Stability of algorithms of phase and amplitude-phase beam control in a nonlinear medium

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Beam control under thermal blooming conditions based on phase conjugation and phase-conjugate reflection algorithms is considered. The stability of these two methods for correction of nonlinear distortions is analyzed. Instability of phase conjugation is shown to be due to violation of the optical reciprocity principle. Rigorous fulfillment of this principle allows achieving the stable correction. The phase-conjugate reflection in nonlinear medium is also accompanied by violation of this principle, and this leads to development of oscillations in parameters of the light field on a focusing object.

As known, compensation for nonlinear distortions of laser radiation performed based on the phase conjugation algorithm is unstable. Hermann, the author of one of the first papers[1] devoted to this problem, mentioned that, starting from some value of the threshold power, application of this algorithm leads to increasing divergence of the beam to be corrected, and this hampers the increase of the light field concentration in the observation plane. Along with the divergence, instability of the oscillating type,[2,4,5] that is, oscillation of the radiation parameters, is observed. The mechanism of development of these oscillations has been described in Ref. 3, where it is believed that the development of self-sustained oscillations is explained by the break of the feedback loop in the adaptive system.

A "strong" defocusing lens on the path causes the feedback break and decreases the field concentration on a focusing object. With time, the local thermal lens moves along the wind, and once it goes beyond the beam the feedback is restored and the radiation parameters in the observation plane increase. More detailed analysis of the process of control, whose results are presented in this paper, allows us to conclude that under conditions considered it is incorrect to speak about the break of the feedback in the system. The oscillations are likely caused by violation of the optical principle of reciprocity, which forms the basis for the phase conjugation algorithm.

Laser radiation propagation in a weakly absorbing homogeneous medium is described by the equation of quasi-optics[6]

$$2ik\frac{\partial E}{\partial z} = \Delta_\perp E + R_0 T E \qquad (1)$$

and the heat transfer equation

$$\rho C_p\left(\frac{\partial T}{\partial t} + V_x\frac{\partial T}{\partial x} + V_y\frac{\partial T}{\partial y}\right) = \alpha I,\ I = \frac{cn_0}{8\pi}EE^*. \qquad (2)$$

Equations (1) and (2) use the following designations: $E = E(x, y, z, t)$ for the complex amplitude of the light field; $T = T(x, y, z, t)$ for the temperature distribution; n_0 and α for the unperturbed values of the refractive index and the extinction coefficient, respectively; ρ for the density; C_p for the specific heat; V_x and V_y for the projections of the wind velocity onto the coordinate axes; z for the propagation direction; $k = 2\pi/\lambda$ for the wave number, λ for the wavelength. The main similarity criterion of the system of equations (1) and (2) – the nonlinearity parameter R_0, characterizes the refractive properties of the induced thermal lens and can be determined from the mean wind velocity V_0:

$$R_0 = \frac{2k^2 a_0{}^2 \alpha I_0}{n_0 \rho C_p V_0}\frac{\partial n}{\partial T}. \qquad (3)$$

The radius of the transmitting aperture a_0 is the spatial scale of the problem in the plane normal to the propagation direction $x' = x/a_0$, $y' = y/a_0$, while the diffraction length $Z_d = ka_0^2$ is the spatial scale in the propagation direction, $z' = z/Z_d$. To characterize the field on the object, the focusing criterion

$$J(x, y, z_0, t) = \frac{1}{P_0}\iint S(x, y) I(x, y, z_0, t)\,dx\,dy, \qquad (4)$$

proportional to the power density of the light field is used. Here P_0 is the total power of the beam; $S(x, y) = \exp(-(x^2 + y^2)/a_0^2)$ is the aperture function; $I(x, y, z_0, t)$ is the beam intensity in the observation plane.

The distributed thermal lens is modeled by a set of distorting phase screens set in the beam propagation path. To determine the characteristic peculiarities of the correction, the problem geometry was simplified and only one screen was placed in the path just at the center of the distance from the source aperture to the observation plane (Fig. 1).

