Aberration of a Gabor hologram at formation of shear interferogram in diffusively scattered light for wavefront control

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Received January 30, 2001

Record and reconstruction a two-exposure Gabor hologram of an amplitude screen for wavefront control are described in the third-order approximation. It is shown that control errors are caused by spherical aberration of the hologram.

In Ref. 1 it was shown that the two-exposure record of a Gabor hologram of an amplitude screen leads, at the stage of its reconstruction, to formation of a shear interferogram in infinitely wide bands that characterizes the wave front of the coherent radiation used for screen illumination. The mechanism of formation of the interference pattern in this case reduces to the need to match objective speckles of the two exposures in the plane of a photographic plate at the stage of hologram recording. The speckles are matched by changing the tilt angle of the controlled wave front and the lateral shift of the photographic plate before the second exposure. The objective speckles of both exposures can be matched both for a divergent and a convergent quasispherical wave fronts.

The interference pattern located in the hologram plane is recorded at the stage of its reconstruction when performing spatial filtering of the diffraction field on the optical axis in the plane of formation of the real image of an amplitude screen. In Ref. 1, to justify the conditions of formation of the interference pattern in coherent diffusely scattered fields, the parabolic approximation was used for the complex field amplitude. This approximation ignores possible control errors caused by monochromatic aberrations of the hologram.

This paper analyzes formation of the shear interferogram characterizing the controlled wave front in the third-order approximation in order to estimate possible control errors due to monochromatic aberrations of the hologram.

According to Fig. 1*a*, the amplitude screen *1* lying in the plane (x_1, y_1) is illuminated by the coherent radiation with the divergent quasispherical wave having the radius of curvature R. The radiation scattered by the screen along with the coherent background is recorded on a photographic plate *2* located in the plane (x_2, y_2) at the distance l for the time of the first exposure. Before the second exposure, as in Ref. 1, the tilt angle of the controlled wave front is changed, for example, in the plane (x, z) by the value α and the photographic plate is displaced by the distance b in the same direction along the axis x. Then, neglecting constant factors, in the third-order approximation the distribution of the complex field amplitude in the plane (x_2, y_2) corresponding to the first exposure can be written in the form:

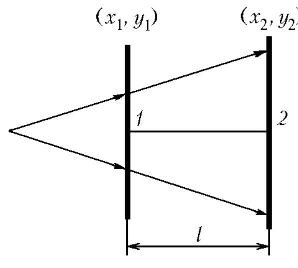
$$u_1(x_2, y_2) \sim \int\int_{-\infty}^{\infty} [1 - t(x_1, y_1)] \times$$

$$\times \exp\left\{i\left\{k\left[\frac{1}{2R}(x_1^2+y_1^2) - \frac{1}{8R^3}(x_1^2+y_1^2)^2\right] - \varphi(x_1, y_1)\right\}\right\} \times$$

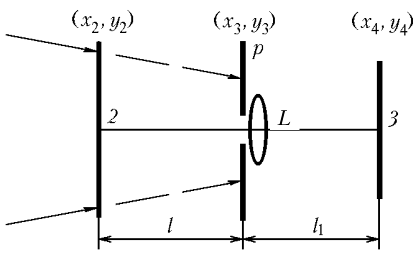
$$\times \exp\left\{\frac{ik}{2l}[(x_1 - x_2)^2 + (y_1 - y_2)^2]\right\} \times$$

$$\times \exp\left\{-\frac{ik}{8l^3}[(x_1 - x_2)^2 + (y_1 - y_2)^2]^2\right\} dx_1\, dy_1, \quad (1)$$

where k is the wave number; $t(x_1, y_1)$ is the screen absorption amplitude, which is a random function of coordinates; $\varphi(x_1, y_1)$ is a deterministic function characterizing wave distortion of the radiation used to illuminate the amplitude screen, for example, because of the aberrations of the optical system that forms it.



a



b

Fig. 1. Schematic of recording (*a*) and reconstruction (*b*) of a two-exposure Gabor hologram: amplitude screen *1*, photographic plate-hologram *2*, plane of interferogram recording *3*; positive lens L, and spatial filter p.

0235-6880/03/02 84-07 $02.00

Equation (1) can be presented in the following form:

$$u_1(x_2, y_2) \sim$$

$$\sim \exp\left\{ik\left[\frac{1}{2l}(x_2^2 + y_2^2) - \frac{1}{8l^3}(x_2^2 + y_2^2)^2\right]\right\} \times$$

$$\times \left\{[\delta(x_2, y_2) - F(x_2, y_2)] \otimes \exp\left[-\frac{ik\mu_1}{2l}(x_2^2 + y_2^2)\right] \otimes\right.$$

$$\left.\otimes \Phi(x_2, y_2) \otimes \Phi_1(x_2, y_2) \otimes \Phi_2(x_2, y_2) \otimes \Phi_3(x_2, y_2)\right\}, \quad (2)$$

where $\otimes$ denotes convolution; $\delta(x_2, y_2)$ is the Dirac delta function; $\mu_1 = R/(R + l)$ is the scaling factor; $F(x_2, y_2)$, $\Phi(x_2, y_2)$, $\Phi_1(x_2, y_2)$, $\Phi_2(x_2, y_2)$, $\Phi_3(x_2, y_2)$ are the Fourier transforms of the functions

