Numerical simulation of laser beam propagation through the turbulent jet from an aero-engine

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A numerical model of the laser beam propagation through the turbulent jet from an aero-engine has been developed based on the Monte Carlo method. The model describes the features observed experimentally that could not be explained by the standard model of a turbulent layer. Among those there is such as the strong dependence on the radiation wavelength and spatial anisotropy of statistical characteristics of the distorted beam. Comparison of the results of computer experiments with the data obtained in the field experiments indicates the correctness of the model and computational algorithms developed. With the allowance made for inevitable measurement errors, there is a good agreement between almost all statistical characteristics of the distorted laser beams obtained in different configurations of the field and imitative experiments.

Introduction

Studies of the spatial characteristics of laser radiation having passed through the zone of strong turbulence, such as the jet from a turbojet engine are urgent both for the development of theoretical models of the influence exerted by the atmosphere on laser radiation and for practical applications. The skill of correctly predicting the characteristics of a distorted laser beam allows one to assess and optimize the conditions for efficient operation of laser information systems under noisy conditions.

For solution of such problems at the stage of a system design, it is important to have a mathematical model adequately describing the laser beam propagation through a turbulent medium. The development of such a numerical model permitting imitation of anticipated experimental situations was just the goal of this study.

This study consisted of three main stages:

– field experiment on laser beam propagation through the jet from a turbojet engine;

– development of a numerical model of the laser beam propagation;

– imitative numerical experiment on verification of the model developed through comparison of the results of the field and computer experiments.

1. Field experiment

In the field experiment, we kept in mind that the primary goal of this study was developing the mathematical model. In this connection, instantaneous images of laser beams were recorded in the digital form. This allowed application of a data processing procedure in both real and computer experiments, if numerical simulation is performed by the Monte Carlo method. Thus, laser beams with the homogeneous intensity distribution and the plane wave front were formed at the entrance into the jet, because such beams are realized in practice in efficient laser systems and they are characterized by the well known coherence function used in analysis of statistical characteristics of the distorted radiation.

The optical layout of the experiment and the results obtained have been described thoroughly elsewhere.[1,2] Therefore, here we briefly remind only the most important details. The aero-engine jet was illuminated by the pulsed laser radiation simultaneously at two wavelengths (0.53 and 1.06 μm) at the distance of ~ 0.5 m from the nozzle section at the angles of 90 and 45° to the jet axis (the radiation path length in the turbulent zone was respectively 0.8 and 1.4 m). The beam diameters were 10 and 30 mm, and the pulse duration did not exceed 50 ns. In every measurement cycle, up to 1500 instantaneous intensity distributions of the distorted beam were recorded in the far zone.

In the experiment it was found that under the jet effect the angular divergence of the beam with $\lambda = 1.06$ μm increased roughly by 5 to 10 times, while that of the beam with $\lambda = 0.53$ μm by 20–30 times. In all experimental situations the angular divergence of the beams at $\lambda = 0.53$ μm two to three times exceeded that of the radiation at $\lambda = 1.06$ μm, which could not be explained within the framework of the standard model of a turbulent layer. In addition, azimuth asymmetry of the angular intensity distributions of laser beams was observed in the experiment.

The experimentally measured variance of random angular displacements of the beam has the same peculiarities, as the average angular distribution of the distorted radiation, namely, strong wavelength dependence and azimuth asymmetry.

The analysis[1,2] showed that the experimentally measured wavelength dependences of the average angular characteristics of the laser beam distorted by the jet agree well with the theoretical model of a randomly inhomogeneous medium, in which the spectrum of refractive index inhomogeneities is described by the combination of the ordinary Karman spectral function with the extra multiscale function increasing the contribution of high-frequency components in the region of spatial frequencies $\geq 10^3\ \text{m}^{-1}$:

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$$\Phi_n(p) = 0.033C_n^2 \left\{ \frac{(L_{0x}L_{0y})^{11/6}}{[1 + (p_xL_{0x})^2 + (p_yL_{0y})^2]^{11/6}} + B\left[\left(\frac{2\pi}{L_s}\right)^2 + p^2\right]^{-11/6} \right\}, \quad (1)$$

where p_x and p_y are the components of the spatial frequency vector; $p^2 = p_x^2 + p_y^2$; C_n^2 is the structure characteristic of the refractive index of the turbulent medium; L_{0x} and L_{0y} are the outer scales of turbulence in the horizontal (along the jet) and vertical (normal to the jet) axis directions, respectively; B and L_s are the numerical coefficient and the scale for the extra high-frequency spectral function found through fitting to the experimental data.

