

# Remote sensing of atmospheric aerosols with a white-light femtosecond lidar. Part 1. Numerical simulation

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The current progress in the technology of high-power femtosecond lasers opens new promises in solution of urgent problems of atmospheric optics. In particular, stable supercontinuum of directed white light generated in the atmosphere by a high-power laser pulse can be used as a white-light lidar for remote analysis of optical and microphysical properties of atmospheric aerosol. Using the experience gained in solving inverse problems of laser sensing, we develop the technique for solution of such problems with the use of a white-light lidar. As a first and necessary stage of the study, a closed numerical experiment was conducted. This paper discusses its results as applied to multifrequency sensing of aerosol along vertical paths in the visible and near-IR spectral ranges. Spectral ranges for sensing are selected, and noise immunity of the retrieval algorithms at wide-aperture reception is analyzed.

## Introduction

The recent progress in nonlinear optics connected with the study of spectral superbroadening of terawatt laser radiation in gas media<sup>1,2</sup> has led to discovery of the phenomenon of filamentation accompanied by directed conical radiation in a wide spectral range from the UV to the mid-IR.<sup>3–5</sup> The most stable results on extended (up to several meters long) collective filamentation are obtained by now with the use of a femtosecond Ti:Sapphire laser with  $\lambda = 775\text{--}800$  nm, pulse duration of 35–100 fs, and peak pulse power of about 2 TW (that is, about  $10^{15}$  W/cm<sup>2</sup> per one filament).<sup>5,6</sup> Precision measurements made in Ref. 5 showed that the spectrum of the emitted supercontinuum extends at least from 300 to 4500 nm.

The high energy density and wide spectral band characteristic of high-power femtosecond pulses in air open principally new possibilities for remote optical sensing of the atmosphere, allowing the combination of the unique properties of a lidar and high-resolution Fourier Transform spectroscopy. In this paper, we dwell on one particular aspect of the problem concerning the lidar sensing of atmospheric aerosols. A white-light lidar allows a new look at the multifrequency sensing of microphysical parameters of aerosol particles, abandoning, in the first turn, the use of bulky and difficult-to-control multilidar systems.<sup>7</sup> It becomes possible to select the sensing wavelengths informative for a given class of problems. Technological advances in miniaturization of Ti:Sapphire and new lasers with similar performance characteristics<sup>8</sup> open the prospects for creating multifunctional aerospace white-light lidars (WLL). However, in this case, some specific problems connected with uncertainty in the lidar

equation arise, and, in our opinion, it is worth solving these problems within the framework of computer experiment.

## 1. Model of numerical experiment

As a physical prototype of WLL, we took the structure and geometry schemes of the active Teramobile mobile femtosecond-terawatt laser and detection system.<sup>9</sup> The sensing scheme is close to the monostatic one, that is, it provides for complete overlap of the field of view of the source and the receiving telescope up to the bottom boundary of the volume under sensing. A series of ultrashort laser pulses (pulse duration  $t_i = 70$  fs, frequency  $f = 10$  Hz, energy  $\sim 350$  mJ, and wavelength  $\lambda = 793$  nm) from a Ti:Sapphire laser produces a cylindrical filament  $\sim 50$  mm in diameter at the distance  $z = 30\text{--}70$  m. It is a source of conically directed white light (supercontinuum).

The propagation direction is vertically upward (along the  $OZ$  axis). According to the available estimates,<sup>6,9</sup> the angle of divergence of the supercontinuum radiation is  $2\phi_s = 1\text{--}30$  mrad. The acceptance angles of traditional lidar systems are much smaller; therefore, they are set as parameters  $2\phi_d = 2, 10, 20,$  and  $35$  mrad. The geometry of the possible experiment is shown in Fig. 1.

The Monte Carlo numerical experiment was aimed at revealing the possibility of retrieving the vertical profiles of optical and microphysical aerosol characteristics under conditions of noticeable multiple scattering background and uncertain optical relations. Therefore, the *a priori* optical model assumed a stratified aerosol layer  $\Delta z = 0.1$  km with the sufficient optical thickness  $\tau_a = 1\text{--}1.5$  and the known model particle size spectrum to be present at

the distance  $z = 0.2$  km from the source. As a well-known reference spectrum, we took the water haze  $H$  (Ref. 10):

$$f(r) = ar^\alpha \exp(-br^\gamma), \quad 0 \leq r \leq \infty, \quad (1)$$

where  $a = 4 \cdot 10^5$ ,  $b = 20.0$ ,  $\alpha = 2.0$ , and  $\gamma = 1.0$ .