$$t(x_1, y_1),\ \exp[-i\varphi(x_1, y_1)],\ \exp\left[-\frac{ik}{8R^3}(x_1^2 + y_1^2)^2\right],$$

$$\exp\left[-\frac{ik}{8l^3}(x_1^2 + y_1^2)^2\right],\ \exp[i\psi_1(x_1, y_1; x_2, y_2)]$$

with the spatial frequencies $x_2/\lambda l$ and $y_2/\lambda l$; λ is the wavelength of coherent radiation used for hologram recording and reconstruction;

$$\psi_1(x_1, y_1; x_2, y_2) =$$

$$-\frac{k}{8l^3}\Big(6x_1^2 x_2^2 + 6y_1^2 y_2^2 - 4x_1^3 x_2 - 4x_1^2 y_1 y_2 + 2x_1^2 y_2^2 -$$

$$- 4x_1 x_2^3 - 4x_1 y_1^2 y_2 + 8x_1 y_1 x_2 y_2 - 4x_1 x_2 y_2^2 + 2y_1^2 x_2^2 -$$

$$- 4y_1 x_2^2 y_2 - 4y_1^3 y_2 - 4y_1 y_2^3\Big)$$

is the phase function characterizing third-order off-axis wave aberrations.

Using the condition $t(x_1, y_1) \ll 1$ (Ref. 2), let us find the complex transmission amplitude $\tau_1(x_2, y_2)$ of the hologram under condition that recording is within the linear portion of the blackening curve of the photosensitive material. Neglecting the regular component occupying a small spatial area in the recording plane of the interference pattern at the stage of hologram reconstruction,[1] it is described by the equation

$$\tau_1(x_2, y_2) \sim \left\{\exp\left[\frac{ik\mu_1}{2l}(x_2^2 + y_2^2)\right] \otimes\right.$$

$$\left.\otimes \Phi^*(x_2, y_2) \otimes \Phi_1^*(x_2, y_2) \otimes \Phi_2^*(x_2, y_2)\right\} \times$$

$$\times \left\{F(x_2, y_2) \otimes \exp\left[-\frac{ik\mu_1}{2l}(x_2^2 + y_2^2)\right] \otimes \Phi(x_2, y_2) \otimes\right.$$

$$\left.\otimes \Phi_1(x_2, y_2) \otimes \Phi_2(x_2, y_2) \otimes \Phi_3(x_2, y_2)\right\} + \text{c.c.}, \quad (3)$$

where c.c. means complex conjugate.

The first term $\tau_1^{(-1)}(x_2, y_2)$ in Eq. (3) determines the following diffraction of the wave in the (−1) order at the stage of hologram reconstruction, while the second term $\tau_1^{(+1)}(x_2, y_2)$ stands for the (+1) order.

The distribution of the complex field amplitude in the photographic plate plane (x_2, y_2) corresponding to the second exposure can be written in the form

$$u_2(x_2, y_2) \sim \int\int_{-\infty}^{\infty} [1 - t(x_1, y_1)] \times$$

$$\times \exp\left\{i\left\{\frac{k}{2R}[(x_1 + R\sin\alpha)^2 + y_1^2] - \varphi(x_1 + a, y_1)\right\}\right\} \times$$

$$\times \exp\left\{-\frac{ik}{8R^3}[(x_1 + R\sin\alpha)^2 + y_1^2]^2\right\} \times$$

$$\times \exp\left\{\frac{ik}{2l}[(x_1 - x_2 - b)^2 + (y_1 - y_2)^2]\right\} \times$$

$$\times \exp\left\{-\frac{ik}{8l^3}[(x_1 - x_2 - b)^2 + (y_1 - y_2)^2]^2\right\} dx_1\, dy_1, \quad (4)$$

where a is the shift of the wave front due to the change in its tilt angle before the second exposure.

If $b = l\sin\alpha$, then the complex transmission amplitude $\tau_2(x_2, y_2) = \tau_2^{(-1)}(x_2, y_2) + \tau_2^{(+1)}(x_2, y_2)$ of the hologram corresponding to the second exposure can be represented as

$$\tau_2(x_2, y_2) \sim \left\{\exp\left[\frac{ik\mu_1}{2l}(x_2^2 + y_2^2)\right] \otimes\right.$$

$$\otimes \exp(-ikax_2/l)\ \Phi^*(x_2, y_2) \otimes \exp(-ikRbx_2/l^2) \times$$

$$\left.\times\ \Phi_1^*(x_2, y_2) \otimes \exp(ikbx_2/l)\ \Phi_2^*(x_2, y_2)\right\} \times$$

$$\times \left\{F(x_2, y_2) \otimes \exp\left[-\frac{ik\mu_1}{2l}(x_2^2 + y_2^2)\right] \otimes \exp(ikax_2/l) \times\right.$$

$$\times\ \Phi(x_2, y_2) \otimes \exp(ikRbx_2/l^2)\ \Phi_1(x_2, y_2) \otimes$$

$$\otimes \exp(-ikbx_2/l)\ \Phi_2(x_2, y_2) \otimes$$

$$\left.\otimes \exp(-ikbx_2/l)\ \Phi_3(x_2, y_2)\right\} + \text{c.c.} \quad (5)$$

As shown in Fig. 1*b*, at the stage of reconstruction, the two-exposure hologram is illuminated by the coherent radiation with the convergent quasispherical wave having the radius of curvature $r = R + l$. Spatial filtering of the diffraction field is performed on the optical axis in the plane (x_3, y_3) with an opaque screen p having a round aperture, and the interference pattern located in the hologram plane is recorded. The positive lens L forms the hologram image in the plane (x_4, y_4).