Based on the experimental data, the following estimates were obtained for parameters of the spectral function (1) [Refs. 1, 2]:

$$C_n^2 = (1.5 \pm 0.2) \cdot 10^{-9}\ \text{m}^{-2/3};$$

$$L_{0x} \approx 0.35\ \text{m and } L_{0y} \approx 0.7\ \text{m},\ B \sim 10 \text{ and } L_s \sim 1\ \text{mm}.$$

2. Mathematical model of laser beam propagation

The general scheme of the mathematical model of laser beam propagation across the jet is as follows: an undistorted laser beam with the preset parameters crosses a thin randomly inhomogeneous phase screen that models a turbulent medium layer and then propagates in free space. The beam path length in the jet in the experiment was about 1 m, and the radiation scintillation index at the exit from the jet β_0^2 did not exceed 0.15, that is, the condition $\beta_0^2 \ll 1$ was fulfilled. Thus, the turbulent jet was modeled by a single random phase screen.[3] This imitative experiment was repeated many times with statistically independent phase screens.

The main factors determining the efficiency of the model under development are the choice of the method for formation of the phase screen providing for an adequate effect of inhomogeneities with both high and low spatial frequencies on the laser beam and the choice of the efficient method for calculation of the beam propagation in the free space that allows repeating the experiment for the short time and obtaining statistically significant results.

Having known the two-dimensional spatial spectrum (1), we can readily obtain[4] the spectrum of randomly inhomogeneous phase change acquired by the laser beam as it crosses the turbulent jet

$$\Phi_s(p) = 2\pi HK^2\Phi_n(p),$$

where $K = 2\pi/\lambda$; H is the length of the turbulent zone along the beam trajectory. The randomly inhomogeneous field of the phase itself $S(x, y)$ can be obtained by the spectral sampling method through summation of the Fourier series with the random coefficients[5] by the following equation:

$$S(x, y) = \text{Re}\left\{ \sum_{n=-N/2}^{N/2} \sum_{m=-N/2}^{N/2} (a_{n,m} + ib_{n,m}) \times \Delta p \sqrt{\Phi(p_n, p_m)}\, e^{[i(n\{2\pi x/L\} + m\{2\pi y/L\})]} \right\}, \quad (2)$$

where N is the total number of summed harmonics; $a_{n,m}$ and $b_{n,m}$ are the pairs of real random values obeying normal distribution with the zero mean and variance equal to unity:

$$\langle a_{n,m}\rangle = \langle b_{n,m}\rangle = 0;\ \langle a_{n,m}^2\rangle = \langle b_{n,m}^2\rangle = 1;$$

L is the linear size of the screen determining the minimum spatial frequency present in the sum of harmonics, and the grid step in the frequency region is

$$\Delta p = 2\pi/L;\ p_n = n\Delta p, \text{ and } p_m = m\Delta p.$$

The imaginary part of this series gives yet another independent random realization of the phase screen.

Solving this problem, we have to determine the number of summed harmonics sufficient for simulation of the screen. As known,[6] the screen size should exceed the outer scale of turbulence, that is, $L \ge L_{0x,0y}$, then the number of harmonics should be about $N = \max(L_{0x,0y})/h$, where h is the minimum size of inhomogeneities in the phase screen, which should be taken into account in calculations. From the form of Eq. (1) it follows only that $h < L_s$. To determine the particular value, we have to compare the experimental and calculated data on intensity fluctuations of the distorted laser beam. The results of such analysis are shown in Fig. 1, which depicts the photos of some single realizations of the far-field images of the distorted laser beam.

