Statistics of photocounts of laser radiation passed through the turbulent atmosphere for non-Gaussian radiation fields

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The results of experimental and theoretical research of statistical and correlation characteristics of laser radiation having passed through a turbulent medium are presented. The relation between the correlation function and the variance of photocounts was established. It was shown that the character of the integral intensity distribution is independent of the sampling time.

Atmospheric turbulence has a significant effect on propagation of optical signals and causes random distortion of radiation beams, as well as random modulation of the signal phase and intensity, thus giving rise to field fluctuations at the receiving point. Field fluctuations caused by atmospheric turbulence considerably deteriorate the quality of information obtained with ground-based radiation receivers and thus restrict the efficiency of laser systems used in optical communication, detection and ranging, and sensing of the atmosphere.[1,2] Therefore, for optimization and determination of the efficiency of laser systems, it is necessary to study laser radiation fluctuations and noise in order to construct their statistical model. Laser radiation in the visible region is usually recorded with photomultiplier tubes (PMT). In this case, the main statistical characteristic is the probability distribution of photocounts (PDP) for the sampling time $T - P(n, T)$.

The probability distribution of photocounts has been studied quite thoroughly for the case of Gaussian field in the reception plane.[2] The result of this study was the development of a number of approximate solutions used depending on the relation between T and τ_c, where τ_c is the correlation time of intensity fluctuations of laser radiation in the atmosphere. In some cases of laser system operation, the field distribution in the reception plane is non-Gaussian. For non-Gaussian fields, PDP are known only in the asymptotic cases of $T \ll \tau_c$ and $T \gg \tau_c$. Thus, for example, introducing the condition of the lognormal intensity distribution

$$\omega(I) = \frac{1}{\sqrt{2\pi}\,\sigma I} \exp\left\{-\left[\ln\frac{I}{I_0} + \frac{\sigma}{2}\right] \Big/ 2\sigma^2\right\}^2, \quad (1)$$

where I_0 is the radiation intensity in the absence of turbulence and

$$\sigma^2 = \left\langle \left(\ln\frac{I}{I_0}\right)^2 \right\rangle - \left\langle \ln\frac{I}{I_0} \right\rangle^2, \quad (2)$$

PDP at $T \ll \tau_c$ can

$$P(n, \langle n\rangle, \sigma) = \frac{M^n\, e^{-M}}{n!} \frac{\exp\,[-0.5\,\sigma^2\,(M-n)^2]}{[1+\sigma^2 M]^{1/2}}, \quad (3)$$

where for every n w be described by the Diament–Teich distribution e have to determine M, solving the transcendent equation of the following form:

$$\ln M = \ln\langle n\rangle + \sigma^2\,(n - M - 0.5).$$

In the inverse asymptote $T \gg \tau_c$, PDP is the Poison distribution. In a wide range of the sampling time between the asymptotes $T \ll \tau_c$ and $T \gg \tau_c$ and at the non-Gaussian field distribution, PDP of laser radiation passed through the turbulent atmosphere is almost unstudied because of cumbersome mathematical calculations and instability of the atmosphere during the time needed to conduct an experiment.

In this paper, we present some results of experimental investigations of PDP at the sampling time roughly equal to the correlation time of radiation intensity. The experiments were conducted under laboratory conditions, and this allowed us to provide for the steady state of the path and to monitor the main path parameters to the extent that is inaccessible in field atmospheric experiments.

The schematic diagram of the experimental setup is shown in Fig. 1. The radiation from an LG-38 laser at the wavelength of 0.63 μm passes along a short path with intense air turbulence induced by a heating system and comes to a photodetector. The radiation intensity is regulated with the use of neutral density and polarization filters. A collimator and a diaphragm of 0.4-mm diameter provide for the "point sampling" mode. The optical radiation was detected with a FEU-147-3 PMT operated in the photon counting mode. Single-electron PMT pulses were standardized on an amplifier-shaper and then sent to a counter. The number of signals accumulated by the counter for the sampling time was entered into a computer responsible for control of the experiment and processing of information. To exclude the effect

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of variation of the signal-to-noise ratio on the results and to improve the experimental reliability at the fixed intensity I = const, we have studied PDP at different values of the sampling time T. In this connection, for estimation of the distribution characteristics, we used normalized parameters, namely, the relative variance of the number of photocounts β_n^2 and the asymmetry coefficient K_{asym}.