In the approximation used, the distribution of the complex wave amplitude of the coherent radiation used at the stage of hologram reconstruction in the plane (x_2, y_2) takes the following form:

$$u_0(x_2, y_2) \sim \exp\left\{i\left[-\frac{k}{2(R + l)}(x_2^2 + y_2^2) +\right.\right.$$

$$\left.\left.+ \frac{k}{8(R + l)^3}(x_2^2 + y_2^2)^2 + \varphi_0(x_2, y_2)\right]\right\}, \quad (6)$$

where $\varphi_0(x_2, y_2)$ is the deterministic function characterizing possible wave distortions, for example, due to aberrations of the optical system forming it.

Let us consider reconstruction of the two-exposure hologram in different diffraction orders separately, since there is no correlation between their speckle fields.[3] Then, ignoring the spatial boundedness of the field because of the limited size of the hologram, the distribution of the complex field amplitude in the plane (x_3, y_3) for the (+1) diffraction order is determined by the equation

$$u^{(+1)}(x_3, y_3) \sim \iint\limits_{-\infty}^{\infty} \left[\tau_1^{(+1)}(x_2, y_2) + \tau_2^{(+1)}(x_2, y_2)\right] \times$$
$$\times\, u_0(x_2, y_2) \exp\left\{\frac{ik}{2l}\left[(x_2 - x_3)^2 + (y_2 - y_3)^2\right]\right\} \times$$
$$\times \exp\left\{-\frac{ik}{8l^3}\left[(x_2 - x_3)^2 + (y_2 - y_3)^2\right]^2\right\} \mathrm{d}x_2\, \mathrm{d}y_2. \quad (7)$$

Substituting the corresponding functions into Eq. (7) and making use of the integral representation of the convolution operation for the reference wave in $\tau_1^{(+1)}(x_2, y_2)$ and in $\tau_2^{(+1)}(x_2, y_2)$, we obtain

$$u^{(+1)}(x_3, y_3) \sim \exp\left\{ik\left[\frac{1}{2l}(x_3^2 + y_3^2) - \frac{1}{8l^3}(x_3^2 + y_3^2)^2\right]\right\} \times$$
$$\times \Big\{\Phi_0(x_3, y_3) \otimes \Phi_2'(x_3, y_3) \otimes \Phi_3'(x_3, y_3) \otimes$$
$$\otimes\, \Phi_4(x_3, y_3) \otimes \Big\{\Phi_5(x_3, y_3) \exp\left[-\frac{ik}{2\mu_1 l}(x_3^2 + y_3^2)\right] \otimes$$
$$\otimes\, t(x_3, y_3)\, A(x_3, y_3; x_2, y_2) \exp\left[-\frac{ik}{2\mu_1 l}(x_3^2 + y_3^2)\right] \times$$
$$\times \exp\left\{i\left[\varphi(x_3, y_3) + \frac{k}{8}\left(\frac{1}{R^3} + \frac{1}{l^3}\right)(x_3^2 + y_3^2)^2\right]\right\} +$$
$$+\, \Phi_5'(x_3, y_3) \exp\left[-\frac{ik}{2\mu_1 l}(x_3^2 + y_3^2)\right] \otimes t(x_3, y_3) \times$$
$$\times\, A(x_3 - b, y_3; x_2, y_2) \exp\left[-\frac{ik}{2\mu_1 l}(x_3^2 + y_3^2)\right] \times$$
$$\times \exp[i\varphi(x_3 + a, y_3)] \exp\left\{\frac{ik}{8R^3}\left[(x_3 + Rb/l)^2 + y_3^2\right]^2\right\} \times$$
$$\times \exp\left\{\frac{ik}{8l^3}\left[(x_3 - b)^2 + y_3^2\right]^2\right\}\Big\}\Big\}, \quad (8)$$

where

$$A(x_3, y_3; x_2, y_2) = \iiiint\limits_{-\infty}^{\infty} \exp\left[-i\psi_1(x_1, y_1; x_2, y_2)\right] \times$$
$$\times \exp\{ik[(x_1 - x_3)x_2 + (y_1 - y_3)y_2]/l\}\, \mathrm{d}x_1\, \mathrm{d}y_1\, \mathrm{d}x_2\, \mathrm{d}y_2$$

is the complex function, being the result of calculations at each hologram point;

$$\Phi_0(x_3, y_3),\ \Phi_2'(x_3, y_3),\ \Phi_3'(x_3, y_3),\ \Phi_4(x_3, y_3),$$
$$\Phi_5(x_3, y_3),\ \Phi_5'(x_3, y_3)$$