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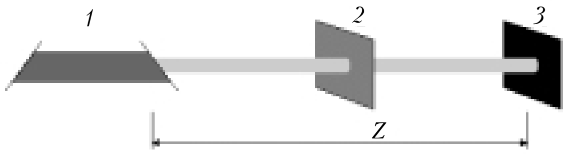
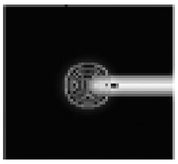
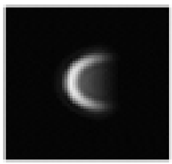
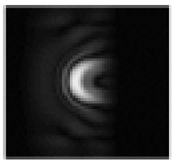
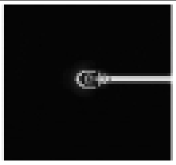
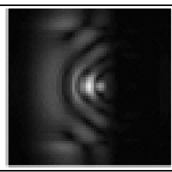
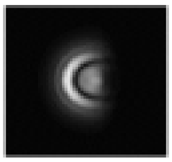
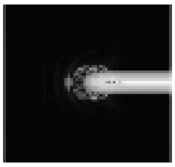
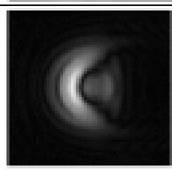
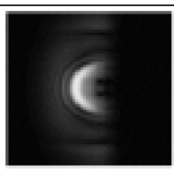
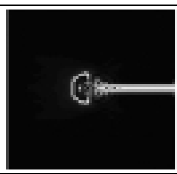
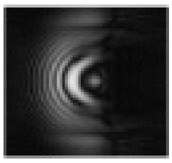
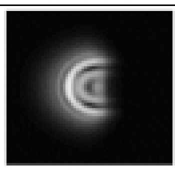
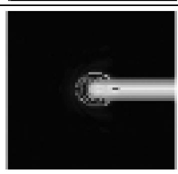
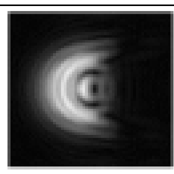
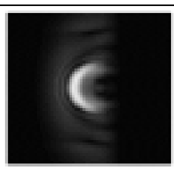
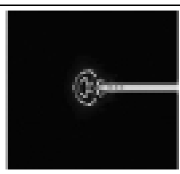
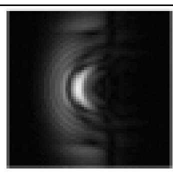
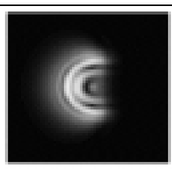
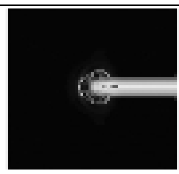
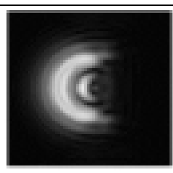


Fig. 1. Numerical experiment on compensation for thermal blooming: laser source *1*, phase screen *2*, and observation plane *3*.

Beam control with the use of the phase conjugation and phase-conjugate reflection (PCR) algorithms is considered in the approximation of the stationary refraction, that is, changes to the phase profile (amplitude or phase profile in the case of PCR) of radiation are introduced with the periods much longer than the duration of transient processes developing in the "beam—nonlinear medium" system. The computer monitor displays the intensity distribution of the radiation to be corrected in the observation plane and on the phase screen, the temperature distribution on the screen, as well as the intensity of the reference radiation in the laser aperture plane. The results obtained in the case of phase conjugation are given in Table 1 (one screen on the path).

Table 1. Distribution of the light and temperature fields in the phase conjugation algorithm (one distorting screen is placed at the center of the propagation path)

Iteration number	Reference beam intensity distribution	Intensity and temperature distribution on the screen *	Intensity distribution in the observation plane	J
1				0.13
2				0.15
3				0.06
4				0.07
5				0.60
6				0.11
7				0.06

*Gray-scale image is for temperature; lines of different level are for the beam.

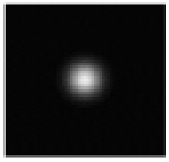
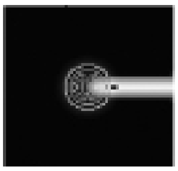
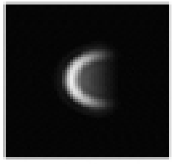
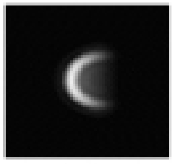
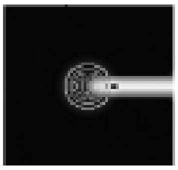
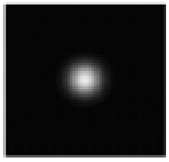
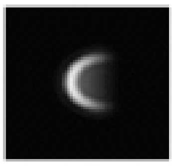
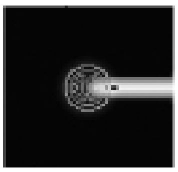
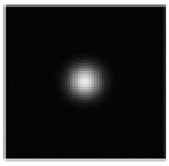
At the first iteration, the initial phase profile of the beam is plane. At the second iteration, the beam focuses due to phase conjugation applied, its radius on the distorting screen decreases, and the field concentration increases. As a result, the criterion $J(t)$ recorded in the object plane increases from 0.13 to 0.15. However, the size of the thermal lens decreases simultaneously, and therefore the reference beam at the third iteration is less defocused. The focusing radius of the beam to be corrected decreases (as can be seen from the increase of the beam diameter on the phase screen and decrease of $J(t)$ down to 0.06 at the third iteration), and the size of the thermal lens increases. Then we again can see the increase in defocusing of the reference radiation causing the increase in focusing of the direct beam. Self-sustained oscillations are actually developed in the system, but it is likely incorrect to speak about the break of the feedback.

