ), and  $\varepsilon_i(k)$  is the measurement error ( $i = 1, 2, \dots, n-1$ ).

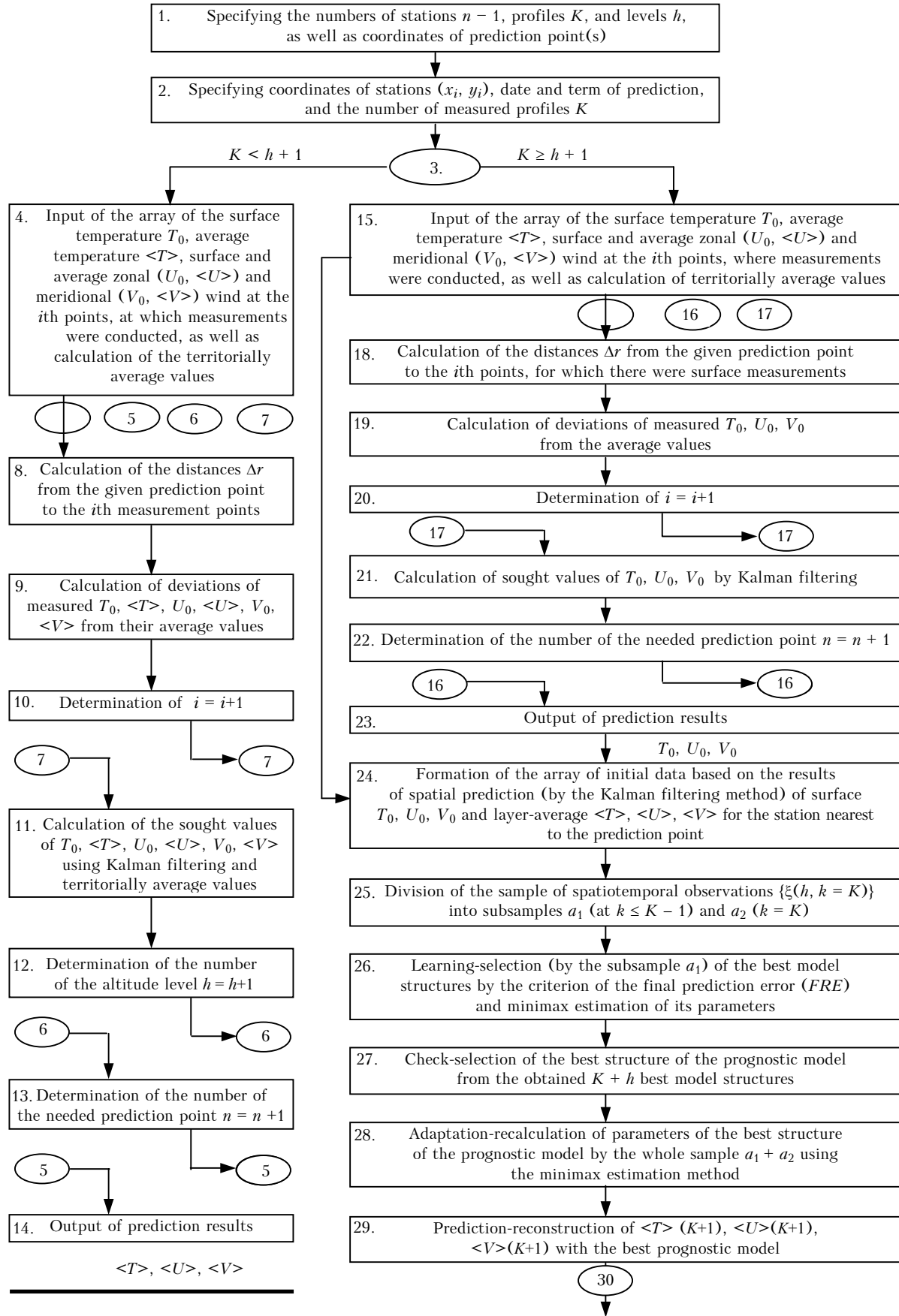
Equations (6) and (9) and the apparatus of Kalman filtering were used for synthesizing the algorithm of estimating the state vector  $\mathbf{X}(k+1)$  (for details see Refs. 9 and 10).