are the Fourier transforms of the functions

$$\exp\left[i\varphi_0(x_2, y_2)\right],\ \exp\left[-\frac{ik}{8l^3}(x_2^2 + y_2^2)^2\right],$$
$$\exp\left[i\psi_2(x_2, y_2; x_3, y_3)\right],\ \exp\left[\frac{ik}{8(R + l)^3}(x_2^2 + y_2^2)^2\right],$$
$$B_1(x_2, y_2) =$$
$$= \exp\left\{-i\left[\varphi(\mu_1 x_2, \mu_1 y_2) + \frac{k\mu_1^4}{8}\left(\frac{1}{R^3} + \frac{1}{l^3}\right)(x_2^2 + y_2^2)^2\right]\right\},$$
$$B_2(x_2, y_2) = \exp\Big\{-i\Big\{\varphi(\mu_1 x_2 + a, \mu_1 y_2) + \frac{k}{8R^3} \times$$
$$\times \left[(\mu_1 x_2 + Rb/l)^2 + \mu_1^2 y_2^2\right]^2 + \frac{k}{8l^3}\left[(\mu_1 x_2 - b)^2 + \mu_1^2 y_2^2\right]^2\Big\}\Big\}$$

with the spatial frequencies $x_3/\lambda l$ and $y_3/\lambda l$; the function $\psi_2(x_2, y_2; x_3, y_3)$ has the form of the function $\psi_1(x_1, y_1; x_2, y_2)$ with the corresponding change of variables.

It follows from Eq. (8) that the light field is an objective speckle field in the plane of formation of the real image of an amplitude screen. In this case, as compared to the diffraction limit determined by the hologram size,[4] the objective speckle is widened by the value determined by the width of the function

$$\Phi_0(x_3, y_3) \otimes \Phi_2'(x_3, y_3) \otimes \Phi_3'(x_3, y_3) \otimes \Phi_4(x_3, y_3) \otimes$$
$$\otimes\, \Phi_5(x_3, y_3) \exp\left[-\frac{ik}{2\mu_1 l}(x_3^2 + y_3^2)\right].$$

Besides, the information on the unknown phase function $\varphi(x_1, y_1)$ is contained, on the one hand, in the limits of each objective speckle in the plane (x_3, y_3), while on the other hand, this phase function modulates the speckle field in this plane.

Coincidence of identical speckles of both exposures in the plane of formation of the real image of an amplitude screen causes location of the interference pattern modulating the objective speckle structure in this plane. This interference pattern has the form

$$I(x_3, y_3) \sim 1 + \cos[\varphi(x_3 + a, y_3) -$$
$$-\, \varphi(x_3, y_3) + \psi(x_3, y_3; b)],$$

where

$$\psi(x_3, y_3; b) = \frac{k}{8}\left[4x_3^3\left(\frac{1}{R^2 l} - \frac{1}{l^3}\right)b + (6x_3^2 + 2y_3^2) \times\right.$$
$$\left.\times \left(\frac{1}{Rl^2} + \frac{1}{l^3}\right)b^2 + 4x_3 y_3^2\left(\frac{1}{R^2 l} - \frac{1}{l^3}\right)b\right].$$

In this case, the spatial size of the interference pattern with high contrast is limited due to the off-axis wave aberrations of the hologram. The diameter of a spot in the plane (x_3, y_3), within which the interference pattern has high contrast, can be estimated using the circumstance that the change of the phase $\psi_1(x_1, y_1; x_2, y_2)$ is the largest at the shift of axis for aberrations like coma.[5]

When performing spatial filtering of the diffraction field on the optical axis (see Fig. 1*b*), if the diameter of the filtering aperture does not exceed the width of an interference fringe, we can assume that $\Phi'_3(x_3, y_3) \simeq \simeq \delta(x_3, y_3)$ taking into account that it is sufficiently small. Then the distribution of the complex field amplitude at the filter exit is determined by the equation

$$u^{(+1)}(x_3, y_3) \sim p(x_3, y_3) \exp\left[\frac{ik}{2l}(x_3^2 + y_3^2)\right] \times$$
$$\times \left\{ \Phi_0(x_3, y_3) \otimes \Phi'_2(x_3, y_3) \otimes \Phi_4(x_3, y_3) \otimes \right.$$
$$\otimes \left\{ [\Phi_5(x_3, y_3) + \Phi'_5(x_3, y_3)] \exp\left[-\frac{ik}{2\mu_1 l}(x_3^2 + y_3^2)\right] \otimes \right.$$
$$\left.\left. \otimes\, t(x_3, y_3) \exp\left[-\frac{ik}{2\mu_1 l}(x_3^2 + y_3^2)\right]\right\}\right\}, \qquad (9)$$

where $p(x_3, y_3)$ is the transmission function of the spatial filter.[6]

