Adaptive algorithm for spatial prediction of meteorological fields based on the Kalman filter and the second-order polynomial model

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An adaptive algorithm for spatial prediction of meteorological fields based on the use of Kalman filter and the second-order polynomial model with coefficients varying in time is considered. This algorithm is capable of accounting for the variable configuration of the local network of aerological stations. The results of optimization (from the viewpoint of the minimum prediction error) of the number of stations involved are discussed. The quality of the adaptive algorithm developed is evaluated for the case of its application to the procedure of objective analysis of mesoscale wind and temperature fields.

Introduction

Among numerous problems in the modern meteorology, of particular interest is the problem of objective analysis of meteorological fields (in the first turn, geopotential, temperature, and wind fields) over a mesoscale region based on the data of a local network of aerological stations. The solution of this problem, unlike the objective analysis of mesoscale meteorological fields, depends in a higher degree on the regular reception of data from all available aerological stations, because their number within mesoscale networks is limited.

In practice, however for many reasons (for example, because of instrumentation failure or storm weather), radiosonde observations may be omitted at some or other station in the network considered. In addition, large errors may appear during data transmission through communication channels due to typing errors, transmission distortions, etc. As a consequence, the data received may be partially or completely not decodable.

In any of these cases it becomes necessary, for correct operation of the spatial interpolation algorithms (the basis of objective analysis), to reconstruct the lost data for one or another station. In the earlier papers,[1–3] the spatial interpolation algorithms were developed using the assumption that observations for a given configuration of a local observation network are being always conducted. The configuration of a local observation network is understood here as the number of aerological stations and their spatial arrangement within a mesoscale region.

In contrast to what has been stated in Refs. 1–3, in this paper we propose a little bit different technique for solving the problem of objective analysis of meteorological fields over a mesoscale region. This technique is also based on the application of Kalman filter and the second-order polynomial model with time dependent coefficients, but it accounts for the varying (for different hours of observation) configuration of a local network. Besides, this technique uses the additional procedure of splitting the initial measurements into regular and fluctuating components. Consider this technique in a more detail.

Let the measurements of a meteorological field ξ be conducted at ith points of a given mesoscale region ($i = 1, 2, \ldots, S$) with coordinates (x_i, y_i). The task is to determine the field ξ_0 at some point (or a node of a regular grid) with coordinates (x_0, y_0), located at the territory not covered by meteorological information within the same mesoscale region, from the data of these measurements. For this purpose, represent the field ξ as a sum of the regular $\bar{\xi}$ and fluctuating ξ' components (i.e., $\xi = \bar{\xi} + \xi'$), and consider the methods for reconstruction of each of the components.

Thus, to estimate the regular component of the field $\bar{\xi}_0$ at the point or node (x_0, y_0) from the data of three nearest (to this point) stations, we can use the following equation:

$$\xi(x_i, y_i) = a_0 + a_1(x_i - x_0) + a_2(y_i - y_0), \qquad (1)$$

or in the matrix form

$$\boldsymbol{\xi}(\mathbf{x},\mathbf{y}) = \mathbf{H}(\mathbf{x},\mathbf{y}) \cdot \mathbf{A}, \qquad (2)$$

where $\boldsymbol{\xi}(\mathbf{x},\mathbf{y}) = |\xi(x_1,y_1),\ \xi(x_2,y_2),\ \xi(x_3,y_3)|^{\mathrm{T}}$ is the measurement vector, T denotes transposition;

$$\mathbf{H}(\mathbf{x},\mathbf{y}) = \begin{vmatrix} 1 & (x_1 - x_0) & (y_1 - y_0) \\ 1 & (x_2 - x_0) & (y_2 - y_0) \\ 1 & (x_3 - x_0) & (y_3 - x_0) \end{vmatrix}$$ is the transfer matrix;

$\mathbf{A} = |a_0, a_1, a_2|^{\mathrm{T}}$ is the vector of unknown coefficients, which is estimated as

$$\hat{\mathbf{A}} = \mathbf{H}^{-1}(\mathbf{x},\mathbf{y}) \cdot \bar{\boldsymbol{\xi}}(\mathbf{x},\mathbf{y}). \qquad (3)$$

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Solving Eq. (3), we can easily find the value of $\hat{a}_0$, which corresponds to the value of the regular component $\bar{\xi}_0$ at the point (x_0, y_0).