At realization of phase conjugation in the nonlinear medium, the optical principle of reciprocity is violated twice. First, it happens because only the beam phase is controlled, and the amplitude at the entrance into the medium differs from the amplitude of the reference radiation (this issue is considered in a more detail in Ref. 7). The second cause is that the reference beam and the beam to be corrected propagate through the path with different inhomogeneities, since the change of the phase profile of the beam to be corrected leads to the change of the temperature distribution and the refractive index of the propagation path. To demonstrate that the stability of optical correction increases if the conditions of optical reciprocity are fulfilled, the following numerical experiment was conducted. The phase-conjugate reflection was conducted for the beam propagating along the path with one phase screen placed at its center. The monitor displayed the same data as in the case of phase conjugation. The results are summarized in Table 2 (one screen).

In this case, the process converges to the maximum value of the criterion ($J = 0.5$) after a single iteration. At the first step, the beam on the distorting screen is almost the same as in the plane of the emitting aperture (slight increase of the radius due to diffraction), and distortions are observed on the focusing object. The process of propagation of the reference radiation is fully symmetric: the initial amplitude is Gaussian and the phase is plane. The amplitude on the screen remains Gaussian, but the phase changes after passage through the screen. Due to the phase shift, the reference beam acquires a crescent shape in the plane of the source aperture. The phase-conjugation reflection is performed here. As a result, the beam to be corrected is Gaussian on the distorting screen, and the thermal lens keeps unchanged. Distortions in the object plane are absent. The complete compensation for distortions is achieved in a single iteration, and then the temperature field along the path and the criterion in the observation plane keep constant. It can be concluded that even in the nonlinear medium fulfillment of the optical reciprocity provides for stability of the control.

The control based on the phase-conjugate reflection algorithm is different in the case of the distributed thermal lens (the criterion $J(t)$ obtained at correction for thermal blooming in this case is shown in Fig. 2).

Table 2. Distribution of the light and temperature fields in the PCR algorithm (one distorting screen at the center of the propagation path)

Iteration number	Intensity distribution at the entrance to the medium	Intensity and temperature distribution on the screen*	Intensity distribution in the observation plane	J
1				0.13
2				0.50
3				0.50

* Gray-scale image is for temperature; lines of different level are for the beam.