Assume that the lens L (Fig. 1*b*) with the focal length f is in the plane (x_3, y_3). Besides, assume for brevity that $f = l/2$ and $l_1 = l$, where l_1 is the spacing between the planes (x_3, y_3) and (x_4, y_4). Then, using the Fresnel approximation, since the allowance for the higher approximation orders leads only to changes in the distribution of subjective speckles in the recording plane *3* of the interference pattern modulating the speckle structure, determine the complex amplitude in the plane (x_4, y_4). Neglecting the factor characterizing the phase distribution of a spherical wave that is insignificant for the further consideration, it takes the form:

$$u^{(+1)}(x_4, y_4) \sim$$
$$\sim \exp\left\{ i\left\{ \varphi_0(-x_4, -y_4) + \frac{k}{8}\left[\frac{1}{(R+l)^3} - \frac{1}{l^3}\right](x_4^2 + y_4^2)^2 \right\}\right\} \times$$
$$\times \left\{ \exp\left[\frac{ik\mu_1}{2l}(x_4^2 + y_4^2)\right.\right. \otimes \left\{ \exp\left\{ -i[\varphi(-\mu_1 x_4, -\mu_1 y_4) + \frac{k\mu_1^4}{8} \times \right.\right.$$
$$\times \left(\frac{1}{R^3} + \frac{1}{l^3}\right)(x_4^2 + y_4^2)^2] \right\} + \exp[-i\varphi(-\mu_1 x_4 + a, -\mu_1 y_4)] \times$$
$$\times \exp\left\{ -\frac{ik\mu_1^4}{8R^3}[(x_4 - Rb/\mu_1 l)^2 + y_4^2]^2 \right\} \times$$
$$\left.\left.\left. \times \exp\left\{ -\frac{ik\mu_1^4}{8l^3}[(x_4 + b/\mu_1)^2 + y_4^2]^2 \right\}\right\}\right\} \times$$
$$\times \left\{ F_1(x_4, y_4) \otimes \exp\left[\frac{ik\mu_1}{2l}(x_4^2 + y_4^2)\right]\right\} \otimes P(x_4, y_4), \quad (10)$$

where $F_1(x_4, y_4)$ and $P(x_4, y_4)$ are the Fourier transforms of the functions $t(x_3, y_3)$ and $p(x_3, y_3)$ with the spatial frequencies $x_4/\lambda l$ and $y_4/\lambda l$.

Since the function

$$\exp[-i\varphi(-\mu_1 x_4, -\mu_1 y_4)] +$$
$$+ \exp[-i\varphi(-\mu_1 x_4 + a, -\mu_1 y_4) - \psi_1(x_4, y_4; \mu_1 b)],$$

where

$$\psi_1(x_4, y_4; \mu_1 b) = \frac{k}{8}\left[4x_4^3\left(\frac{\mu_1^2}{l^3} - \frac{\mu_1^2}{R^2 l}\right)(\mu_1 b) + (6x_4^2 + 2y_4^2) \times \right.$$
$$\left. \times \left(\frac{1}{l^3} + \frac{1}{Rl^2}\right)(\mu_1 b)^2 + 4x_4 y_4^2 \left(\frac{\mu_1^2}{l^3} - \frac{\mu_1^2}{R^2 l}\right)(\mu_1 b)\right],$$

changes slowly with the coordinate, it can be factored out of the convolution integral signs in Eq. (10). Then the distribution of illumination in the plane (x_4, y_4) in the (+1) diffraction order is determined by the equation

$$I^{(+1)}(x_4, y_4) \sim \{1 + \cos[\varphi(-\mu_1 x_4 + a, -\mu_1 y_4) -$$
$$- \varphi(-\mu_1 x_4, -\mu_1 y_4) + \psi_1(x_4, y_4; \mu_1 b)]\} \times$$
$$\times \left| \exp\left\{ i\left\{ \varphi_0(-x_4, -y_4) + \frac{k}{8}\left[\frac{1}{(R+l)^3} - \frac{1}{l^3}\right](x_4^2 + y_4^2)^2 \right\}\right\} \times \right.$$
$$\times \left\{ \exp\left[\frac{ik\mu_1}{2l}(x_4^2 + y_4^2)\right] \otimes \exp\left[-\frac{ik\mu_1^4}{8}\left(\frac{1}{R^3} + \frac{1}{l^3}\right)(x_4^2 + y_4^2)^2\right]\right\} \times$$
$$\left. \times \left\{ F_1(x_4, y_4) \otimes \exp\left[\frac{ik\mu_1}{2l}(x_4^2 + y_4^2)\right]\right\} \otimes P(x_4, y_4) \right|^2 . (11)$$

As follows from Eq. (11), the subjective speckle structure in the plane of hologram image formation is modulated by interference fringes. The interference pattern has the form of a shear interferogram in infinitely wide bands that characterizes the controlled wave front. Due to spherical aberration of the hologram it can be distorted, if the function $\psi_1(x_4, y_4; \mu_1 b)$ is nonzero.