At the same time, the fluctuating component ξ' is estimated by use of Kalman filtering algorithm and a few-parameter model describing the spatial variability of this field. This model is represented by a second-order polynomial with time dependent coefficients, namely,

$$\xi_i'(k) = a_0(k) + a_1(k)x_i + a_2(k)y_i + $$
$$+ a_3(k)x_iy_i + a_4(k)x_i^2 + a_5(k)y_i^2, \quad (4)$$

where $\xi_i'(k)$ is the value of the fluctuating component of the meteorological field at the point i with the coordinates (x_i, y_i) at the time k; $a_0(k)$, $a_1(k)$, ..., $a_5(k)$ are the unknown coefficients of the polynomial.

For estimation of the polynomial coefficients, introduce the vector of states of a dynamic system in the following form:

$$\mathbf{X}(k) = |a_0(k), a_1(k), ..., a_5(k)|^{\mathrm{T}} =$$
$$= |X_1(k), X_2(k), ..., X_6(k)|^{\mathrm{T}}. \quad (5)$$

In this case, the dynamics of the components of the state vector (5) can be represented as a system of difference equations[1]:

$$\begin{cases} X_1(k+1) = X_1(k) + \omega_1(k) \\ X_2(k+1) = X_2(k) + \omega_2(k) \\ \cdots \\ X_6(k+1) = X_6(k) + \omega_6(k), \end{cases} \quad (6)$$

where $\mathbf{\Omega}(k) = |\omega_1(k), \omega_2(k), \ldots, \omega_6(k)|$ is the vector of random perturbations of the system (generating noise, state noise).

In the vector form, the system of equations (6) takes the form

$$\mathbf{X}(k+1) = \mathbf{X}(k) + \mathbf{\Omega}(k). \quad (7)$$

Assume that the fluctuating field ($\xi' = \xi - \bar{\xi}$) is measured at the ith points and at k moments in time with some error. Then the model of its current measurements $Y_i(k)$ can be specified as a sum of the true value $\xi_i'(k)$ and the measurement error $\varepsilon_i(k)$, that is,

$$Y_i(k) = \xi_i'(k) + \varepsilon_i(k). \quad (8)$$

The whole set of measurements $Y_i(k)$ forms the vector of measurements $\mathbf{Y}(k) = |Y_1(k), Y_2(k), ..., Y_S(k)|^{\mathrm{T}}$, where S is the number of measurement stations involved.

Represent the model (8) in terms of the state variables (5) by introducing the transfer measurement matrix $\mathbf{H}^*(\mathbf{x}, \mathbf{y})$ and write the relationship between the measurement vector $\mathbf{Y}(k)$ and the state vector $\mathbf{X}(k)$ in the matrix form

$$\mathbf{Y}(k) = \mathbf{H}^*(\mathbf{x}, \mathbf{y}) \cdot \mathbf{X}(k) + \mathbf{E}(k). \quad (9)$$

Note that the dimension of the vector $\mathbf{E}(k)$ is also determined by the number of stations S.

The transfer measurement matrix $\mathbf{H}^*(\mathbf{x}, \mathbf{y})$ is an $(S\times 6)$ matrix, which is specified through the coordinates of the stations as follows:

$$\mathbf{H}^*(\mathbf{x}, \mathbf{y}) = \begin{vmatrix} 1 & x_1 & y_1 & x_1y_1 & x_1^2 & y_1^2 \\ 1 & x_2 & y_2 & x_2y_2 & x_2^2 & y_2^2 \\ - & - & - & - & - & - \\ 1 & x_s & y_s & x_sy_s & x_s^2 & y_s^2 \end{vmatrix}. \quad (10)$$