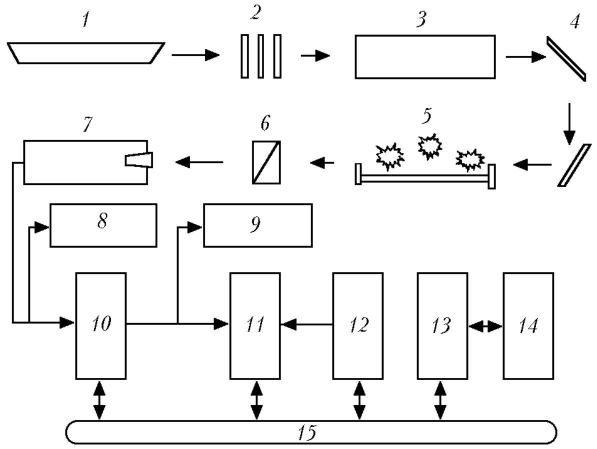

Fig. 1. Experimental setup: laser *1*, neutral density filters *2*, collimator *3*, reflector *4*, heater *5*, polarization filters *6*, PMT with a diaphragm *7*, oscilloscope *8*, frequency meter *9*, amplifier-shaper *10*, counter *11*, timer *12*, CAMAC crate controller *13*, computer *14*, CAMAC data bus *15*.

The studies were conducted for three different states of the induced turbulence, which, judging from the values of β_n^2, corresponded to the weak and moderate turbulence. This conclusion was based on the following reasoning: at $T \ll \tau_c$ $\beta_n^2 = \beta^2$, where β^2 is the variance of relative intensity fluctuations.[3] On the other hand, in the absence of saturation $\beta^2 \approx \beta_0^2$, where $\beta_0^2 = 1.23\, C_n^2 k^{7/6} z^{11/6}$ is the parameter characterizing the turbulence along the path; C_n^2 is the structure characteristic of the refractive index; k is the wave number; z is the path length. The experimental values of β_n^2 did not exceed 1.5, and the control experiments showed no clear saturation of β_n^2 values, thus suggesting that the induced turbulence corresponded to regions with weak and moderate turbulence in real atmosphere.

For every state of the path turbulence, PDP was determined at 10 values of the sampling time. The number of photocounts in each experiment on PDP determination was ~10^6, thus providing for sufficient statistical averaging. In order to exclude random errors and provide for sufficient accuracy, PDP was determined ten times for every sampling time. The parameters characterizing the distribution of photocounts were determined through averaging over all realizations at the same value of the sampling time.

During the experiment for one state of the induced turbulence at every value of the sampling time, a sequence of 256 readouts was recorded with the sampling time less than the presumed correlation time. This sequence was used to determine the autocorrelation function of photocounts, which at $T \ll \tau_c$ can be believed equivalent to the correlation function of intensity. Thus, for one state of turbulence the correlation function was determined ten times with different time intervals.

Figure 2 depicts the correlation functions of photocounts averaged over all realizations for three different states of the induced turbulence.

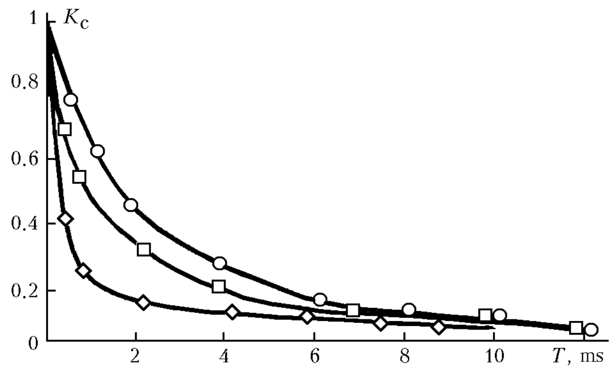

Fig. 2. Correlations functions of photocounts K_c at different states of turbulence: averaged values (solid line), standard deviations (signs) at τ_c = 3.63 (○), 2.48 (□), and 0.68 ms (◊).

The correlation time of intensity was thought equal to the average value of the experimentally obtained correlation times of photocounts. The standard deviation for the correlation time of intensity did not exceed 7% for every state of turbulence along the path, which is indicative of a sufficiently steady-state character of optical characteristics of the propagation path.

Careful analysis of numerous experimental data obtained with the experimental setup described suggested that the Diament–Teich distribution can be applied to description of experimental PDP under the condition that the parameter σ^2 in the Diament–Teich distribution depends on the sampling time. Indeed, as the sampling time increases from $T \ll \tau_c$ to $T \approx \tau_c$, the averaging time for intensity increases as well, and the variance of intensity fluctuations in every sample decreases.