The operation of the combined estimation algorithm is generally described in introduction. It should be emphasized that its development involved the main advantages of the methods considered above. As was noted earlier, the problem of spatial prediction is solved in two stages. At the first stage, the Kalman filtering method was used for measurements obtained at all  $(n - 1)$  aerological stations to predict the centered value of the weather parameter in the horizontal plane to the point with the given coordinates  $(x_n, y_n)$  at the surface level. At the second stage, the MGMDH based on the best model for the point  $i$  (nearest to the prediction point) and the prognostic value of the weather parameter at the surface level (obtained at the first stage) is used to reconstruct the whole altitude profile. Reconstruction is completed by adding the territorially average value  $\bar{\xi}(k)$  at the same level  $h$  to the obtained centered values of the weather parameter.

It should be noted that for correct operation of the MGMDH, the number of measurements (the number of measured altitude profiles) should exceed the number of altitude levels.<sup>6</sup> Therefore, as long as  $K < h + 1$ , spatial prediction is performed using the Kalman filters realized for each altitude level. Then, at  $K \geq h + 1$ , the combined algorithm described above is used.

The block diagram of the estimation algorithm is shown in Fig. 1. Consider operation of its component blocks.

*Block 1* provides for input of the data about the number of stations  $n - 1$  conducting aerological measurements, the needed number of vertical profiles (i.e., the number of measurements)  $K$ , and the number of altitude levels  $h$ , as well as specifies the coordinates of the prediction point (or points)  $(x_n, y_n)$ .



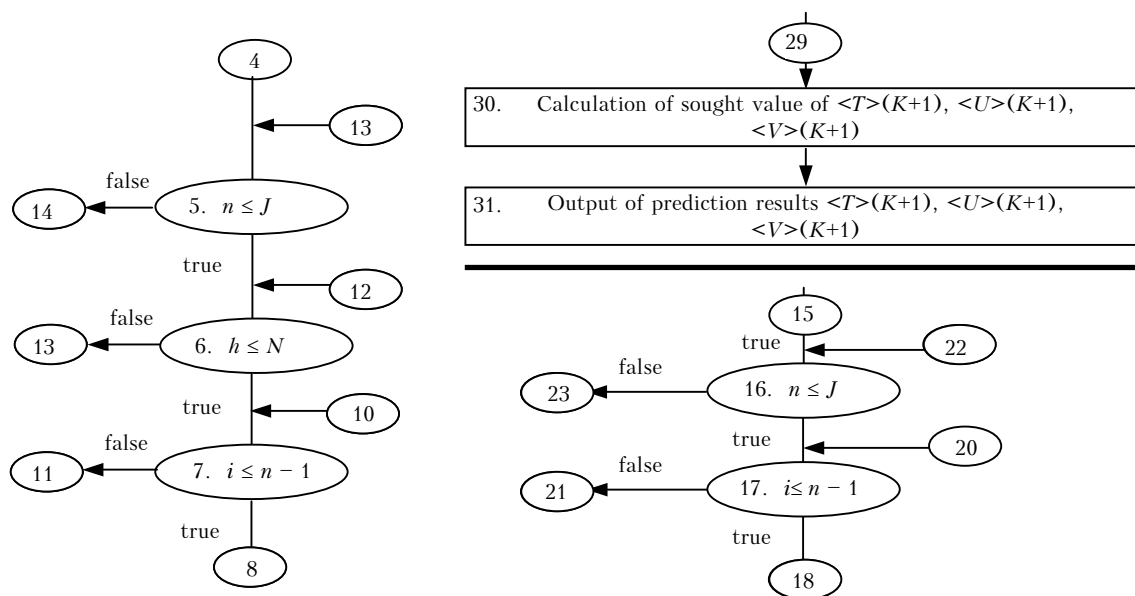


Fig. 1. Block diagram of spatial prediction by the technique combining the Kalman filtering and MGMDH.