The models of the dynamic system (7) and measurements (9) are linear, therefore the estimation problem is solved with the aid of the linear Kalman–Bucy filter,[4,5] ensuring the estimation of the state vector $\mathbf{X}(k)$ with the minimum variance. Note that in this case the following conditions are imposed onto the elements of the models (7) and (9):

$$M\,[\mathbf{\Omega}(k)\,\mathbf{\Omega}^{\mathrm{T}}(j)] = \mathbf{Q}_k\delta_{kj}$$

for the covariance matrix of the state noise (δ_{kj} is the Kronecker delta);

$$M\,[\mathbf{E}(k)\,\mathbf{E}^{\mathrm{T}}(j)] = \mathbf{R}_k\delta_{kj}$$

for the covariance matrix of measurement errors;

$$M\,[\mathbf{\Omega}(k)\,\mathbf{E}(k)^{\mathrm{T}}] = 0$$

for random processes $\mathbf{\Omega}(k)$, and $\mathbf{E}(k)$ are mutually uncorrelated; and

$$M\,[\mathbf{X}_0\,\mathbf{\Omega}^{\mathrm{T}}(k)] = M\,[\mathbf{X}_0\,\mathbf{E}(j)^{\mathrm{T}}] = 0$$

that means that the initial state $\mathbf{X}_0$ is uncorrelated with the perturbations $\mathbf{\Omega}(k)$ and $\mathbf{E}(k)$.

The algorithm for estimation of the polynomial coefficients has the following form:

$$\hat{\mathbf{X}}(k+1) = \hat{\mathbf{X}}(k+1|k) + \mathbf{G}(k+1)\cdot\mathbf{J}_0(k+1)\cdot[\mathbf{Y}(k+1) -$$
$$-\mathbf{H}^*(\mathbf{x}, \mathbf{y})\cdot\hat{\mathbf{X}}(k+1|k)], \quad (11)$$

where

$$\hat{\mathbf{X}}(k+1) = |\hat{X}_1(k+1), \hat{X}_2(k+1), \ldots, \hat{X}_6(k+1)|^{\mathrm{T}}$$

is the estimate of the state vector at the time $(k+1)$;

$$\hat{\mathbf{X}}(k+1|k) = \hat{\mathbf{X}}(k)$$

is the vector of estimates predicted for the time $(k+1)$ from the data at the step k; $\mathbf{G}(k+1)$ is the $(6\times S)$ matrix of weighting coefficients; $\mathbf{J}_0(k+1)$ is the diagonal matrix that allows for the presence or absence of data from a measurement station.

Unlike the classical Kalman filter,[6] an additional factor – matrix $\mathbf{J}_0(k+1)$ – is introduced in Eq. (11). This matrix is formed at the stage of preliminary analysis of current measurements; $\mathbf{J}_0(k+1)$ is an $(S\times S)$ matrix with the diagonal elements taking the values:

0 denotes the absence of measurements or low quality data from the measurement station i;

1 denotes the data from the station i with no abnormal errors.

Thus, the mathematical operations (11) exclude the components of the measurement discrepancy

$$[\mathbf{Y}(k+1) - \mathbf{H}^*(\mathbf{x},\mathbf{y})\cdot\hat{\mathbf{X}}(k+1|k)]$$

corresponding to incorrect or missing measurements from the formation of the estimate $\hat{\mathbf{X}}(k+1)$.

In other respects, the algorithm corresponds to the classical linear Kalman–Bucy filter. The calculation of the weighting coefficients $\mathbf{G}(k+1)$ is a recurrence procedure independent of Eq. (11), which involves solution of matrix equations for covariances of estimation errors[4]:

$$\mathbf{G}(k+1) = \mathbf{P}(k+1|k)\,\mathbf{H}^{*\mathrm{T}}(\mathbf{x},\mathbf{y})\times$$

$$\times[\mathbf{H}^*(\mathbf{x},\mathbf{y})\,\mathbf{P}(k+1|k)\,\mathbf{H}^{*\mathrm{T}}(\mathbf{x},\mathbf{y})+\mathbf{R}_\mathbf{E}(k+1)]^{-1}; \quad (12)$$

$$\mathbf{P}(k+1|k) = \mathbf{P}(k|k) + \mathbf{R}_\mathbf{\Omega}(k); \quad (13)$$