Figure 1 demonstrates clear dependence of the calculated speckle structure in the beam cross section on the step of the computational grid in the plane of the randomly inhomogeneous phase screen. The grid step needed for correct simulation of the beam propagation was chosen based on the degree of similarity of the fluctuation characteristics:

$$h \sim 0.2L_s \text{ for } \lambda = 1.06\ \mu\text{m and } h \sim 0.15L_s \text{ for } \lambda = 0.53\ \mu\text{m}.$$

Since $L \sim 1$ m and $L_s \sim 1$ mm, it can easily be seen that the number of harmonics needed, $N \sim 10^4 \times 10^4$, turns out to be catastrophically large even when using Fast Fourier Transform (FFT). However, we can use the fact that the laser beam diameter is much smaller than the outer scales of turbulence and simulate the turbulent layer by two successive random phase screens $S1(x, y)$ and $S2(x, y)$ generated as refractive index inhomogeneities with the size, respectively, smaller and larger than the beam diameter D. Then the screen $S1(x)$ is represented by the series of the form (2), where $L = L_D \approx (2–3)D$, and for calculation of the filtering function determining the series coefficients we can use only a part of the spectrum (1) with the frequencies higher than $p_D = 2\pi/D$, that is,

$$\Phi1_n(p_n, p_m) = 0.033C_n^2 \left\{ \left[\left(\frac{2\pi}{L_D}\right)^2 + (p_n^2 + p_m^2)\right]^{-11/6} + B\left[\left(\frac{2\pi}{L_s}\right)^2 + (p_n^2 + p_m^2)\right]^{-11/6} \right\}. \quad (3)$$

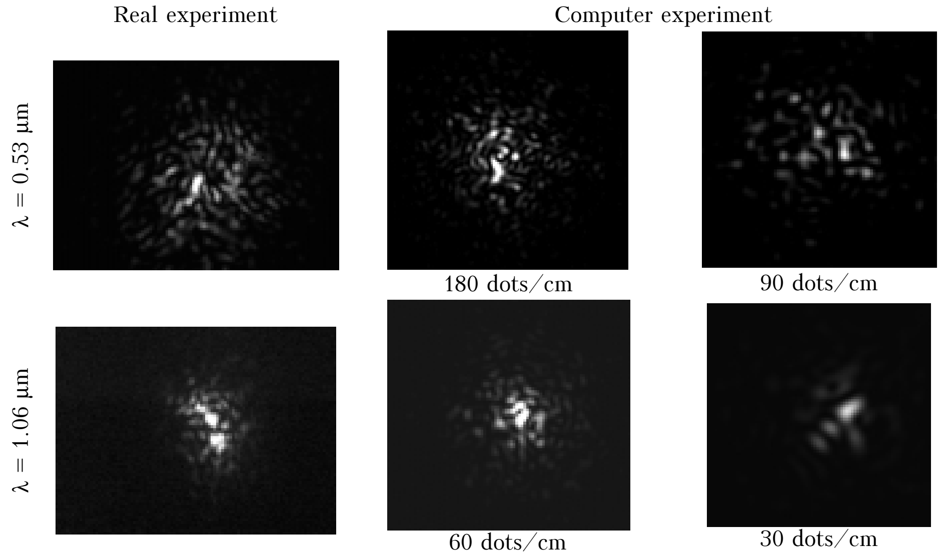

Fig. 1. Realizations of the far-field images of the distorted laser beam obtained in the real and computer experiments at different steps of the computational grid in the plane of a randomly inhomogeneous phase screen.

In the experiment the maximum value is $D = 30$ mm; therefore, for calculation of such a screen it is sufficient to use $N = 512$ harmonics. The phase was calculated by use of the FFT procedure. For this purpose, the Fourier series was reduced to the form

$$S(k\delta,\ j\delta) = \mathrm{Re}\left\{ \sum_{n=1}^{N} \sum_{m=1}^{N} (a1_{n,m} + ib1_{n,m}) \times \right.$$

$$\left. \times \Delta p \sqrt{\Phi 1(p_n,\ p_m)}\, e^{[i(2\pi nk/N\, +\, 2\pi mj/N)]} \right\}, \qquad (4)$$

where $\delta = L_D/N$ is the step of the computational grid on the phase screen; $k = 1, 2, ..., N$ and $j = 1, 2, ..., N$.

The rest part of the spectrum $\Phi 2(p) = \Phi(p) - \Phi 1(p)$ was used for calculation of the screen $S2(x, y)$ by direct summation of low-frequency harmonics, and the series of the form (2) stopped at the harmonics with the maximum spatial frequency $\sim 1/D$ (the number of harmonics turns out equal to $\sim L/D$). In this case, the spatial frequencies of harmonics forming the screens $S1(x, y)$ and $S2(x, y)$ almost do not overlap and the screens prove to be uncorrelated,[5] which allows them to be calculated independently. The exact number of low-frequency harmonics is determined by comparing the analytical approximation of the structure function of a random phase corresponding to the determined spectrum (1) with the calculated structure function of the sum of random screens $S1(x, y) + S2(x, y)$. Figure 2 shows the results of such a comparison.