Block 2 provides for specifying rectangular coordinates of the stations  $(x_i, y_i)$  conducting measurements (if they are lacking, rectangular coordinates are obtained from the geographical ones), date and term of prediction, as well as the number of measured altitude profiles  $K$ .

Block 3 checks fulfillment of the condition  $K \leq h + 1$  (fulfillment of  $K = h + 1$  permits operation of the MGMDH algorithm in combination with the Kalman filter).

If  $K < h + 1$ , then only the Kalman filtering procedure is performed for prediction of both surface values of temperature  $T_0$ , zonal  $U_0$  and meridional  $V_0$  wind and the layer-average values  $\langle T \rangle_{h_0, h}$ ,  $\langle U \rangle_{h_0, h}$ , and  $\langle V \rangle_{h_0, h}$  (hereinafter for simplicity the index  $h_0, h$  of  $\langle \bullet \rangle$  is omitted).

Block 4 provides for the input of the array including the data on the surface ( $T_0$ ) and average  $\langle T \rangle_{h_0, h}$  temperature, surface and average zonal ( $U, \langle U \rangle_{h_0, h}$ ) and meridional ( $V, \langle V \rangle_{h_0, h}$ ) wind at the  $i$ th points, where measurements were conducted, as well as calculation of average over the territory values  $(\bar{\xi})$  and standard deviations  $(\sigma_{\xi})$  for the mesometeorological territory chosen.

Blocks 5, 6, and 7 provide for the organization of cycles for the given prediction points  $n \leq J$ , altitude levels (layers)  $h \leq K$ , and stations  $i \leq n - 1$ .

Block 8 calculates the distances  $\Delta r_{ni}$  from the given prediction point  $n$  to the points, at which the weather parameters  $T, U, V$ , and  $\langle T \rangle, \langle U \rangle$ , and  $\langle V \rangle$  were measured, through the following equation:

$$\Delta r_{ni} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}. \quad (10)$$

Block 9 calculates deviations of the measured values of  $T_0, U, V$  and  $\langle T \rangle, \langle U \rangle, \langle V \rangle$  from the norm (territorially mean), that is, performs calculations by Eq. (11).

Block 10 provides for repetition of calculation cycles for the used stations  $i = i + 1$ .

Block 11 calculates the sought values of  $T_0, \langle T \rangle, U_0, \langle U \rangle$  and  $V_0, \langle V \rangle$  at the point with the coordinates  $(x_n, y_n)$  using the Kalman filtering algorithm and territorially average norms by the equation

$$\xi_n = \bar{\xi} + \hat{\xi}_n, \quad (11)$$

where  $\hat{\xi}_j$  is an estimate (prediction) of the centered value of a weather parameter.

Block 12 determines the number of the altitude level  $h = h + 1$ .

Block 13 determines the number of the needed prediction point  $n = n + 1$ .

Block 14 outputs the results of spatial prediction (sought values of  $T_0, \langle T \rangle, U_0, \langle U \rangle, V_0, \langle V \rangle$ ).

Block 15 provides, at  $K = h + 1$ , i.e., under conditions of operation of the MGMDH algorithm, input of the array of initial data on the surface and layer-average temperature ( $T_0, \langle T \rangle$ ), surface and layer-average zonal ( $U_0, \langle U \rangle$ ) and meridional ( $V_0, \langle V \rangle$ ) wind at the  $i$ th point, as well as calculation of territorially average values (norms)  $\bar{\xi}$  and standard deviations  $\sigma_{\xi}$  for the mesoscale territory chosen.