$$\mathbf{P}(k+1|k+1) =$$

$$[\mathbf{I} - \mathbf{G}(k+1)\mathbf{H}^*(\mathbf{x},\mathbf{y})\cdot\;\mathbf{P}(k+1|k), \quad (14)$$

where $\mathbf{P}(k+1\,|\,k)$ is the *a posteriori* (6×6) covariance matrix of prediction errors; $\mathbf{P}(k+1\,|\,k+1)$ is the *a priori* (6×6) covariance matrix of estimation errors; $\mathbf{R}_\mathbf{E}(k+1)$ is the diagonal $(S\times S)$ covariance matrix of observation noise; $\mathbf{R}_\mathbf{\Omega}(k)$ is the diagonal (6×6) covariance matrix of the state noise; $\mathbf{I}$ is the unit (6×6) matrix.

The final calculation of the predicted value of a meteorological parameter $\hat{\xi}_0(k+1)$ at the point (x_0, y_0) and time $(k+1)$ is performed by the equation

$$\hat{\xi}_0(k+1) = \bar{\xi}_0 + \hat{X}_1(k+1) + \hat{X}_2(k+1)\,x_0 + \hat{X}_3(k+1)\,y_0 +$$

$$+\;\hat{X}_4(k+1)\,x_0y_0 + \hat{X}_5(k+1)\,x_0^2 + \hat{X}_6(k+1)\,y_0^2. \quad (15)$$

To start the algorithm (11), (15) at the time $k = 0$ (initiation time), it is necessary to set the following initial conditions: the initial estimation vector $\hat{\mathbf{X}}(0) = M[\mathbf{X}(0)]$, the initial covariance matrix of estimation errors

$$\mathbf{P}(0\,|\,0) = M\{[\mathbf{X}(0) - \hat{\mathbf{X}}(0)]\cdot[\mathbf{X}(0) - \hat{\mathbf{X}}(0)]^\mathrm{T}\},$$

as well as the value of the elements of the noise covariance matrices $\mathbf{R}_\mathbf{E}(k)$ and $\mathbf{R}_\mathbf{\Omega}(k)$.

In practice, the values of $\hat{\mathbf{X}}(0)$ and $\mathbf{P}(0|0)$ can be specified from the minimum of information on real properties of the system, and in the case of complete absence of useful information they are set to be $\hat{\mathbf{X}}(0) = 0$ and $\mathbf{P}(0|0) = \mathbf{I}$.

The adaptive Kalman filtering algorithm considered uses the second-order polynomial model. It was tested for efficiency in application to the objective analysis of mesoscale temperature and wind fields. For this purpose, we used the data of two-year (2002–2003) observations at thirteen aerological stations: Schleswig (54°32′ N, 09°33′ E), Emden (53°23′ N, 07°14′ E), Greifswald (54°06′ N, 13°24′ E), Bergen (52°49′ N, 09°56′ E), Lindenberg (52°13′ N, 14°07′ E), Essen (51°24′ N, 06°58′ E), Meiningen (50°34′ N, 10°23′ E), Idar-Oberstein (49°42′ N, 07°20′ E), Stuttgart (48°50′ N, 09°12′ E), Kummersbruck (49°26′ N, 11°54′ E), Prague (50°00′ N, 14°27′ E), Munich (48°15′ N, 11°33′ E), Brno-Sokolnice (49°07′ N, 16°45′ E), which form a typical mesoscale network (Fig. 1). This network was chosen, because it is represented by the largest number of aerological stations separated by the minimum (as compared to other networks) distances.

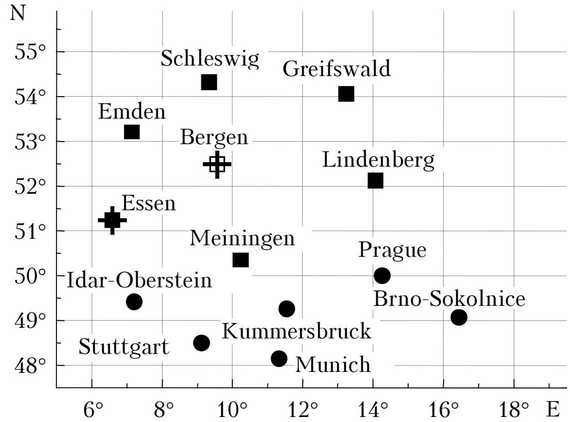

Fig. 1. Mesoscale network.