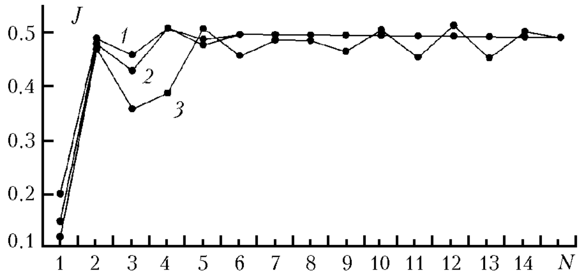
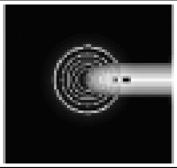
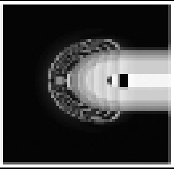
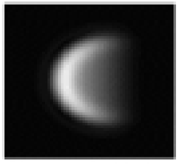
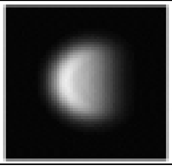
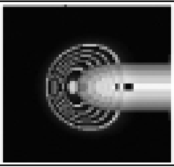
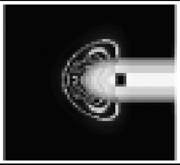
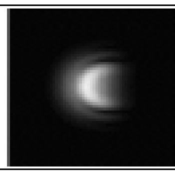
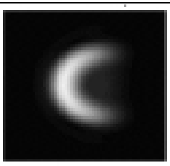
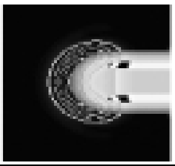
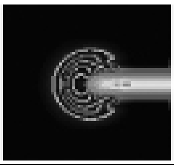
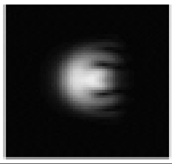
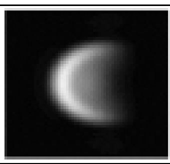
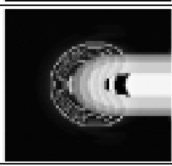
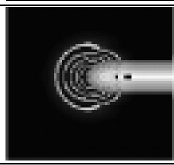
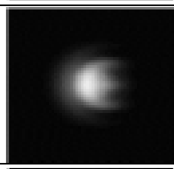
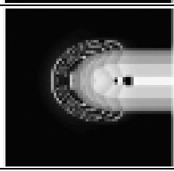
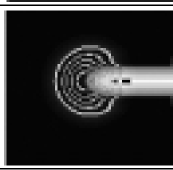
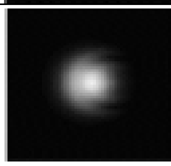
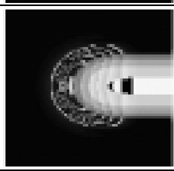
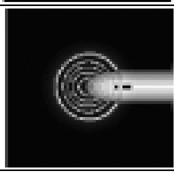
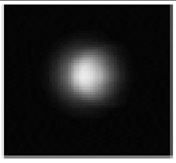
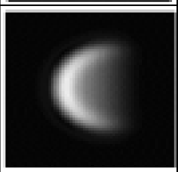
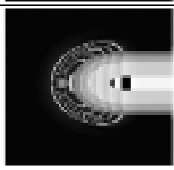
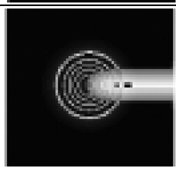
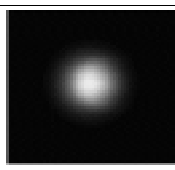


Fig. 2. Variation of the focusing criterion in the case of control based on PCR. Parameters: path length $Z = 0.50$, $R_0 = -20$ (curve *1*), -25 (curve *2*), and -30 (curve *3*).

At the initial iterations here, we again can observe oscillations of the field concentration that decay at $|R_0| \leq 25$, and the lower is the medium nonlinearity, the smaller number of iterations are needed for the decay. If $|R_0| > 25$, oscillations of the criterion take place at a low amplitude.

In the above, simplified (one screen along the path) consideration of the PCR algorithm, no explanation to the oscillations of the criterion $J(t)$ at the initial iterations are given. For a more detailed analysis, experiments with the distributed lens were conducted, and the distribution of the temperature and the radiation amplitude on the screen closest to the laser and on the screen closest to the focusing object was recorded. The data are summarized in Table 3.

Table 3. Distribution of light and temperature fields in the PCR algorithm; five distorting screens are placed along the path ($Z = 0.5$, $R_0 = -25$)

Iteration number	Distribution of reference beam intensity	Distribution of intensity and temperature on the screen $Z = 0.1$	Distribution of intensity and temperature on the screen $Z = 0.4$	Distribution of beam intensity in the observation plane	J
1					0.17
2					0.48
3					0.44
4					0.51
5					0.49
6					0.50
7					0.50

At the first iteration, the beam on the first screen is Gaussian, and the thermal lens has small cross size. The beam on the last screen has the crescent shape, and the lens is much wider than at the beginning of the path. At the seventh iteration, the situation is quite opposite: the wide beam and the thermal lens at $Z = 0.1$ and the Gaussian beam of smaller radius (at the corresponding decrease of the lens size) in the plane $Z = 0.4$. The distribution of light and temperature fields on the first screen at the first iteration are identical to the corresponding field distributions on the last screen at the last iteration. As a result, the control yields that thermal lens "overturns" and then remains unchanged. The values of the criterion oscillate at the iterations when this process occurs. After stabilization of the temperature field along the propagation path, the criterion remains constant. After some threshold value of the intensity, temperature stabilization is not likely achieved, and slight oscillations of the radiation parameters continue in the system.

Comparison of the control algorithms allows us to conclude that the loss of stability of phase conjugation in the nonlinear system is caused by violation of the optical principle of reciprocity at realization of the algorithm. In media with low nonlinearity ($|R_0| \leq 25$), phase-conjugate reflection provides for complete compensation for the beam distortions, and as the radiation power increases, oscillations of the field parameters develop in the observation plane. Appearance of oscillations is also explained by violation of the optical principle of reciprocity.

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