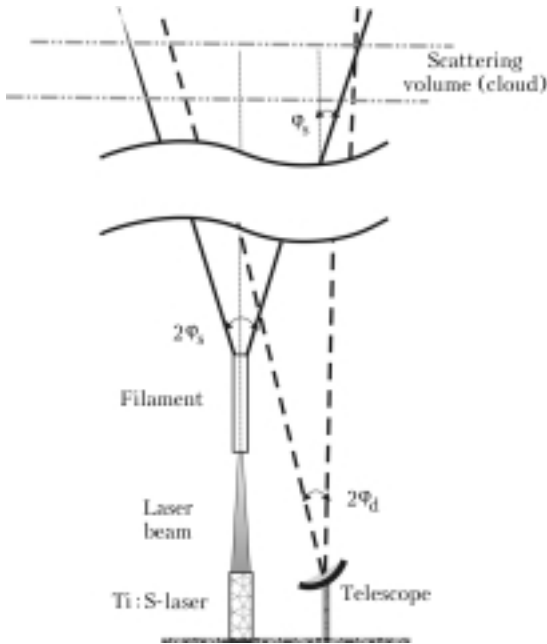


Fig. 1. Experimental geometry.

The corresponding coefficients of optical interaction for the selected array of wavelengths  $\lambda_i$ ,  $i = 1-4$ , are calculated through the Mie diffraction equations.<sup>10</sup> Selecting a set of wavelengths informative from the viewpoint of retrieval of the aerosol particle size spectrum is an independent problem. When sensing in wide spectral ranges, we should select, first of all, the working wavelengths, that is, the most informative range for sensing a given disperse medium. If it is known, for example, that the radii of particles in the medium  $r \in [0, R]$ , and the optical sensing can be carried out in principal in any wavelength range  $\lambda_i$ , then the most informative is the range, for which the functional  $F(K_r)$  reaches the extreme value.<sup>11</sup> Here  $K_r$  is the factor of optical interaction. The functional  $F$  characterizes the smoothness of the  $K_r(r)$  behavior. For the chosen unimodal model (1), this problem is not difficult and gives the range of  $\lambda$  from 400 to 800 nm with the roughly uniform arrangement of subranges  $\lambda_i$ .

Another factor determining the proper selection of  $\lambda_i$  is the presence of  $O_2$ ,  $O_3$ , and  $H_2O$  absorption lines even in the visible transmission spectrum. It is commonly known (see, for example, Ref. 12), that strong frequency dispersion in the region of optical resonances or in narrow absorption lines leads to significant transformation of integral characteristics of an optical signal. These effects intensify as the

pulse shortens even under the linear propagation conditions. The effects of spectral and temporal transformation of a signal obviously can have an independent significance for remote determination of spectral line parameters of minor gaseous constituents under field conditions. This issue calls for separate consideration, and in this paper the active spectral ranges are selected based on *a priori* exclusion of spectroscopic phenomena.

Figure 2 depicts the transmission spectrum of the gas atmosphere in the selected spectral range as calculated by the well-known  $k$ -distribution method<sup>13</sup> with the step  $\Delta\lambda = 10$  nm.

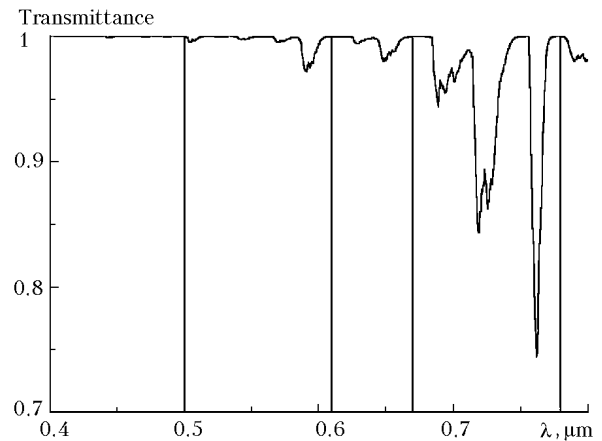


Fig. 2. Spectral ranges selected for sensing.