All the sampled data were reduced (through interpolation with the allowance for the data of singular points) to the system of geometric heights: 0 (ground level), 0.2, 0.4, 0.8, 1.2, 1.6, 2.0, 2.4, 3.0, 4.0, 5.0, 6.0, and 8.0 km.

Since the chosen mesoscale network includes quite a large number of aerological stations, it was interesting what number of stations can ensure the maximum accuracy of spatial prediction based on the technique proposed. Therefore, at the first stage (that is, before estimating the quality of the adaptive algorithm), we have conducted numerical experiments on selection of the optimal number of aerological stations, providing for minimum error in the spatial interpolation of the temperature and zonal and meridional wind fields. For this purpose, Bergen station (see Fig. 1) was taken as a control station, to which the interpolation was performed. The optimization procedure consisted in the sequential reduction of the number of stations involved (depending on their separation from Bergen) and the estimation, at every step, of the quality of interpolation of the meteorological field with the algorithm proposed. The number of aerological stations was thus reduced from 12 to 3.

The results of this estimation are shown in Fig. 2, which presents, as an example, the standard error (δ_ξ) of spatial interpolation of temperature and zonal and meridional wind at the levels of 0.4, 3.0, and 8.0 km as a function of the number of stations involved N. It follows from the plots that the optimal number of stations is $N = 6$ regardless of a height level and a meteorological parameter. Indeed, the use of measurements from six stations (shown by squares in Fig. 1) provides for minimum standard error of interpolation. Similar results were also obtained in the case of spatial extrapolation of the field ξ to Essen station.

At the second stage, having in mind these results, we have realized the procedure of statistical estimation of the quality of an adaptive algorithm based on the use of Kalman filter and the second-order polynomial model. The spatial interpolation and extrapolation of the fields of temperature and orthogonal wind velocity components were performed to the same control stations (Bergen and Essen) from the data of six stations determined in the process of optimization.

Here it should be emphasized that the estimation of the quality of spatial interpolation (extrapolation) of these meteorological fields was performed as applied to prediction of the spread of an industrial pollutant cloud. In this case, according to Ref. 7, we used the layer mean temperature and wind values calculated by the equation

$$\langle \xi \rangle_{h_0,h} = \frac{1}{h - h_0} \int_{h_0}^{h} \xi(h) \mathrm{d}h, \qquad (16)$$

where $\langle \cdot \rangle$ denotes averaging over a given atmospheric layer, for the height ranges $h - h_0$ (here h_0 corresponds to the ground level, and h is the height of the top boundary of the layer chosen): 0–0.2, 0–0.4, 0–0.8, 0–1.2, 0–1.6, 0–2.0, 0–2.4, 0–3.0, 0–4.0, 0–5.0, 0–6.0, and 0–8.0 km.

As an example, Fig. 3 shows the summer plots of the standard errors of spatial interpolation δ_1 (or extrapolation δ_2) of layer-averaged values of temperature $\langle T \rangle$, zonal $\langle U \rangle$ and meridional $\langle V \rangle$ wind as functions of height along with the corresponding standard deviations σ obtained from the data of six stations of the mesoscale network.

The analysis of Fig. 3 shows that the adaptive algorithm proposed yields higher accuracy of the results if used for spatial interpolation. In this case, the standard errors vary within 1.1–1.4°C (for $\langle T \rangle$) and 1.5–2.0 m/s (for $\langle U \rangle$ and $\langle V \rangle$) regardless of