Let the diameter D of the wave front controlled fall within the domain of applicability of the approximation used, that is,

$$2\sqrt[4]{0.8\lambda l^3} \le D(2R + l)/R \le 2\sqrt[6]{1.6\lambda l^5}.$$

Then, to exclude the control error, we can find the maximum permissible value of the lateral shift, which does not exceed the optimal value and for which we can assume that $\psi_1(x_4, y_4; \mu_1 b) = 0$. Starting from the criterion of accuracy in determination of the phase equal to $0.1 \cdot 2\pi$ and its maximum variation on the shift axis, from the equation for the function $\psi_1(x_4, y_4; \mu_1 b)$, the maximum permissible value of the lateral shift is the result of solution of the equation

$$6\left(\frac{1}{l^3} + \frac{1}{Rl^2}\right)(D/2)^2(\mu_1 b)^2 +$$
$$+ 4\left(\frac{\mu_1^2}{l^3} - \frac{\mu_1^2}{R^2 l}\right)(D/2)^3(\mu_1 b) - 0.8\lambda = 0.$$

For the (−1) diffraction order, the distribution of the complex amplitude of the two-exposure field in the plane (x_3, y_3) (see Fig. 1*b*), when $R > l$, takes the form

$$u^{(-1)}(x_3, y_3) \sim \exp\left\{ ik\left[\frac{1}{2l}(x_3^2 + y_3^2) - \frac{1}{8l^3}(x_3^2 + y_3^2)^2\right]\right\} \times$$
$$\times \left\{ \Phi_0(x_3, y_3) \otimes \Phi'_2(x_3, y_3) \otimes \Phi'_3(x_3, y_3) \otimes \Phi_4(x_3, y_3) \otimes \right.$$

$$\otimes \exp\left[-\frac{ik}{4\mu_1 l}(x_3^2+y_3^2)\right] \otimes \left\{\tilde{\Phi}_5(x_3, y_3)\exp\left[\frac{ik}{2\mu_1 l}(x_3^2+y_3^2)\right] \otimes\right.$$

$$\otimes\, t(-x_3, -y_3)\, A^*(-x_3, -y_3; x_2, y_2)\exp\left[\frac{ik}{2\mu_1 l}(x_3^2+y_3^2)\right] \times$$

$$\times \exp\left\{-i\left[\varphi(-x_3, -y_3) + \frac{k}{8}\left(\frac{1}{R^3}+\frac{1}{l^3}\right)(x_3^2+y_3^2)^2\right]\right\} +$$

$$+\, \tilde{\Phi}'_5(x_3, y_3)\exp\left[\frac{ik}{2\mu_1 l}(x_3^2+y_3^2)\right] \otimes t(-x_3, -y_3) \times$$

$$\times A^*(-x_3 - b, -y_3; x_2, y_2)\exp\left[\frac{ik}{2\mu_1 l}(x_3^2+y_3^2)\right] \times$$

$$\times \exp[-i\varphi(-x_3+a, -y_3)]\exp\left\{-\frac{ik}{8R^3}[(x_3 - Rb/l)^2 + y_3^2]^2\right\} \times$$

$$\left.\times \exp\left\{-\frac{ik}{8l^3}[(x_3+b)^2+y_3^2]^2\right\}\right\}\Bigg\}, \qquad (12)$$

where $\tilde{\Phi}_5(x_3, y_3)$ and $\tilde{\Phi}'_5(x_3, y_3)$ are the Fourier transforms of the functions $B_1^*(x_2, y_2)$ and $B_2^*(x_2, y_2)$ with the spatial frequencies $x_3/\lambda l$ and $y_3/\lambda l$.

Taking into account that the function $\exp[-i\varphi \times (-x_3, -y_3)] + \exp\{-i[\varphi(-x_3+a, -y_3) + \psi(x_3, y_3; -b)]\}$ varies slowly with the coordinate as compared with the function $\exp\left[-\frac{ik}{4\mu_1 l}(x_3^2+y_3^2)\right]$ and the assumption that within the diameter of the filtering aperture

$$\varphi(-x_3 + a, -y_3) - \varphi(-x_3, -y_3) + \psi(x_3, y_3; -b) \le \pi,$$

the distribution of the complex field amplitude at the exit from the spatial filter is determined by the equation

$$u^{(-1)}(x_3, y_3) \sim p(x_3, y_3)\exp\left[\frac{ik}{2l}(x_3^2+y_3^2)\right] \times$$

$$\times \left\{\Phi_0(x_3, y_3) \otimes \Phi'_2(x_3, y_3) \otimes \Phi_4(x_3, y_3) \otimes\right.$$

$$\otimes \exp\left[-\frac{ik}{4\mu_1 l}(x_3^2+y_3^2)\right] \otimes \left\{[\tilde{\Phi}_5(x_3, y_3) + \tilde{\Phi}'_5(x_3, y_3)] \times\right.$$

$$\left.\left.\times \exp\left[\frac{ik}{2\mu_1 l}(x_3^2+y_3^2)\right] \otimes t(-x_3, -y_3)\exp\left[\frac{ik}{2\mu_1 l}(x_3^2+y_3^2)\right]\right\}\right\}. \qquad (13)$$