Blocks 16–20 correspond to blocks 5–10, but only applied to calculation of the surface values of  $T_0, U_0, V_0$ .

Block 21 provides for calculation of the sought values of centered fields of the surface temperature  $T_0$ , surface zonal  $U_0$  and meridional  $V_0$  wind at the point

with the coordinates  $x_n, y_n$  by means of the Kalman filtering.

*Block 22* provides for determination of the order number of prediction point  $n = n + 1$  needed.

*Block 23* provides for output of the results of spatial prediction of the centered values of temperature, zonal and meridional wind ( $T_0, U_0, V_0$ ) by the Kalman filtering method.

*Block 24* provides for sampling spatiotemporal observations based on the results of spatial prediction of the surface temperature ( $T_0$ ), surface zonal ( $U_0$ ) and meridional ( $V_0$ ) wind by the Kalman filtering method, as well as the array of the data on the mean temperature  $\langle T \rangle$ , mean zonal  $\langle U \rangle$  and meridional  $\langle V \rangle$  wind for the previous (with respect to prediction) terms for the nearest (to the sought prediction point  $x_n, y_n$ )  $i$ th point (station).

*Block 25* divides the sample of spatiotemporal observations  $\{\xi(h, k = K)\}$  into two subsamples:  $a_1$  (at  $k \leq K - 1$ ) and  $a_2$  (at  $k = K$ ).

*Block 26* provides for learning-selection (by the subsample  $a_1$ ) of the best prognostic MGMDH models by the criterion of final prediction error (*FRE*) and using the minimax estimation of its parameters.

*Block 27* provides (by the subsample  $a_2$ ) check-selection of the best structure of the prognostic MGMDH model.

*Block 28* provides for adaptation-recalculation of parameters of the best structure of the prognostic model for the whole sample  $a_1 + a_2$  by the minimax estimation method.

*Block 29* is responsible for prediction-reconstruction, by the best prognostic MGMDH model, of the values of  $\langle T \rangle(K + 1)$ ,  $\langle U \rangle(K + 1)$ , and  $\langle V \rangle(K + 1)$  at the point  $(x_n, y_n)$ .

*Block 30* is responsible for calculation of the sought profile of the given weather parameter, including the values of the surface ( $T_0$ ) and average temperature  $\langle T \rangle$ , surface and average zonal  $U_0, \langle U \rangle$  and meridional  $V_0, \langle V \rangle$  wind, for the point with the coordinates  $(x_n, y_n)$  by the equation

$$\xi(h^* < h \leq h_{\max}, K + 1) = \bar{\xi}(h^* < h \leq h_{\max}) + \hat{\xi}(h^* < h \leq h_{\max}, K + 1) \quad (12)$$

(here  $\bar{\xi}(h^* < h \leq h_{\max})$  is the mean profile of layer-average values of the weather parameter;  $\hat{\xi}(h^* < h \leq h_{\max}, K + 1)$  is the profile of random deviations for the same weather parameter calculated by the best prognostic MGMDH model for the time  $k = K + 1$ ).

*Block 31* provides for output of predicted values of  $\langle T \rangle(K + 1)$ ,  $\langle U \rangle(K + 1)$ , and  $\langle V \rangle(K + 1)$ .

## 2. Estimation of the accuracy of the algorithm of spatial prediction

To estimate the performance of the combined algorithm at its use in the problem of spatial prediction

of mesoscale temperature and wind fields, we used five-year two-term (0 am and 12 am GMT) observations of five aerological stations: Warsaw (52°11'N, 20°58'E), Kaunas (54°53'N, 23°53'E), Brest (52°07'N, 23°41'E), Minsk (53°11'N, 27°32'E) and Lvov (49°49'N, 23°57'E) which form a typical mesometeorological territory. The Warsaw station spaced by 185 km from the nearest Brest station, for which aerological observations are available, was taken as a control station, for which spatial prediction was performed.