Thus, σ^2 decreases with the increasing sampling time T, and the law of this decrease depends on the form of the correlation function of intensity fluctuations. It should be noted that if the condition $T \ll \tau_c$ is not fulfilled, σ^2 loses the meaning of the variance of the relative log intensity and acquires the meaning of the variance of the relative log integral (averaged over the sampling time) intensity $\sigma_{\ln U}$. To find the relation between $\sigma_{\ln U}$ and experimentally determinable parameters, let us use the equation for determination of distribution moment. Taking into account that the distribution of the integral intensity is lognormal, we have

$$\langle U\rangle = U_0,\ \langle U^2\rangle = U_0^2 \exp(\sigma^2_{\ln U})$$

and

$$\sigma^2_U = U_0^2 [\exp(\sigma^2_{\ln U}) - 1)],$$

where U_0 is the integral intensity of radiation in the absence of turbulence. Consequently,

$$\sigma^2_{\ln U} = \ln(1 + \sigma^2_U/\langle U\rangle^2). \quad (4)$$

On the other hand, according to the Mandel's formula,[4] the variance of photocounts can be written as

$$\sigma^2_n = \langle n\rangle + \sigma^2_U. \quad (5)$$

Taking into account that $\langle n\rangle = \langle U\rangle$ and the normalized variance of photocounts is

$$\beta^2_n = (\sigma^2_n - \langle n\rangle)/\langle n\rangle^2,$$

from Eqs. (4) and (5) we have

$$\sigma^2_{\ln U} = \ln(1 + \beta^2_n). \quad (6)$$

Consequently, having determined the value of β^2_n from experimental PDP, we can find the variance of integral intensity. If we now substitute the obtained value of $\sigma^2_{\ln U}$ and the mean number of photocounts from the corresponding experiment into the Diament–Teich distribution, then it will describe the experimental distribution of photocounts obtained at $T \approx \tau_c$ with high accuracy. It should be noted that $\sigma^2_{\ln U} \to \sigma^2$ at $T \to 0$.

The dependence of K_{asym} of the experimental distributions on sampling time at $T \approx \tau_c$ is plotted in Fig. 3.

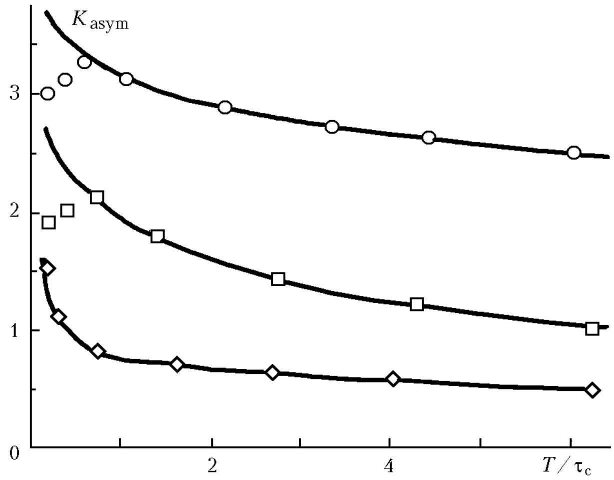

Fig. 3. Dependence of the asymmetry coefficients on the sampling time. The signs are the same as in Fig. 2.

The solid curve corresponds to the characteristics of the Diament–Teich distribution obtained through numerical calculations from Eq. (3) using Eq. (6) with the values of $\langle n\rangle$ equal to the experimental one. As seen from Fig. 3, at $\tau_c = 0.68$ ms (corresponding to the weakest turbulence in our experiment) the values of K_{asym} obtained from the Diament–Teich distribution describe the experimental results very well. In the case of $\tau_c = 2.48$ ms and, especially, at $\tau_c = 3.63$ ms the asymmetry coefficients obtained from the Diament–Teich distribution at $T/\tau_c \le 1$ widely differ from the experimental values. The increase of the sampling time leads to a decrease of this discrepancy and at $T/\tau_c \ge 1$ the discrepancy between the calculated and experimental data is within the experimental uncertainty. The further increase of T leads to a close agreement between the experimental and calculated data.

This discrepancy at $T/\tau_c \le 1$ is explained by the effect of cutoff of experimental distributions. Because of some technical restrictions, some information is usually lost in the experimental distribution due to the finite total number of samples. Samples with low probability (usually they are the samples with large number of photocounts) usually have no time to manifest themselves during the sampling time, but the probability of recording of these samples is nonzero. In this connection, the experimental distribution turns out to be cutoff. The more pronounced is the cutoff effect, the longer is the tail of the studied distribution. Thus, the distributions, in which the probabilities of large numbers decrease more slowly, are subject to larger cutoff in the experiments.

Numerical calculations have no such restrictions. In our experiment at the sampling time $T/\tau_c \le 1$ the effect of turbulence turns out to be stronger, the distributions have more flat tails, and the cutoff effect is stronger. As the sampling time T increases, intensity fluctuations become smoother, the probabilities of large numbers of photocounts decrease far faster, and thus introduced distortions decrease. At $\tau_c = 0.68$ ms the effect of cutoff on the experimental data did not show itself at neither $T/\tau_c \le 1$ nor at $T/\tau_c \ge 1$ because of small intensity fluctuations. To check the correctness of this reasoning, summation in numerical calculations when determining moments was restricted to the channel with the number N_{max}, where N_{max} is the maximum number of a channel in experimental PDP, in which a nonzero count was recorded. In this case, the values of K_{asym} agreed well with the values obtained from the experimental distributions.