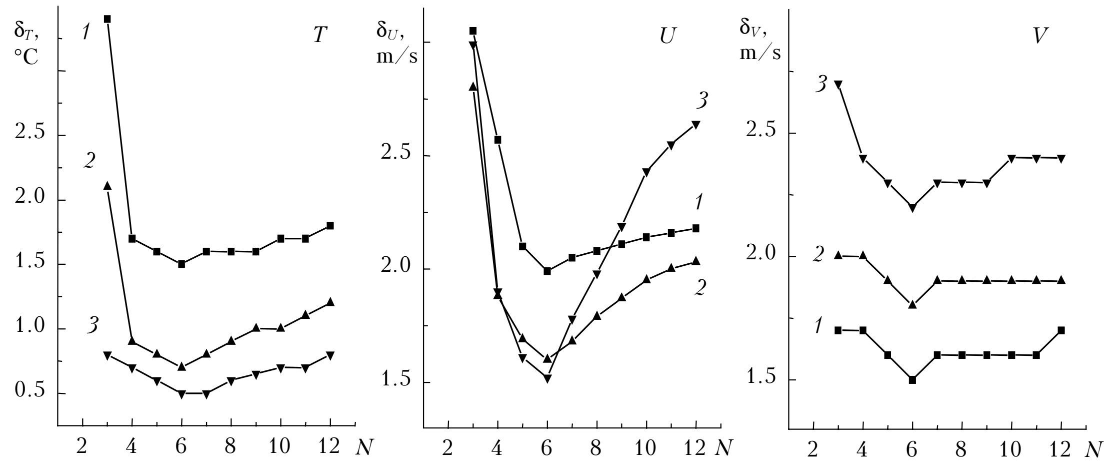

Fig. 2. Standard error in estimation of temperature (T), zonal (U) and meridional (V) wind vs. the number of stations involved N; height of 0.4 (curve *1*), 3.0 (*2*), and 8.0 km (*3*).

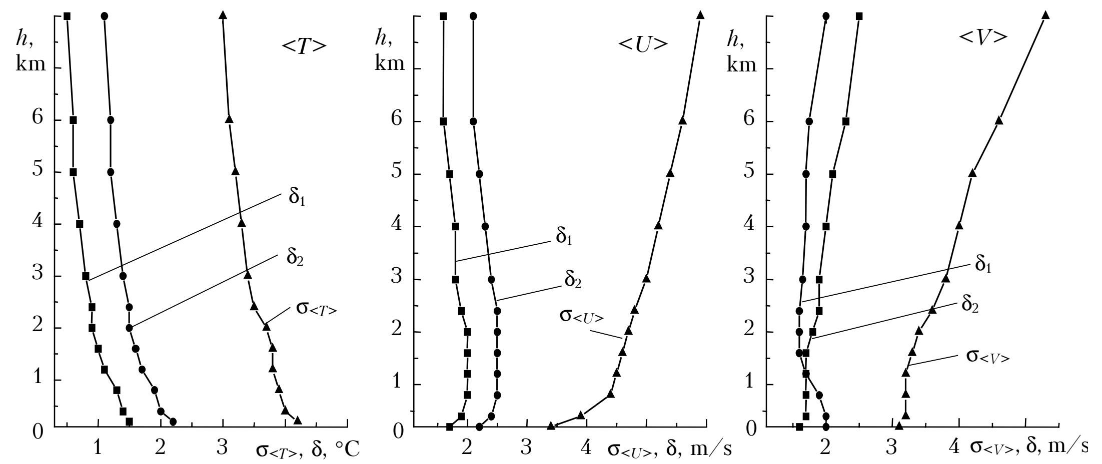

Fig. 3. Height dependence of standard errors of spatial interpolation (δ_1) and extrapolation (δ_2) of layer mean values of temperature $\langle T \rangle$, zonal $\langle U \rangle$ and meridional $\langle V \rangle$ wind, as well as standard deviations σ averaged over the network.

an atmospheric layer. Second, this algorithm gives results, which are acceptable in practice if it is used for spatial extrapolation within a mesoscale region, yielding the standard error of such extrapolation to the distance of 230 km no higher than 1.5°C (for <*T*>) and 2.0 m/s (for <*U*> and <*V*>) over entire atmospheric boundary layer.

Thus, the adaptive algorithm considered, which is based on Kalman filter and the polynomial model with time dependent coefficients can successfully be used for meteorological support (with the forecast data) of assessment and prediction of local atmospheric pollution.

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