Temperature, pressure, and concentration of gases and  $H_2O$  vapor correspond to the tropospheric model of the mid-latitude summer. The joint consideration of the formulated requirements has led to selection of the following particular sensing wavelengths:  $\lambda_i = 0.5, 0.61, 0.67, 0.78 \mu\text{m}$ , which are shown by vertical lines in Fig. 2.

## 2. Solution of the direct problem

Correct statement of the direct problem of lidar sensing assumes a solution of the nonstationary radiative transfer equation under complicated boundary and initial conditions representing the actual experimental scheme. The rigorous analytical solution of such problems is not obtained yet. Among numerical methods, the most rational is the Monte Carlo method. Within the framework of this method, the radiative transfer equation is usually written in the integral form<sup>14</sup>:

$$f(x) = \int_{\bar{x}} k(x',x)f(x')dx' + \psi(x); \quad (2)$$

$$k(x',x) = \frac{\Lambda(r')g(\mu,r')\exp[-\tau(r',r)]\sigma(r)}{2\pi|r-r'|^2} \times$$

$$\times \delta\left(\omega - \frac{r-r'}{|r-r'|}\right) \delta\left[t' - \left(t + \frac{|r'-r|}{c}\right)\right] \quad (3)$$

with the stochastic kernel determining the transition density of the Markovian chain of random collisions of a particle (photon) in the disperse medium.

In Eqs. (2) and (3):

$$f(x) = \sigma(r)I(x)$$

is the collision density;

$$I(x) = I(r, \omega, t)$$

is the radiation intensity at a point of the phase space  $x \in X$ ;

$$X = \{r \in R \subset R^3, \omega = (a, b, c) \in \Omega = (a^2 + b^2 + c^2 = 1)\};$$

$g(\mu, r)$  is the scattering phase function satisfying the normalization condition

$$\int_{-1}^1 g(\mu) d\mu = 1;$$

$\mu = (\omega, \omega')$  is the cosine of the scattering angle;

$$\tau(r, r') = \int_0^l \sigma(r, t) dt$$

is the optical length of the segment  $[r, r']$ ;

$$l = |r' - r|;$$

$$\sigma(r) = \sigma_a(r) + \sigma_s(r)$$

is the extinction coefficient;  $\sigma_a, \sigma_s$  are the absorption and scattering coefficients including the additive parts of the aerosol and molecular components;  $\psi(x)$  is the source distribution density describing the spatial-angular distribution of radiation from the source, the initial pulse shape

$$\Psi(x) = p(r_0)p(\omega_0)p(t_0); \int_R p(r_0) dr = 1$$

and so on. Among numerous modifications of the Monte Carlo method, the method of local estimation is efficient for lidar problems with localized sources and receivers of radiation. Essentially, this method consists in the following. Let  $x^*(r^*, \omega^*, t^*) \in D$ , where  $D$  is the phase volume of some optical detector,  $D \in X$ ,  $D \ll X$ . Assume that in Eq. (2)  $x = x^*$ ,  $\psi(x^*) = 0$ , and  $\sigma_s = \sigma$ . Write Eq. (2) in the form of a linear functional of the collision density  $f(x)$ :

$$I(x) = \int_x \frac{k(x', x^*)}{\sigma} f(x') dx'. \quad (4)$$

Then, following Ref. 14, the statistical estimate of the functional

$$J(r^*) = \int_{D_{ij}} I(r, \omega, t) d\Omega dt \quad (5)$$

at the point  $r^*$  over some domain  $D_{ij} = \Omega_i T_j$  of directions and the times of recording takes the form

$$\tilde{J}(r^*) = M \sum_{n=0}^N Q_n \xi_{ij}(x_n, x^*), \quad (6)$$

where  $M$  denotes the mathematical expectation;  $n = 1, 2, \dots, N$  is the number of a random photon collision;  $Q_n$  is the statistical weight of a photon compensating for the fictitious character of transitions  $k(x_n, x^*)$ ;

$$\xi_{ij} = \frac{\exp[-\tau(r, r^*)] g(\mu^*, r) \Delta(\omega^*) \Delta_j(t^*)}{2\pi |r - r^*|^2}; \quad (7)$$

$$\mu^* = (r^* - r) / |r^* - r|;$$

$\Delta_i, \Delta_j$  are the indicators of the domain  $D_{ij}$ .