To finally check the possibility of applying the Diament–Teich distribution to description of the statistics of photocounts of laser radiation passed through the turbulent atmosphere at the sampling time comparable with the correlation time of intensity fluctuations, we decided to determine the distribution of the integral intensity in our experiment.

The distribution of the integral intensity can be obtained from experimental PDP[4] by solving the inverse problem in the form

$$\omega(x) = \sum_{n=0}^{\infty} a_n(x) P(n, N), \quad (7)$$

where $x = U/\langle U \rangle$ and

$$a_n(x) = 2N(-2)^n \sum_{m=n}^{\infty} \binom{m}{n} l_m(2Nx) =$$

$$= 2N(-2)^n \sum_{k=0}^{\infty} \binom{n+k}{n} l_{n+k}(2Nx).$$

Here $l_n(\nu)$ is the Laguerre function; $P(n, N)$ is the experimental distribution of photocounts with the mean equal to N. Thus, solving Eq. (7) numerically, we obtain the distribution histogram of the integral intensity corresponding to one specific realization of PDP. Analysis of numerous distributions of the integral intensity obtained by the method described above showed[5] that they all are described by the lognormal distribution with the corresponding parameter $\sigma^2_{\ln U}$, which can be determined from experimental PDP using Eq. (6). Thus, we see that the character of distribution of the integral intensity keeps unchanged. As the sampling time changes, only the value of $\sigma^2_{\ln U}$ varies. The lognormal character of the integral intensity distribution confirms our assumption on the possibility of applying the Diament–Teich distribution to description of experimental PDP at $T \approx \tau_c$, since this distribution was obtained just at the lognormal intensity fluctuations.

Conservation of the character of the integral intensity distribution makes one realization of the studied process far more valuable. Indeed, in this case the distributions of photocounts are fully determined by the mean number of photocounts, which is directly related to the radiation intensity, and the correlation function intensity, since it is related to the normalized variance of photocounts as:

$$\beta_n^2(T) = \frac{2}{T^2} \int_0^T (T - \tau)\, \rho(\tau)\, d\tau. \qquad (8)$$

Thus, a long experiment aimed at obtaining the distribution of photocounts reduces to measurement of the correlation function of intensity fluctuations $\rho(\tau)$. Then, if the characteristics of the path and the initial radiation are stable enough, the distribution of photocounts in a wide range of the sampling time, including the sampling time comparable with the correlation time, can be calculated using the Diament–Teich distribution and the relation between the correlation function and the normalized variance of photocounts. Figure 4 depicts the dependence of β_n^2 on the sampling time that was obtained from the correlation function (solid curve) and from PDP measurements for the corresponding sampling times (dots) for three states of turbulence realized in our experiment. From Fig. 4 one can see quite close agreement between the curves obtained in different ways. It should be noted that Eq. (8) was integrated numerically, since we failed to describe experimental correlation functions by elementary functions.

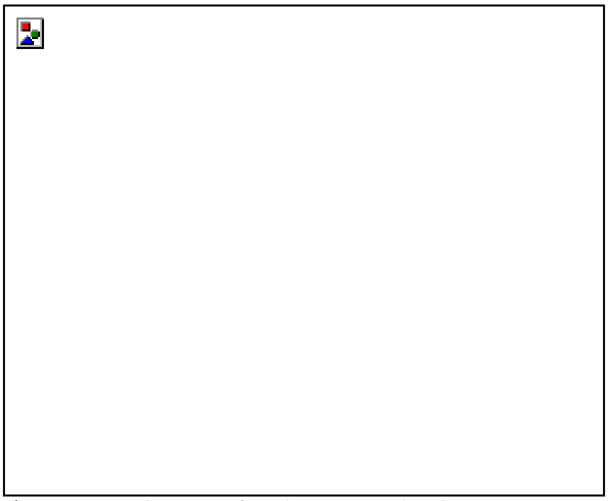

Fig. 4. Dependence of the normalized variance of photocounts on the sampling time. Signs are the same as in Fig. 2.

Thus, given the lognormal intensity distribution and the characteristics of the path and radiation intensity stable, measurement of only one correlation function of intensity gives complete information about statistical characteristics of photocounts of laser radiation passed through the turbulent atmosphere in a wide range of sampling times, including $T \approx \tau_c$. This fact allows the comprehensive information on the statistics of photocounts of laser radiation to be obtained during a short time, when the state of the atmosphere does not change significantly.

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