Interpretation of the photometry data on cloud fields by use of the compound-signal model

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Received December 26, 2003

The approximation of statistical characteristics of brightness fluctuations observed in the spatial cross section of a cloud field is considered using the compound-signal model.

Introduction

The cloud fields have a stochastic structure, and any realization of the observed characteristic of radiation under cloudy conditions is random too.[1] The stochasticity manifests itself in the random appearance and random lifetime of the structure elements of a cloud layer (cloud and break in a cloud layer) along the line of sight.

This paper considers statistical characteristics of brightness fluctuations in a linear cross section of a cloud layer: mean values, variances, and correlation functions. It is commonly accepted that a cloud layer is characterized by the relative velocity of an observer and the cloud layer, cloud amount, mean dimensions of the cloud and a break in the cloud layer (broken clouds, that is, cumulus, stratocumulus, altocumulus, stratus, and other clouds) and the cloudless sky is observed through breaks in the cloud layer (single-layer cloudiness). The elements of the cloud layer are characterized by different optical properties, and fluctuations of (zenith) brightness are caused by the motion of the cloud layer relative to the line of sight.

Model

One of the mathematical models used to interpret the observations, as well as to model and predict the processes under study is the model of time-multiplex signals, which forms a compound signal by combining two or more signal processes.[4,7] In the analog representation, it is a system with two (or more) inputs, one output, and a key switching the output to different inputs. The signal processes are most often represented by statistically independent random processes. The parameters in the model described are the values of the combined processes characterizing the process of variations of the brightness elements of a cloud layer. The stochastic nature of a cloud layer is represented by the characteristics of the steady-state, statistically independent switching function.

Process of state switching

The time series corresponding to the spatial (linear) cross sections of the cloud layer can be represented by the process of state switching $q(t)$ using a flow of switching points t_k distributed over the real axis according the Poisson law with the parameter λ_q (Ref. 4). In the interval between the neighboring switching points, $q(t)$ is a constant equal to unity (cloud) with the probability p or 0 (a break in the cloud layer) with the probability $(1 - p)$. The observer acquires the brightness of the cloud element in the state 1 and the brightness of the break in the cloud layer in the state 0. It is a model of a random phototelegraphic signal, which is characterized by two parameters: p and λ_q. The frequency λ_q is interpreted as the band width of the process, and the probability p is interpreted as the probability of the presence of a cloud on the line of sight. The statistical characteristics of the random phototelegraphic signal $q(t)$ are well known: the mean value and the mean square are equal to the probability $P\left[q(t) = 1\right] = p$, the variance is $\sigma_q^2 = p(1-p)$, the mean lifetime in the state 0 is $\overline{\Theta}_0 = 1/\lambda_q p$, and that in the state 1 is $\overline{\Theta}_1 = 1/[\lambda_q(1-p)]$, the mean time between any two transitions is $\overline{\Theta} = 1/[2p(1-p)\lambda_q]$ (from here the bar above the quantity denotes the averaging procedure). All the above parameters have clear physical meanings.

The autocorrelation function of the (non-centered) process $q(t)$ is

$$k_{qq}(\tau) = \overline{[q(t)q(t+\tau)]} = p(1-p)e^{-\lambda_q|\tau|} + p^2, \quad (1)$$

where the mean square is $\overline{q^2} = k_{qq}(0) = p$; and $e^{-\lambda_q|\tau|} = R_q(\tau)$ is the time behavior of the correlation coefficient (correllogram).

The parameters of the cloud layer, namely, $\overline{V}$ (the mean velocity of the cloud layer relative to the observer), $\overline{L}_1$ and $\overline{L}_0$ (the mean dimensions of the cloud layer elements: cloud and a break in the cloud layer), are related to the statistical characteristics of $q(t)$ by simple relations[8]:

$$p = \overline{L}_1/(\overline{L}_1 + \overline{L}_0), \qquad \lambda_q = \overline{V}/(p\overline{L}_0). \quad (2)$$

0235-6880/04/04 305-05 $02.00

In Eq. (2), the parameter λ_q is directly proportional to the velocity of the cloud layer and inversely proportional to the probability of the presence of a cloud on the section line and the mean value of the break in the cloud layer.