We have considered the commonly known aspects of Eqs. (5) and (6), because application of the local estimate (6), in spite of its efficiency, is connected with certain, generally insuperable difficulties. The factor  $1/|r - r^*|^2$  in Eq. (7) leads to the square divergence at  $|r - r^*| \rightarrow 0$  and, consequently, to the uncontrollable variance of the estimate. Formal removal of the phase volume of the detector  $D$  outside the scattering medium, as in the standard practice, is not always allowable. This takes place in the case of the considered scheme of a white-light lidar.

However, just the specificity of a lidar signal recording allows us to propose a simple method stabilizing the estimate (6). As will be noticed below, most existing methods for inversion of the lidar equation, i.e., Eq. (2) in the single scattering approximation, do not employ the intensity of the incoming radiation  $I(r, \omega, t)$  as input physical functional, but the relative, square amplified signal of the form

$$S(z) = \frac{I(z)}{I_0} z^2, \quad (8)$$

where  $I_0$  is the intensity of the emitted signal;  $z$  is the reduced height of the sensed area.

The results of numerical experiments have shown that for statistical estimation of  $\tilde{S}(z)$  Eq. (7) can be used without the factor  $1/|r - r^*|^2$ . The shift of the result is compensated by *a posteriori* processing of the histogram  $\tilde{S}(T_j)$ ; the result has a finite invariance.

The method considered was used for calculation of relative backscattering signals from the supercontinuum in the selected spectral ranges and for solving the corresponding system of lidar equations under the conditions of possible noise.

### 3. Solution of the inverse problem

#### 3.1. Retrieval of the vertical profile of extinction coefficients

The information about the character of aerosol formations in the atmosphere is needed for analysis of

weather formation and monitoring of environmental pollution. Practical application of new-generation meteorological white-light lidars, which allow obtaining of detailed information about the atmospheric aerosol microstructure, requires a particular attention to methodological problems concerning the retrieval of the information from measurements. In spite of significant progress in this sphere, the requirements to increase the efficiency of the interpretation methods remain valid. The range of applicability of classical methods<sup>15</sup> is significantly limited because of the ambiguous mutual behavior of the coefficients of total scattering and backscattering that are connected by the well-known lidar equation

$$P(z) = BN_0G(z)z^{-2}\beta_\pi \exp\left\{-2\int_0^z\sigma(z')dz'\right\}, \quad (9)$$

where  $P(z)$  is the amplitude of the backscattering signal as a function of distance;  $N_0 = P_0(c\tau/2)$  is the energy of the initial sensing pulse;  $B$  is the instrumental function including characteristics of transceiving apertures;  $\beta_\pi(z) = \sigma_s(z)g_\pi(z)$  is the volume backscattering coefficient;  $g_\pi(z)$  is the absolute value of the vector of the scattering phase function for the scattering angle of  $180^\circ$ ;  $G(z)$  is the geometric function accounting for the mutual overlap of the fields of view of the source and the receiving telescope (detector). A characteristic feature of boundary conditions for this problem is, as can be seen from Fig. 1, the fact that the detector's fields of view can only partly cover the single scattering volume. The information about microphysical characteristics of the scattering medium is contained in the functions  $\sigma_s(z)$ ,  $\sigma(z)$ , and  $\beta_\pi(z)$  to be estimated.

Equation (9) cannot be solved for these variables without simplifying assumptions or *a priori* information about the relation between them. Thus, in the selected wavelength range the assumption  $\sigma_s(z) = \sigma(z)$  is justified. At the single-frequency sensing, the profile of the molecular extinction coefficient  $\sigma_m(z)$  is also assumed known from the standard model, although it depends on the meteorological parameters of the atmosphere. The aerosol scattering coefficient and the scattering phase function can vary quite widely. This is explained both by the various origin of atmospheric aerosol and the interaction of aerosol particles with random humidity and wind fields in the atmosphere. The methods analyzed in Ref. 15 assume that the scattering phase function remains constant with height. Along with the multiple scattering, this assumption leads to the uncontrollably high error in determination of optical parameters of scattering particles and, consequently, their microstructural characteristics.

Possible ways to compensate for the shift of the results due to the multiple scattering background were discussed in Ref. 15. Along with them, the processing algorithm proposed in this paper allows

one to approximately take into account the spatial variations of the scattering phase function, as well as the errors of recording and square amplification of the signal. An advantage of this method is the possibility of operating with arbitrary comparable units, that is, the prior calibration of the measurement instrumentation is not needed.