Since we consider spatial prediction as applied to estimation of the spread of a pollution cloud, according to Ref. 11, we did not took the direct measurements of temperature and wind at some atmospheric levels, but their layer-average values determined by the equation

$$\langle \xi \rangle_{h_0, h} = \frac{1}{h - h_0} \int_{h_0}^h \xi(z) dz, \quad (13)$$

where  $\langle \cdot \rangle$  denotes vertical averaging over an atmospheric layer  $\Delta H = h - h_0$  (here  $h$  and  $h_0$  are the altitudes of the layer top and bottom, and  $h_0$  corresponds to the ground level);  $\xi$  is the value of the weather parameter.

Figure 2 depicts the results of extrapolation of the field of the layer-average temperature and zonal and meridional components of the wind velocity. The dependences are plotted for the combined algorithm and the optimal extrapolation method. It follows from the plots that the combined algorithm provides the gain in accuracy in comparison with the optimal extrapolation method. Thus, for example, regardless of the season, atmospheric layer, and the weather parameter, the rms errors of the spatial prediction  $\delta$  are 1.3–1.8 times lower than those for the optimal extrapolation method.

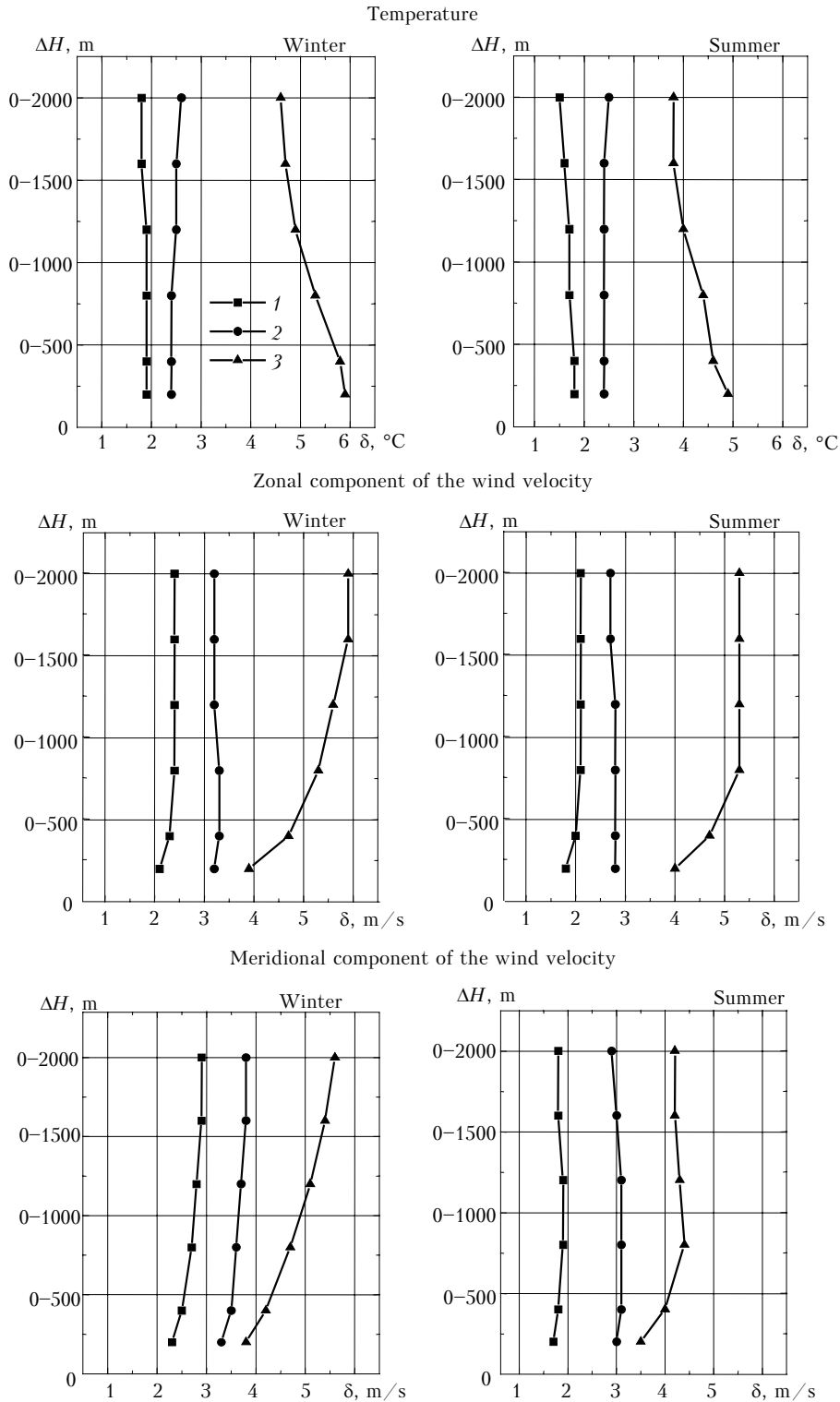
Besides, it should be noted that the proposed combined dynamic-stochastic algorithm has other remarkable advantages over the method of optimal extrapolation, namely:

- spatial prediction of the fields of weather parameters on the mesoscale is performed in real time (as soon as the observation data are coming), that is, without invoking archived information;

- in the gap between the time of income of actual measurement data (that is, at the intervals shorter than the interval between synoptic terms), it is possible to perform spatial prediction due to solution of difference (differential) of the equations of state with smaller discretization step;

- the combined algorithm allows adaptation to unknown parameters of the dynamic model (for example, to the preset intervals of spatial and temporal correlation).

All the above-said allows us to conclude that the algorithm combining the Kalman filtering with MGMDH is quite efficient and can be successfully used in the problems of estimating the spatial spreading of pollutants over local territories.



**Fig. 2.** Altitude dependence of rms errors of extrapolation of the layer-average values of temperature, zonal and meridional wind velocity components to the distance of 185 km as estimated by the combined algorithm (1) and the optimal extrapolation method (2), as well as the corresponding standard deviations (3).

**References**

1. V.I. Akselevich, "Physical and statistical methods for reconstructing physical parameters of the troposphere in

problems of atmospheric monitoring and military geophysics," Author's Abstract of Cand. Phys.-Math. Sci. Dissert., St. Petersburg (1994), 16 pp.

2. V.S. Komarov, S.A. Soldatenko, and O.M. Sobolevskii, Atmos. Oceanic Opt. **9**, No. 4, 278-281 (1996).

3. L.S. Gandin and R.L. Kagan, *Statistical Methods for Interpretation of Meteorological Data* (Gidrometeoizdat, Leningrad, 1976), 359 pp.
4. V.A. Gordin, *Mathematical Problems of Hydrometeorological Weather Forecast* (Gidrometeoizdat, Leningrad, 1987), 264 pp.
5. V.S. Komarov and A.V. Kreminskii, Atmos. Oceanic Opt. **8**, No. 7, 488–495 (1995).
6. V.S. Komarov, *Statistics in Application to Problems of Applied Meteorology* (Spektr, Tomsk, 1997), 256 pp.
7. H. Akaike, in: *Trends and Progress in System Identification*, ed. by P. Eykoff (Pergamon Press, Paris, 1981).
8. Yu.L. Kocherga, *Avtomatika*, No. 2, 65–71 (1991).
9. V.S. Komarov and Yu.B. Popov, Atmos. Oceanic Opt. **14**, No. 4, 230–234 (2001).
10. K. Brammer and G. Siffling, *Kalman–Bucy Filter* (Oldenbourg Verlag, 1975).
11. F.F. Bryukhan, *Methods for Climatic Processing and Analysis of Aerological Information* (Gidrometeoizdat, Moscow, 1983), 112 pp.