The correllogram $R_q(\tau)$ in Eq. (1) is usually characterized by the correlation length τ_R, that is, the time for which $R_q(\tau)$ decreases by e times. Upon substitution of $\lambda_q = 1/\tau_R$ obtained from the processed results into Eq. (2), we obtain the relation between the correlation length, the relative velocity of the cloud layer, the probability, and the characteristic (for the given cross section) dimensions of the cloud layer elements. In particular, the autocorrelation function at the zero time is $k_{qq}(0) = p$. If V is known, for example, for the observer moving with a preset velocity with respect to the cloud layer, then L_0 can be estimated. Thus we can establish the relations between the statistical characteristics of the function of state switching with the physical parameters of the cloud layer.

Observed brightness fluctuations

The observed brightness fluctuations $B_o(t)$ for a two-component process (single-layer cloudiness against the background of the cloudless sky) can be described by the model

$$B_o(t) = q(t)B_1(t) + [1 - q(t)]B_0(t) = \\ = q(t)\left[B_1(t) - B_0(t)\right] + B_0(t), \qquad (3)$$

where $q(t)$ is the random phototelegraphic signal; $B_1(t)$ and $B_0(t)$ are the steady-state, statistically independent processes of the brightness fluctuation of a cloud and of the clear sky, $\overline{B_1} \neq \overline{B_0}$.

Mean value and variance

Consider statistical characteristics for the (steady-state) processes of brightness fluctuation observed on the linear cross sections of the cloud layer.

If $B_1(t)$, $B_0(t)$, $q(t)$ are steady-state and statistically independent processes of brightness fluctuations of the cloud elements, cloudless sky (a break in the cloud layer), and the state switching process, then using the obvious (from the mathematical point of view) transformations we can represent the mean value and the mean square of the observed brightness (3) as linear dependences of the mean value of the process $\overline{q(t)} = p$:

$$\bar{B}_o(p) = p\bar{B}_1 + (1 - p)\bar{B}_0 = p(\bar{B}_1 - \bar{B}_0) + \bar{B}_0, \qquad (4)$$

$$\overline{B_o^2(p)} = p\overline{B_1^2} + (1 - p)\overline{B_0^2} = p(\overline{B_1^2} - \overline{B_0^2}) + \overline{B_0^2}, \qquad (5)$$

and the variance of the observed brightness σ_o^2 as a square dependence of the probability:

$$\sigma_o^2(p) = -p^2(\bar{B}_1 - \bar{B}_0)^2 + p[(\overline{B_1^2} - \overline{B_0^2}) - \\ - 2\bar{B}_0(\bar{B}_1 - \bar{B}_0)] + \sigma_0^2, \qquad (6)$$

where σ_0^2 is the variance of brightness fluctuations of the cloudless sky.

The dependence $\sigma_o^2(p)$ is a descending parabola. The vertex of the parabola corresponds to the maximum variance of the observed process and the probability (cloud amount), at which it is achieved, is:

$$[p|\sigma_{max}^2] = 0.5\left[\frac{(\overline{B_1^2} - \overline{B_0^2}) - 2\overline{B}_0(\overline{B_1} - \overline{B_0})}{(\overline{B_1} - \overline{B_0})^2}\right]. \qquad (7)$$

If we neglect fluctuations in every class of the characteristics (as is done, for example, in considering the cloud fields with elements of constant brightness), then $[p|\sigma_{max}^2] = 0.5$. The symmetry of the $\sigma_o^2(p)$ plot about the probability of 0.5 may be caused by the fact that the variances of brightness fluctuations (in, particular, zero ones) in every class of the characteristics are equal: $\sigma_1^2 = \sigma_0^2$, which does not contradict the physical sense. If the variance in the class "cloud" is greater than that in the class "break," then the vertex of the parabola (maximum value of the variance) is markedly shifted from $p = 0.5$ toward the probability higher than 0.5.

Reference 8 gives empiric dependences of the mean values of the zenith brightness on the cloud amount. Such an analytical representation of the dependences of the mean value and the variance in the model of compound signal has a general character. Similar dependences of, for example, the mean flux of the direct radiation on the coverage of the solar disk, the mean value and the variance of the net radiation flux on the cloud amount, the variance and the standard deviation of the direct radiation flux on the cloud amount are known. In these cases, the fluxes of the direct and net radiation can be used as signal processes, and the variability of the cloud amount can serve the characteristic of the switching process. In Refs. 1 and 2 the empirical dependences are approximated: the mean values by the linear function and the variances by the square function (Figs. 1 and 2). In Ref. 12 the similar dependence was used to describe the radiation of the sea surface. In this case, the brightness of the atmosphere along a given direction and the brightness of the black body at the temperature of the sea surface, with due regard of the reflection coefficient, can be the parameters.