The procedure of the signal processing is based on the iterative solution of Eq. (9), which assumes prior basic measurements of  $\sigma(z_0)$  and  $g_\pi(z_0)$  at the initial part of the path. Some prerequisites for this approach were formulated in Ref. 16. Thus, for the discrete scheme of processing of the signal  $P(z)$  by the readouts  $z_j$ ,  $j = 1, 2, \dots$ , Eq. (9) can be transformed as follows:

$$S(z_j) = S(z_{j-1}) \frac{\beta_\pi(z_j)}{\beta_\pi(z_{j-1})} T^2(\Delta z_j), \quad (10)$$

where

$$S(z) = P(z)z / [AN_0G(z)]; \quad \beta_\pi(z) = \beta_a(z) + \beta_m(z);$$

$$T^2(\Delta z_j) = \exp\{-\Delta z_j[\sigma(z_{j-1}) + \sigma(z_j)]\}, \quad j = 1, 2, \dots, k.$$

As was noted in Ref. 17, for a femtosecond lidar with a characteristic wide (up to  $2^\circ$ ) radiation cone the effect of the geometric function  $G(z)$  is significant. Correct consideration of  $G(z)$  is nontrivial. Therefore, we assume that the ratio of the solid angles of emission and reception in the neighboring strobes  $\Delta z_j = z_j - z_{j-1}$  remains constant, then

$$\frac{S(z_j)}{S(z_{j-1})} = \frac{P(z_j)z_j^2}{P(z_{j-1})z_{j-1}^2}, \quad (11)$$

and the unknown, generally speaking, values of the instrumental function  $B(\lambda)$  are cancelled at such an approach. In the case if the radiation in the cone is isotropic, i.e.,  $p(\omega) = \text{const}$ , this assumption is correct, otherwise additional refinements are needed.

The solution of Eq. (10) for  $\sigma_a(z)$  and  $\beta_a(z)$  results from the iteration procedure carried out at each part of the path:

$$\left\{ \begin{aligned} \beta_a^{(m)}(z_j) &= F(z_j) \exp\left\{\Delta z_j \left[ \frac{\beta_m(z_j)}{g_m} + \frac{\beta_a^{(m-1)}(z_j)}{g_a(z_j)} \right]\right\} - \beta_m(z_j), \\ F(z_j) &= \frac{S(z_j)}{S(z_{j-1})} \beta_\pi(z_{j-1}) \exp\{\Delta z_{j-1}[\sigma_m(z_{j-1}) + \sigma_a(z_{j-1})]\}, \end{aligned} \right. \quad (12)$$

where  $g_m$  and  $g_a(z)$  are the molecular and aerosol scattering phase functions in the direction of  $180^\circ$ ;  $\beta_m$  and  $\beta_a$  are the corresponding components of the backscattering coefficients;  $m$  is the iteration number.

It can be shown<sup>15</sup> that such iteration procedure converges with the rate of a geometric series provided that  $\tau(\Delta z_j) < 1$ , which can be easily obtained in the experiment. The method represented by Eqs. (10)

and (11) is the basic one, that is, it requires *a priori* estimation (measurement) of the coefficients  $\sigma(z_0)$  at a certain part of the path. In our experiment, this problem is solved by invoking a parallel method for retrieval – the so-called method of asymptotic signal,<sup>15</sup> which is optimal for retrieval of  $\sigma(z_0)$  at the closest boundary of the scattering layer.

Some difficulties connected with the need of *a priori* model prediction of  $g_a(z)$  values remain valid. Here it is possible to use a modification of the iteration method (10), (11) developed in Ref. 18 and using the heuristic dependence

$$\log \beta_\pi = K_\pi \log \tau + \log C_\pi / 4\pi \quad (13)$$

with  $K_\pi = 0.69$  and  $C_\pi = 0.51$ .

On the other hand, as was shown in Ref. 18, if we neglect the errors of the instrumental origin and the errors in determination of basic values, that is,  $\delta F_0$  and  $\delta \sigma_0$ , then for the mutual relative errors in determination of the unknown values of the extinction coefficient and the scattering phase function  $g_x(z)$  we have

$$\delta g_x = \delta \sigma_x (\tau_x - 1), \quad (14)$$

where  $\tau_x = \sigma_x \Delta z_i$ . It follows herefrom that  $\delta \beta \rightarrow 0$  at  $\tau_x \rightarrow 0$ . This fact was earlier mentioned in Ref. 11, but in this algorithm it shows itself in the situation that at the reasonable processing interval  $2\tau_x = 0.05$  the error in *a priori* setting  $\delta g_\pi \cong 50\%$  leads to the error  $\delta \beta_a = 1\text{--}2\%$  in determination of  $\beta_a$ .