It can be seen that to take into account the influence of large-scale inhomogeneities on the characteristics of laser beams in the low-frequency screen, the number of harmonics should be rather large (about 100). Nevertheless, this method is not extremely time-consuming in computation of the phase screen.

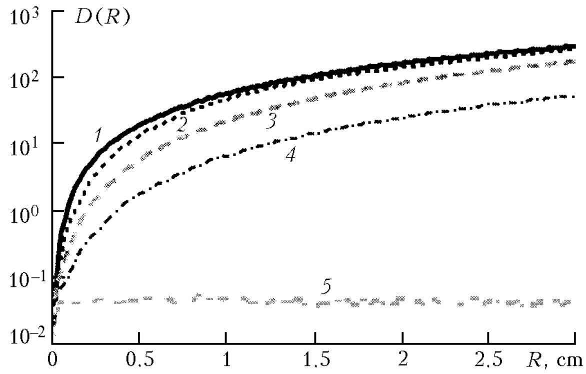

Fig. 2. Structure function of phase at different number of low-frequency harmonics taken into account in calculating the phase screen (spatial frequencies from $2\pi/L$ to $2\pi N/L$). $C_n^2 = 1.4 \cdot 10^{-9}$ m$^{-2/3}$, L_{0x}, $L_{0y} = 0.7$ m, $l_0 = 3$ mm, $H = 1.5$ m; R is cross coordinate: theoretical structure function for the Karman spectrum (curve *1*); $N = 100$ (*2*); 10 (*3*); 2 (*4*); and structure function of the screen $S1(r)$ (*5*).

Consider the algorithm for numerical simulation of propagation of the laser beam distorted by the turbulent jet.

Usually, in problems of this kind, the propagation of a laser beam through the free space is calculated through solving the wave equation in the parabolic approximation.[5,6] For this purpose, the complex amplitude of the laser beam is transformed by applying the Fourier transform: direct in the initial plane of the beam path and inverse at the end of the path, that is, in the observation plane. However, in our case for simulation of a narrow laser beam propagation it is possible to use the Huygens–Fresnel integral.

If the complex amplitude of the beam just behind the phase screen has the form

$$U(x,\ y,\ z) = U_0(x,\ y,\ z)\exp[iS(x,\ y)],$$

then, according to the Huygens–Fresnel principle, the field amplitude in the plane with the coordinate z' can be found from the integral relationship:

$$U(x', y', z') = \frac{e^{[ik(z'-z)]}}{i\lambda(z'-z)} e^{[ik(x'^2 + y'^2)/\{2(z'-z)\}]} \times$$
$$\times \iint_D \left\{ U(x, x, z,)\, e^{[ik(x^2 + y^2)/\{2(z-z')\}]} \right\} \times$$
$$\times e^{[-i(q_x x + q_y y)]} \,\mathrm{d}x\mathrm{d}y, \qquad (5)$$

where $q_x = kx'/(z' - z)$ and $q_y = ky'/(z' - z)$.

As can be seen from this relationship, the field amplitude in the observation plane can be considered as Fourier transform of the expression in square brackets, and the variables q_x and q_y in this case play the role of spatial frequencies. To pass on to the coordinates of the observation plane in the final equation, the Fourier transform arguments q_x and q_y should be modified appropriately. Thus, when this method is used, the FFT procedure should be applied only once in the calculation.

Application of the FFT procedure imposes restrictions on the acceptable variations of the phase in the integrand within the step of the computational grid. These restrictions are determined by the condition $(x_{j+1} - x_j)\psi'(x_j, y) < \pi$ (Nyquist condition[6]), where $\psi'(x_j, y)$ is the partial derivative of the function

$$\psi(x, y) = S(x, y) + \pi\,(x^2 + y^2)/\lambda(z' - z).$$

in terms of x.

Hence, it follows that the simulated path length should exceed some minimum value:

$$z' - z \geq 2D(x_{j+1} - x_j)/\lambda.$$

In our case $(z' - z)_{\min} \sim 10$ m. Note for a comparison that in the case of simulation by the method of solution of the parabolic equation with application of the FFT the opposite restriction takes place[6]:

$$z' - z \leq 2L(x_{j+1} - x_j)/\lambda,$$

which complicates somewhat the simulation of laser beam propagation to long distances. This served as an additional argument in favor of the simulation algorithm using the Huygens–Fresnel integral.