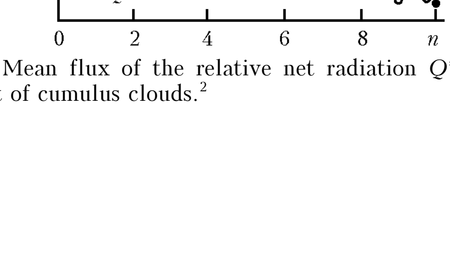

Fig. 1. Mean flux of the relative net radiation Q^* vs. the amount of cumulus clouds.[2]

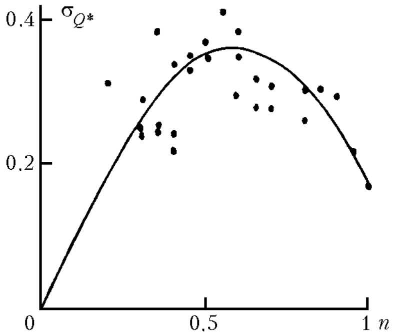

Fig. 2. Standard deviation of the flux of relative net radiation vs. the amount of cumulus clouds.[1]

Autocorrelation function

The autocorrelation function $k_{oo}(\tau)$ as a mathematical expectation of the product $\overline{B_o(t)B_o(t+\tau)}$ can be represented in the form

$$k_{oo}(\tau) = k_{qq}(\tau)[k_{11}(\tau) + k_{00}(\tau) - 2\overline{B_1}\overline{B_0}] + +2p(\overline{B_1}\overline{B_0} - k_{00}) + k_{00} \qquad (8)$$

or

$$c_{oo}(\tau) + \left(\overline{B_o}\right)^2 = [c_{qq}(\tau) + p^2][(\bar{B}_1 - \bar{B}_0)^2 + c_{11}(\tau) + c_{00}(\tau)] + + 2p[(\bar{B}_1 - \bar{B}_0)\bar{B}_0 - c_{00}(\tau)] + c_{00}(\tau) + (\bar{B}_0)^2, \qquad (9)$$

where $k(\tau)$ (subscripts o, q, 1, 0) are the autocorrelation functions of the observed process, the process of state switching, the process of brightness fluctuations of the cloud, and the break in the cloud layer; $c_{oo}(\tau)$, $c_{qq}(\tau)$, $c_{11}(\tau)$, and $c_{00}(\tau)$ are the corresponding covariance functions.

Approximation of the empirical correlation function

The correlation function in the form (8) and (9) contains the information about the spatial structure of the cloud layer and its elements, as well as on the related brightness fluctuations. The correlation function (1) contains the information not only on the spatial structure of the cloud layer. When processing the observations, to obtain the correlation function (1), it is necessary to transform the initial time series into the series of binary signals (classification of signals) and then the statistical processing with estimation of the probability p that the cloud is present on the line of sight, the correllogram $R_q(\tau)$, and the correlation length τ_R. The possible algorithms for classification of signals can be found in Refs. 8–10. It should be noted that the model (3) is applicable to steady-state input processes; consequently, the initial series must be free of the effect of trends.[6] The main causes for the trends of the mean value are the change of the Sun position during observations, the inhomogeneous structure of the cloud layer elements, the alternation of their phase composition, the variation of the velocity and the cloud amount. The problem of taking the trends into account is quite complicated and will be considered separately.

The periodic and ordered spatial structure is inherent in some types of clouds. The variations of the optical properties due to the dynamics of spatial inhomogeneity of cloud elements and the cloud field as a whole can have a mixed character both random and pseudoperiodic. Reference 3 gives some characteristic examples of the normalized correlation functions of the fluxes of direct and net radiation under cloudy conditions and notes that the correlation functions can be both periodic and monotonic even under almost identical conditions of observations (Fig. 3).