### 3.2. Method for retrieval of aerosol microstructure

Assume that the function  $s(r)$  describes the aerosol particle size distribution in the unit volume of the scattering medium. In the problems of laser sensing, the form of  $s(r)$  can change depending on the spatial coordinates. Mathematically, the problem of  $s(r)$  determination from the spectral dependence of the aerosol extinction coefficient  $\sigma(\lambda)$  consists in solving the first-kind integral equation

$$\int_0^R K(\lambda, r) s(r) dr = \sigma(\lambda), \quad (15)$$

where  $K(\lambda, r)$  is the extinction efficiency factor for an individual particle of the radius  $r$  at the wavelength  $\lambda$ , which also depends on the complex refractive index of the particulate matter  $m - ik$ .

In the general case, the complex refractive index and the upper boundary of the particle size  $R$  can be also unknown. Inversion of Eq. (15) is an ill-posed problem. Construction of approximate solutions for ill-posed inverse problems, stable to small changes in initial data, requires the use of special mathematic methods.<sup>19</sup> The theory and efficient methods for

solution of this-class problems are now well developed. The comparison of different methods of inversion of the spectral aerosol extinction coefficients was carried out in Ref. 20.

The methods and algorithms for solving the inverse problems of laser sensing of the atmospheric aerosol developed in the IAO SB RAS were successfully tested in practice.<sup>11</sup> Consider briefly the particular version<sup>21</sup> of constructing the regularized solution of Eq. (15) based on the Tikhonov method that was used in computer experiment on laser sensing of a model aerosol medium.

At the first stage, the transition from the integral equation (15) to the discrete analog of the problem is performed. For the given system of nodes  $r_j$ ,  $j = 1, 2, \dots, n$ , the effective finite-difference approximation of the integral in the left-hand side of Eq. (15) is achieved by replacing  $s(r)$  at each subinterval  $[(r_{j-1} + r_j)/2, (r_j + r_{j+1})/2]$  with the Lagrange interpolation polynomial constructed by the nodes  $r_{j-1}$ ,  $r_j$ ,  $r_{j+1}$ . In practice, it is usually believed that the input data are presented by measurements of the extinction coefficient  $\sigma_i = \sigma(\lambda_i)$  at a finite set of wavelengths  $\lambda_i$ ,  $i = 1, 2, \dots, m$ . As a result, the integral equation (15) is replaced by the ill-conditioned system of linear algebraic equations  $As = \sigma$  for the vector  $s$  with the components  $s_j = s(r_j)$ . The vector of the approximate regularized solution  $s_\alpha$  can be found from solution of the following system of linear algebraic equations:

$$(A^*A + \alpha D)s_\alpha = A^*\sigma, \quad (16)$$

where  $A^*$  is the matrix transposed to the matrix  $A$ ;  $D$  is the smoothing matrix;  $\alpha$  is the regularization parameter. In Eq. (16),  $\alpha$  should be selected in accordance with the value of the error in the initial data, for example, by the discrepancy criterion.<sup>19</sup> When solving inverse problems of lidar sensing, the errors connected, for example, with the presence of multiple scattering background, nonsphericity of scattering particles, or insufficiently accurate value of the refractive index hamper estimation of the error in the input data in Eq. (15). Under these conditions, the most efficient method is to use such criteria for  $\alpha$  selection, which do not depend explicitly on errors in the input information. We have considered two such criteria, namely, the selection of the regularization parameter by the criterion of the quasioptimal value of  $\alpha$  [Ref. 19] from the condition of minimum of the functional

$$F_1(\alpha) = \|\alpha s'_\alpha\| \quad (17)$$

or based on the principle of minimal discrepancies<sup>22</sup> from the condition of minimum of the functional

$$F_2(\alpha) = \|As_\alpha - \sigma\| + \|APs_\alpha - \sigma\|, \quad (18)$$

where  $P$  is the operator of projection onto the set of non-negative functions

$$P_{S_{\alpha}}(r) = \begin{cases} S_{\alpha}(r), & S_{\alpha}(r) \geq 0 \\ 0, & S_{\alpha}(r) < 0 \end{cases} \quad (19)$$

It was found in the numerical experiments that these criteria give close results.