As for calculations of the distribution of the complex field amplitude in the plane (x_4, y_4) (see Fig. 1b) in the (+1) diffraction order, let us use the Fresnel approximation. Then based on Eq. (13) in the (−1) diffraction order the distribution of the complex field amplitude in this plane takes the form

$$u^{(-1)}(x_4, y_4) \sim$$

$$\sim \exp\left\{i\left\{\varphi_0(-x_4, -y_4) + \frac{k}{8}\left[\frac{1}{(R+l)^3} - \frac{1}{l^3}\right](x_4^2+y_4^2)^2 +\right.\right.$$

$$\left.\left.+ \frac{k\mu_1}{l}(x_4^2+y_4^2)\right\}\right\}\left\{\exp\left[-\frac{ik\mu_1}{2l}(x_4^2+y_4^2)\right] \otimes\right.$$

$$\otimes \left\{\exp\left\{i\left[\varphi(-\mu_1 x_4, -\mu_1 y_4) + \frac{k\mu_1^4}{8}\left(\frac{1}{R^3}+\frac{1}{l^3}\right)(x_4^2+y_4^2)^2\right]\right\} +\right.$$

$$+ \exp[i\varphi(-\mu_1 x_4 + a, -\mu_1 y_4)] \times$$

$$\times \exp\left\{\frac{ik\mu_1^4}{8R^3}[(x_4 - Rb/\mu_1 l)^2 + y_4^2]\right\}^{2}) \times$$

$$\left.\left.\times \exp\left\{\frac{ik\mu_1^4}{8l^3}[(x_4 + b/\mu_1)^2 + y_4^2]^2\right\}\right\}\right\} \times$$

$$\times \left\{F_2(x_4, y_4) \otimes \exp\left[-\frac{ik\mu_1}{2l}(x_4^2+y_4^2)\right]\right\} \otimes P(x_4, y_4), \qquad (14)$$

where $F_2(x_4, y_4)$ is the Fourier transform of the function $t(-x_3, -y_3)$ with the spatial frequencies $x_4/\lambda l$ and $y_4/\lambda l$.

Based on Eq. (14) and the above statements, in calculating $I^{(+1)}(x_4, y_4)$, the illumination distribution in the plane (x_4, y_4) in the (−1) diffraction order is described by the equation

$$I^{(-1)}(x_4, y_4) \sim \{1 + \cos[\varphi(-\mu_1 x_4 + a, -\mu_1 y_4) -$$

$$- \varphi(-\mu_1 x_4, -\mu_1 y_4) + \psi_1(x_4, y_4; \mu_1 b)]\} \times$$

$$\times \left| \exp\left\{i\left\{\varphi_0(-x_4, -y_4) + \frac{k}{8}\left[\frac{1}{(R+l)^3} - \frac{1}{l^3}\right](x_4^2+y_4^2)^2 +\right.\right.\right.$$

$$\left.\left.+ \frac{k\mu_1}{l}(x_4^2+y_4^2)\right\}\right\}\left\{\exp\left[-\frac{ik\mu_1}{2l}(x_4^2+y_4^2)\right] \otimes\right.$$

$$\left.\otimes \exp\left[\frac{ik\mu_1^4}{8}\left(\frac{1}{R^3}+\frac{1}{l^3}\right)(x_4^2+y_4^2)^2\right]\right\} \times$$

$$\left.\times \left\{F_2(x_4, y_4) \otimes \exp\left[-\frac{ik\mu_1}{2l}(x_4^2+y_4^2)\right]\right\} \otimes P(x_4, y_4)\right|^2. \qquad (15)$$

A feature of the interference pattern located in the hologram plane and corresponding to the (−1) diffraction order is that its spatial size is twice as small as in the (+1) order. This follows from Eqs. (10) and (14), according to which the light field is nonzero within the domain of existence of the functions $\exp[-ik\mu_1[(x_4^2+y_4^2)/2l] \otimes t(-\mu_1 x_4, -\mu_1 y_4)$ for the (+1) diffraction order and $\exp[ik\mu_1[(x_4^2+y_4^2)/l] \otimes \otimes\, t(-2\mu_1 x_4, -2\mu_1 y_4)\exp[ik\mu_1[(x_4^2+y_4^2)/l]$ for the (−1) order. For the spatial size of the interference pattern in the (−1) diffraction order to correspond to its size in the (+1) order, at the stage of hologram reconstruction it is necessary to illuminate the screen by a coherent radiation of a converging quasispherical wave having the radius of curvature $r < (R + l)l/R$. In this case, it should be kept in mind that in the plane of formation of the real image of the amplitude screen spaced by $l' < l(R + l)/2R$ from the hologram, the scale of the interference pattern decreases. In this connection, it is necessary to decrease the maximum allowed diameter of the filtering aperture, and this leads to an increase of the subjective speckle size in the recording plane of the interference pattern. As a results, the visibility of the interference pattern may decrease down to zero, when the speckle size becomes comparable with the width of an interference fringe.[7]

If $R < l$ at the stage of recording the two-exposure hologram, then the phase distribution of the convergent spherical wave is present at the stage of its reconstruction with a coherent radiation of a converging quasispherical wave having the radius of curvature $r = R + l$ in the distribution of the complex field amplitude in the (−1) diffraction order at the hologram output. As a result, the spatial size of the interference pattern recorded in the (−1) diffraction order corresponds to its size in the (+1) diffraction order if performing the spatial filtering of the diffraction field at the distance l from the hologram.