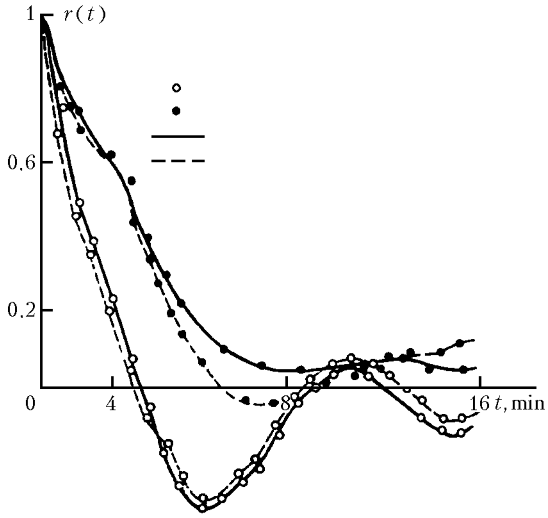

Fig. 3. Normalized autocorrelation functions of the fluxes of net (*1*) and direct (*2*) radiation[3]; *s* is the sunshine duration; *n* is the total cloud amount.

In deep cumulus clouds, clouds of vertical development, and stratus-like clouds, the "internal" inhomogeneities arising due to the dynamics of the processes inside clouds are observed as well.[11]

The empiric covariance functions of the physical processes are usually approximated by the dependences like

$$D\exp[-\alpha t]\cos\omega t, \qquad (10)$$

where D is the variance; α is the parameter of randomness of the observed process; ω is the parameter of relation to the periodicity (hidden periodicity) in the observed process. Assuming the presence of not only random, but also of periodic components in the structure of a cloud layer, in general, apply the approximation (10) to the correlation functions of the observed process of the zenith brightness fluctuations under cloudy conditions.

The Table gives an analytical description of the additive components of the correlation function of the observed process $k_{oo}(\tau)$ with the approximation (10). The last row of the Table presents the general form of the correlation function of the observed brightness.

In this approximation, $\alpha_{0,1}$ are the parameters of the relation to the random space–time inhomogeneity

0235-6880/04/04 305-05 $02.00

of the cloud field elements; in the physical meaning, these parameters coincide with the parameter λ in Eq. (2) and depend on the dimensions, relative number, velocity, and the probability of observation of the internal spatial optical inhomogeneities of the field elements[11] crossing the observer's line of sight; $\omega_{q,0,1}$ are the parameters of relation of the observed process to the periodicity in the space–time structure of a cloud field and its elements; if both of these parameters are zero, then the corresponding components have the exponential form (in the first five terms).

Table

Component	Additive components of approximation of the autocorrelation function of the observed process of brightness fluctuations $k_{oo}(\tau)$ (single-layer cloudiness)
1	$(\overline{B_1}-\overline{B_0})^2 p(1-p)\exp[-\lambda_q\tau]\cos\omega_q\tau$
2	$p^2\sigma_1^2\exp[-\alpha_1\tau]\cos\omega_1\tau$
3	$(1-p)^2\sigma_0^2\exp[-\alpha_0\tau]\cos\omega_0\tau$
4	$p(1-p)\sigma_1^2\exp[-(\lambda_q+\alpha_1)\tau]\times\left[\frac{\cos(\omega_q+\omega_1)\tau+\cos(\omega_q-\omega_1)\tau}{2}\right]$
5	$p(1-p)\sigma_0^2\exp[-(\lambda_q+\alpha_0)\tau]\times\left[\frac{\cos(\omega_q+\omega_0)\tau+\cos(\omega_q-\omega_0)\tau}{2}\right]$
6	$\left[p(\overline{B_1}-\overline{B_0})+\overline{B_0}\right]^2$
	$k_{oo}(\tau) = (\overline{B_1}-\overline{B_0})^2 p(1-p)\exp[-\lambda_q\tau]\cos\omega_q\tau + p^2\sigma_1^2\exp[-\alpha_1\tau]\cos\omega_1\tau + (1-p)^2\sigma_0^2\exp[-\alpha_0\tau]\cos\omega_0\tau + p(1-p)\sigma_1^2\exp[-(\lambda_q+\alpha_1)\tau]\times\left[\frac{\cos(\omega_q+\omega_1)\tau+\cos(\omega_q-\omega_1)\tau}{2}\right] + p(1-p)\sigma_0^2\exp[-(\lambda_q+\alpha_0)\tau]\times\left[\frac{\cos(\omega_q+\omega_0)\tau+\cos(\omega_q-\omega_0)\tau}{2}\right] + \left[p(\overline{B_1}-\overline{B_0})+\overline{B_0}\right]^2$

In the additive components (see the Table), the variables are grouped so that the first three terms have the form of the covariance functions of variability of the cloud field structure and the brightness of the field elements, the fourth and fifth terms are the combinations of the covariance functions of brightness fluctuations of the field elements and the field of the inhomogeneous structure, and the sixth term is the mean square value of the observed brightness (4).