### 4. Results of computer experiment

Backscattering signals calculated through numerical solution of Eq. (1) were then used for retrieval [within the method (10), (11)] of the vertical profiles of  $\sigma(z)$  and  $\beta_a(z)$  for the given boundary and optical conditions of the possible physical experiment at  $\lambda_i = 0.5, 0.61, 0.67,$  and  $0.78 \mu\text{m}$ . Figure 3 depicts the results of retrieval of  $\sigma(z, \lambda_i)$  for three of four wavelengths depending on the aperture conditions of detection  $\varphi_d = 2, 10, 20, 35 \text{ mrad}$ .

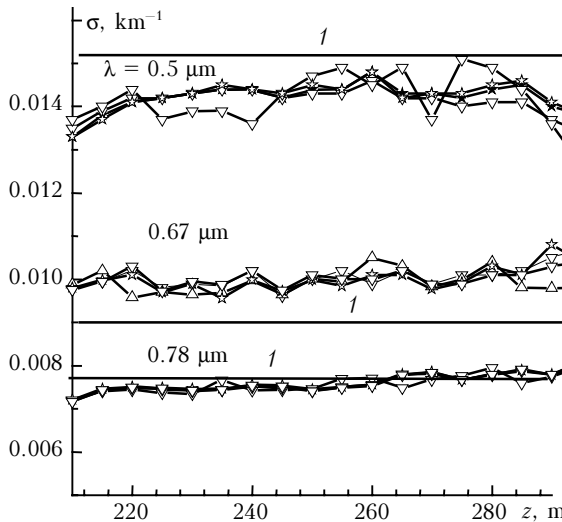


Fig. 3. Retrieval of extinction coefficients from model signals for optically homogeneous medium.

For the model of a homogeneous medium  $\sigma(z, \lambda_i) = \text{const}$  for  $2.0 \leq z \leq 2.3 \text{ km}$ . The model profiles  $\sigma(z, \lambda_i)$  are shown by the straight lines  $\sigma(z, \lambda_i) = C_{\lambda_i}, C_{\lambda_i} = \text{const}$ . The dependence on the angle of reception is almost absent until  $\varphi_d < \varphi_s$ . This indicates that at small optical thickness  $\tau \approx 1-1.5$  the multiple scattering background increases quite uniformly all over the sensing area, rather than exponentially as usually. The accuracy of retrieval increases with the increasing wavelength, and this correlates with the decrease of the medium optical density and the role of molecular scattering. In Fig. 4 the similar results are depicted for the model of an inhomogeneous medium. The model profiles  $\sigma(z, \lambda_i)$  are shown by curve 1. The physical character of the dependences keeps the same. A principally new feature of the presented results is that the optical parameters of the medium under sensing are retrieved from a small part of the single scattering signal under the noise conditions.

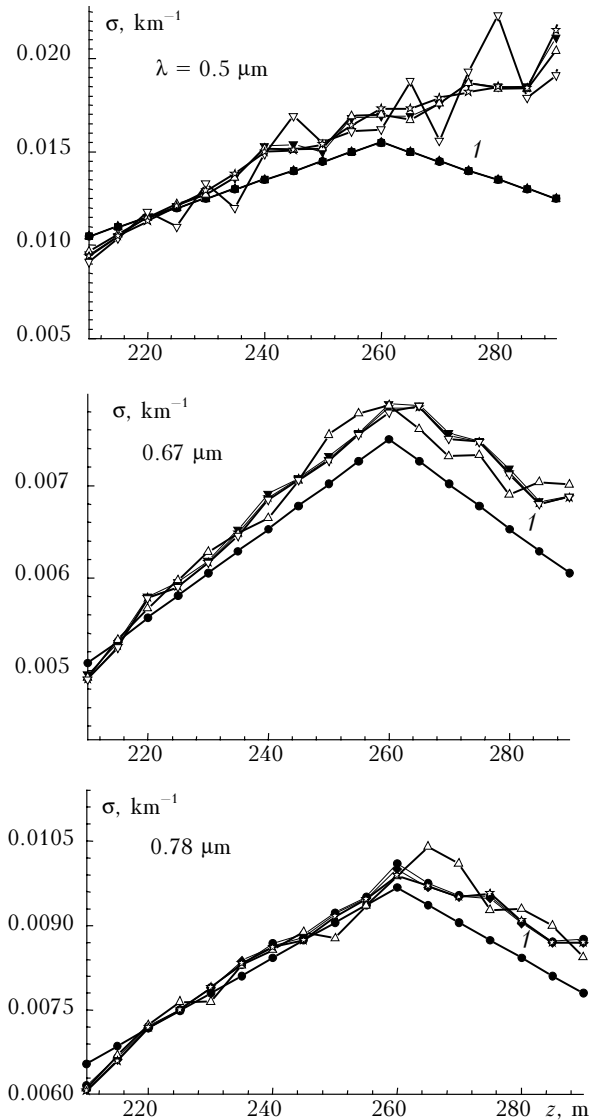
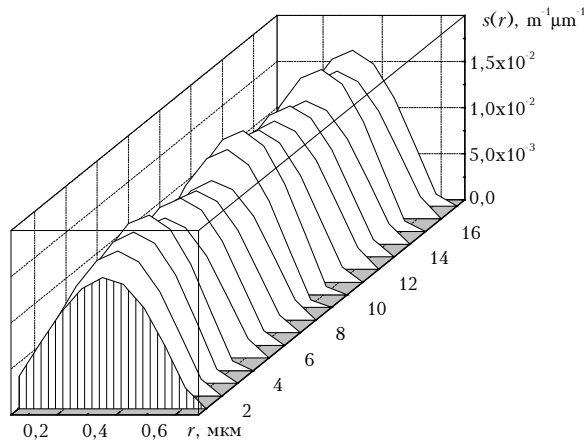