3. Results of numerical experiment

The model software was developed in Fortran-90. In addition to the procedures for calculation of single realizations of the complex amplitude of the distorted laser beam field at the given path length it includes the programs for calculating the radiation intensity distributions in the observation plane. Also it enables determination of the intensity distribution averaged over the ensemble of realizations, as well as the intensity fluctuations at every point of the beam cross section and the centroid of the intensity distribution for every realization. These parameters are the main characteristics of laser beams distorted by a turbulent medium, which are of interest for many practical applications.

As was already noted, the field experiment was conducted in such a way that the same programs were used to obtain these characteristics from the experimental beam images. This allowed direct comparison of the results of field and computer experiments and verification of the mathematical model developed. Some examples of the average intensity distributions in the far zone of the beam (angular distributions) are depicted in Fig. 3, from which we can see rather good agreement between the experimental results and that of computer simulations.

The computer experiment (Fig. 3*b*) reflects the experimentally observed strong wavelength dependence of the angular width of the intensity distribution only in the case, when the spectrum of inhomogeneities of the random phase screen is complemented with high-frequency components [second term in Eq. (1)].

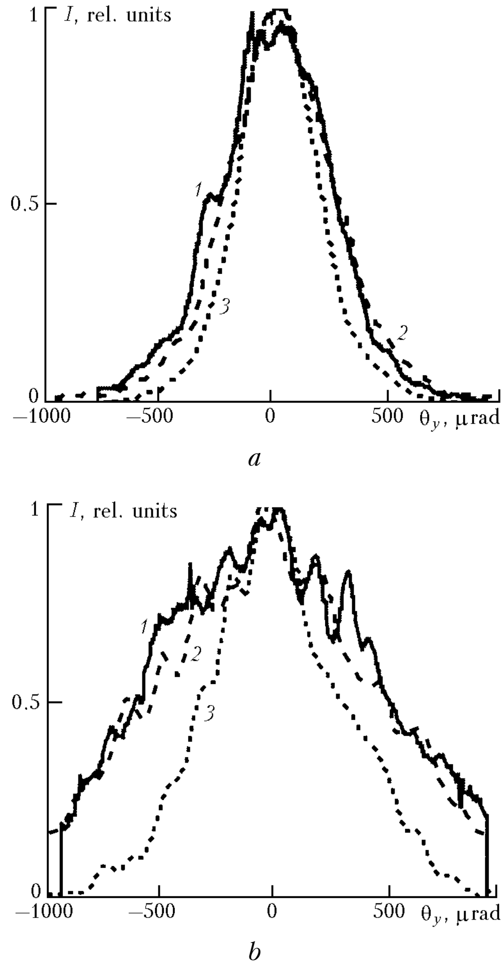

Fig. 3. Example of average angular intensity distribution in the far zone: laser beam 10 mm in diameter crosses the turbulent jet at the angle of 45°, $\lambda = 1.06$ μm (*a*) and 0.53 μm (*b*): field experiment (curve *1*); computer experiment with the use of the Karman spectrum (*2*); computer experiment with the use of the Karman spectrum and an additional high-frequency function (*3*).

The influence of high-frequency spectral components on the intensity fluctuations is illustrated in Fig. 4, which shows some examples of the distribution of the index of intensity fluctuations (scintillation index) over the cross section of a distorted beam.

Comparison of angular $1/e$ level halfwidth θ_x and θ_y, variance of the beam centroid wandering σ_x and σ_y, and the scintillation index at the beam axis $\beta^2 = [<I^2> - <I>^2]/<I>^2$ obtained in the experiment and on a computer