The first component, i.e., the covariance function of the field structure, has the weighting coefficient in the form of the squared difference of the mean brightness values of the cloud field elements.

The following four components are the covariance functions of brightness fluctuations of the cloud field elements and their combination with the covariance function of the field structure. Each of these components has the weighting probability coefficient in the form p^2, $(1-p)^2$ or $p(1-p) = \sigma_q^2$ (the effect of the stochastic structure of the cloud layer).

Examples of approximation

As was noted in Ref. 5 it is usually assumed that the brightness of all clouds is the same and constant within a cloud, as well as the brightness of the cloudless zenith. The analytical equations for the variance and the correlation function of the observed zenith brightness were derived in Ref. 5 with the allowance for the fluctuations of the cloud brightness. The above equations allow one to take into account the random character of the cloud layer structure, as well as the variability of the brightness of the cloud and the break in the cloud layer.

The model (3) can be also applied to description of the brightness fluctuations of both the cloud itself and a break in the cloud layer. As an example, Fig. 4 presents the correllogram of the observed process of brightness fluctuations of the cloud base, the approximation made with the account of the hidden periodicity of the form (10), and the exponential envelope (approximation neglecting the periodicity), which demonstrate the dependence of the brightness fluctuations of the cloud on its spatial structure.

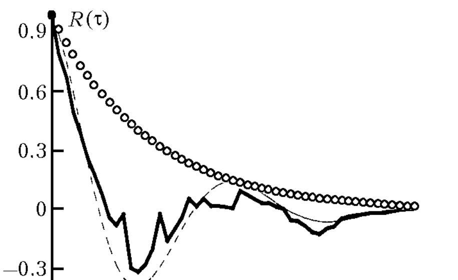

Fig. 4. Correllograms of brightness fluctuations of the cloud base: empirical (solid line) and approximation (dashed line); exponential envelope (circles) is the correllogram $R_1(\tau)$ (relation to the spatial, internal cloud structure).

The processing invoked the record of brightness fluctuations of a large cumulus cloud at zenith observations at the wavelength of 0.69 μm with the interval of 2.6 s and the duration of realization of 2250 s. The velocity measured from the ground was 3.5 m/s. With the model of the form (3), the observed process has been represented by the combination of brightness fluctuations of spatial inhomogeneities inside the cloud. Since the cloud inhomogeneity manifests itself in the inhomogeneous brightness structure, the value 1 of the function of state switching was assigned to the more bright part of the cloud base (by the rule[8]: the current value in excess of the value $\tilde{B}(t)+3\tilde{\sigma}$, where $\tilde{B}(t)$ is the result of exponential smoothing, $\tilde{\sigma}$ is the standard deviation of the current readouts from the smoothed ones), while the value 0 was assigned to the less bright part.

The empirical correlation function was used to determine the exponent α_1 equal to $0.0286\ s^{-1}$, which corresponds to the correlation radius about 35 s, and the "frequency" parameter in the harmonic component equal to $0.0924\ s^{-1}$. The correlation function varies with the period of about one minute. With the allowance for the measured cloud velocity and the duration of realization, this period corresponds to the spatial scale of 210 m.

In recording, the mean lifetime of the process in the state 1 (the more bright part of the cloud) was estimated to be $\Theta_1 = 267$ s. Based on the results of data processing, the probability of observation of more bright (with respect to the mean value) cloud parts was assessed as $p_1 = 1 - 1/(\lambda\Theta_1) \approx 0.9$.

Similar processing procedure can be also applied to the breaks in a cloud layer. The comparison of the results of data processing and theoretical estimation of the corresponding statistical characteristics gives the answer to the question about the dependence of the processes of brightness fluctuations in the breaks of a cloud layer and in the cloud itself with a subsequent correction of the model.

Conclusions

The use of the mathematical model of time-multiplex processes allows the derivation of analytical equations to be done applicable to the approximation of statistical characteristics of fluctuations of the brightness signals in the cross section of a single-layer cloud field. The results obtained do not contradict the commonly accepted idea and may prove useful for interpretation of observations of brightness fluctuations along a fixed angular direction under conditions of single-layer cloudiness.

Acknowledgments

This work was done using the materials obtained under the financial support of the Russian Foundation for Basic Research (grant No. 02–05–65169).

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