Converging quasispherical wavefront

In the case of a two-exposure recording of the hologram with the amplitude screen illuminated by a coherent radiation with a converging quasispherical wave and $R > l$, to reconstruct the hologram, coherent radiation of a diverging quasispherical wave having the radius of curvature $r = R - l$ is used.[1] Spatial filtering of the diffraction field on the optical axis in the plane of formation of the real image of the amplitude screen allows the recording of the interference pattern located in the hologram plane and characterizing the wave front controlled to be done. For the used order of approximation, when the diameter of the wave front controlled obeys the condition

$$2\sqrt[4]{0.8\lambda l^3} \le D(2R - l)/R \le 2\sqrt[6]{1.6\lambda l^5},$$

the interference pattern modulating the subjective speckle structure has the form

$$I(x_4, y_4) \sim 1 + \cos[\varphi(-\mu_2 x_4 + a, -\mu_2 y_4) - \varphi(-\mu_2 x_4, -\mu_2 y_4) + \psi_2(x_4, y_4; \mu_2 b)],$$

where $\mu_2 = R/(R - l)$ is the scale factor;

$$\psi_2(x_4, y_4; \mu_2 b) = \frac{k}{8}\left[4x_4^3\left(\frac{\mu_2^2}{R^2 l} - \frac{\mu_2^2}{l^3}\right)(\mu_2 b) + (6x_4^2 + 2y_4^2)\left(\frac{1}{Rl^2} - \frac{1}{l^3}\right)(\mu_2 b)^2 + 4x_4 y_4^2\left(\frac{\mu_2^2}{R^2 l} - \frac{\mu_2^2}{l^3}\right)(\mu_2 b)\right]$$

is the phase function caused by spherical aberration of the hologram, which determines the wave front control error.

As in the case with the control of a diverging quasispherical wave front for $R > l$, the spatial size of the interference pattern recorded in the (−1) diffraction order is twice as small as in the (+1) order. For the spatial size of the interference pattern located in the hologram plane in the (−1) diffraction order to correspond to its size in the (+1) order, at the stage of hologram reconstruction, it should be illuminated by a coherent radiation of a converging quasispherical wave having the radius of curvature $r < (R - l)l/R$. Then the spatial filtering of the diffraction field should be performed in the plane of formation of the reduced real image of the amplitude screen spaced by $l'' \le l(R - l)/2R$ from the hologram.

If double-exposure recording of the hologram is performed when $R < l$, then it should be reconstructed with a coherent radiation of a converging quasispherical wave having the curvature length $r = l - R$. Then at the distance l from the hologram, where the real image of the amplitude screen is formed with unit magnification, spatial filtering of the diffraction field is performed on the optical axis and the interference pattern located in the hologram plane and characterizing the wave front controlled is recorded. For the approximation order used, when the diameter of the wave front controlled satisfies the condition

$$2\sqrt[4]{0.8\lambda l^3} \le Dl/R \le 2\sqrt[6]{1.6\lambda l^5},$$

the interference pattern modulating the subjective speckle structure has the form

$$I(x_4, y_4) \sim 1 + \cos[\varphi(\mu_3 x_4 + a, \mu_3 y_4) - \varphi(\mu_3 x_4, \mu_3 y_4) + \psi_3(x_4, y_4; \mu_3 b)],$$

where $\mu_3 = R/(l - R)$ is the scaling coefficient;

$$\psi_3(x_4, y_4; \mu_3 b) = \frac{k}{8}\left[4x_4^3\left(\frac{\mu_3^2}{l^3} - \frac{\mu_3^2}{R^2 l}\right)(\mu_3 b) + (6x_4^2 + 2y_4^2)\left(\frac{1}{l^3} - \frac{1}{Rl^2}\right)(\mu_3 b)^2 + 4x_4 y_4^2\left(\frac{\mu_3^2}{l^3} - \frac{\mu_3^2}{R^2 l}\right)(\mu_3 b)\right]$$

is the phase function caused by spherical aberration of the hologram, which determines the wave front control error. Besides, in the case considered, the spatial size of the interference pattern is the same in the (+1) and (−1) diffraction orders. This is explained by the fact that in the (−1) diffraction order the reduced real image of the amplitude screen is formed at the distance $l(l - R)/2R < l$ from the hologram.[8]

If the spatial filtering of the diffraction field is performed in this plane, then the maximum permissible diameter of the filtering aperture should be decreased and, besides, in the recording plane of the interference pattern the spatial size of the area caused by the constant component of the hologram transmission increases.

Thus, the results of this analysis have shown that the two-exposure record of the hologram of the amplitude screen by the Gabor scheme for the wave front control is accompanied by control errors because of spherical aberration of the hologram, when the wave front diameter increases. In the third-order approximation, for the complex amplitude of the field it is possible to determine the systematic error, whose value depends on the hologram recording geometry, lateral shift, and wavelength of radiation used for recording and reconstruction of the hologram. In this case, at the known diameter of the wave front controlled it is possible to determine the maximum permissible value of the lateral shift, which does not exceed the optimal value and permits the control error to be excluded.

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