λ, μm	Beam parameter	Angle of the jet intersection, φ							
		90° (H = 0.8 m)				45° (H = 1.4 m)			
		Beam diameter, mm							
		10		30		10		30	
		experiment	computer	experiment	computer	experiment	computer	experiment	computer
1.06	θ_x μrad	145 ± 20	180	140 ± 20	165	225 ± 30	253	210 ± 30	265
	θ_y μrad	220 ± 30	241	205 ± 30	235	320 ± 40	345	310 ± 40	360
	σ_x μrad	85 ± 10	103	50 ± 5	53	130 ± 15	132	85 ± 10	77
	σ_y μrad	130 ± 15	143	80 ± 10	79	160 ± 15	197	115 ± 10	102
	β^2	2.1 ± 0.2	2	2 ± 0.2	1.8	2.7 ± 0.2	2.2	2.2 ± 0.2	1.6
0.53	θ_x μrad	420 ± 50	390	415 ± 50	405	560 ± 70	600	530 ± 70	660
	θ_y μrad	505 ± 60	460	470 ± 60	525	750 ± 90	715	725 ± 90	730
	σ_x μrad	175 ± 20	178	115 ± 5	94	245 ± 25	235	175 ± 20	157
	σ_y μrad	195 ± 20	200	135 ± 5	140	295 ± 30	350	220 ± 20	250
	β^2	1.9 ± 0.2	1.7	1.8 ± 0.2	1.5	1.5 ± 0.2	1.5	1.5 ± 0.2	1.5

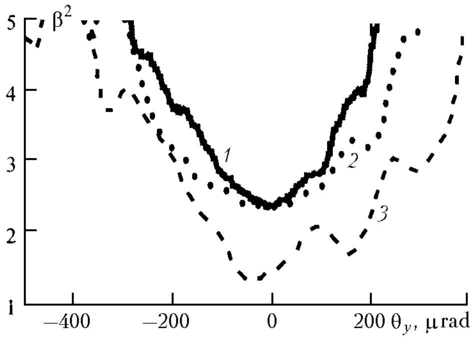

Fig. 4. Examples of the distributions of the scintillation index $\beta^2 = [<I^2> - <I>^2]/<I>^2$ over the cross section of a distorted 10-mm diameter beam, $\lambda = 1.06$ μm, the beam propagates at the angle of 45° with respect to the jet: field experiment (curve *1*); computer experiment with the use of the Karman spectrum (curve *2*); computer experiment with the use of the combined spectrum (1) (curve *3*).

The Table summarizes the results of a comparison between the statistical characteristics of distorted beams obtained for all experimental situations in the field and computer experiments. It can be noted that within the data spread in field experiment a good agreement is observed between the data.

Thus, detailed comparison of the results of statistical processing of images of distorted laser beams obtained experimentally and in computer simulations proves the reliability of the developed numerical model of the laser beam propagation through the aero-engine turbulent jet and its adequacy to the process simulated.

Conclusion

Thus, based on the Monte Carlo method we have developed a numerical model that describes the experimentally observed features, such as strong wavelength dependence and spatial anisotropy of the statistical characteristics of the distorted beam, which were not explained within the standard model of a turbulent layer.

For a beam with a wide spectrum, the random screen was simulated through its division into two screens: high-frequency one (with the frequencies higher than $2\pi/D$) and low-frequency one (with the frequencies lower than $2\pi/D$). Both of the screens were formed by the spectral sampling method traditional for the optics of turbulent atmosphere in the form of Fourier series with random amplitudes of spatial harmonics, which allows application of the FFT procedure. The process of beam propagation behind the phase screen is simulated based on the Huygens–Fresnel principle with application of the FFT, which shortens the computational procedure as compared to the traditional method for solution of the parabolic equation.

For the geometrical configurations realized in the experiments, a cycle of computer simulations was conducted, and the data of field experiments and computer simulations were processed by the same technique. The comparison of the results demonstrates correctness of the computational techniques and algorithms put in the foundation of the mathematical model. With regard for inevitable measurement errors, almost all statistical characteristics of the distorted laser beams obtained in different configurations of the field and computer experiments are in a good agreement.

References

1. V.S. Sirazetdinov, D.I. Dmitriev, I.V. Ivanova, and D.G. Titterton, Atmos. Oceanic Opt. **14**, No. 10, 824–829 (2001).
2. V.S. Sirazetdinov, D.I. Dmitriev, I.V. Ivanova, and D.G. Titterton, Atmos. Oceanic Opt. **14**, No. 10, 830–834 (2001).
3. V.P. Kandidov, Usp. Fiz. Nauk **166**, No. 12, 1309–1338 (1996).
4. V.I. Tatarskii, *Wave Propagation in a Turbulent Medium* (McGraw Hill, New York, 1961).
5. V.P. Lukin and B.V. Fortes, *Adaptive Beaming and Imaging in the Turbulent Atmosphere* (SPIE Press, 2002).
6. D.L. Knepp, Proc. IEEE **71**, 722–737 (1983).