Fig. 4. Retrieval of extinction coefficient for optically inhomogeneous medium.

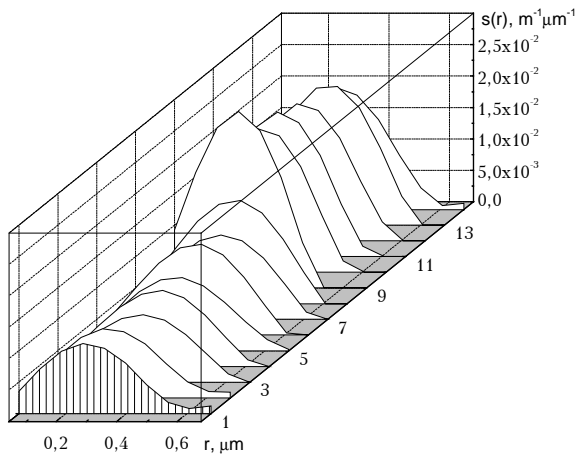
The profiles of the extinction coefficient  $\sigma(z, \lambda)$  retrieved in the numerical experiment from solution of the lidar equation based on the algorithm (10), (11) were then used for solution of the inverse problem of reconstructing the particle size distribution  $s(r)$  along the lidar path. As an example, Figs. 5 and 6 show families of the aerosol size distributions  $s(r)$  reconstructed from inversion of the spectral dependences  $\sigma(z, \lambda)$  depicted in Figs. 3 and 4 for the homogeneous and inhomogeneous scattering media at the angle of reception  $\varphi_d = 2 \text{ mrad}$ .

As can be seen from the results presented, the reconstructed distributions  $s(r)$  keep a rather stable shape and quite adequately correspond to the initial haze model  $H$ . In the case of a homogeneous medium, the modal value of  $s(r)$  also keeps rather stable level. For the inhomogeneous medium, the modal value in the aerosol size distributions  $s(r)$  has a nonmonotone

behavior, following the nonmonotone profiles  $\sigma(z, \lambda)$ .



**Fig. 5.** Aerosol particle size distributions reconstructed in the numerical experiment from lidar signals at a homogeneous path.



**Fig. 6.** Aerosol particle size distributions reconstructed in the numerical experiment from lidar signals at an inhomogeneous path.

## Conclusion

Our numerical investigations have confirmed the wide potentialities of the broadband directed supercontinuum radiation generated by the femtosecond lidar as applied to remote monitoring of microphysical and optical characteristics of atmospheric aerosol. The computer experiment was conducted within the framework of the Monte Carlo method with the use of the new modification excluding a divergence of the local flux estimate. The iteration algorithm for retrieval of the vertical profiles of the extinction coefficient proved itself to be good under the conditions of deficient information about optical sensing channels. The results of

reconstruction of the vertical stratification of the aerosol particle size spectrum adequately reflect the chosen optical model under the conditions of the rather high multiple scattering background.

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