Refraction errors in the visual positioning of surface objects from aircraft

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Received September 3, 2003

The effect of vertical distribution of the refractive index, depending on weather conditions, in the surface atmospheric layer on the trajectory of visual observation of surface objects from an aircraft, as well as the difference from the straight-line observation geometry, are analyzed. Analytical equations are derived and used to calculate refraction errors in visual ranging of objects with respect to the aircraft location. It is shown that the curvature of the earth's surface should necessarily be taken into account.

In visual determination of locations of surface objects from aboard an aircraft, the state of the surface atmospheric layer can significantly affect the visual contact of the aircraft crew with the objects observed.[1,2] Variations of the weather conditions and, consequently, the vertical profiles of air temperature, humidity, and atmospheric pressure cause variations of the vertical distribution of the atmospheric refractive index and give rise to refraction errors in observation of surface objects. In Ref. 3, we considered the errors in the visual determination of the horizontal range to a runway from an aircraft due to the presence of inversion parts in the vertical distribution of temperature in the surface atmospheric layer on the assumption of the plane earth's surface, which is not always correct.

The main objective of this work was to estimate the refraction errors in determination of the range to surface objects viewed from an aircraft along the surface from the data of temperature and wind sensing with the allowance for the earth's surface curvature.

If the aircraft is at the point À at the height h_0 above the surface at the time of observation of a surface object (Fig. 1), the horizontal range of the surface object is determined by the projection of the aircraft trajectory from the point À onto the surface, that is, by the line BD of intersection of the earth's surface by the plane passing through the viewing point, the observation trajectory AD, and the center of the globe O. The origin of the vertical axis coincides with the point À, and the axis is directed downward, normally to the earth's surface. The surface object is observed at the angle φ_0 to the plane perpendicular to the vertical axis at the point À. In this case, the angle between the tangent to the observation trajectory and the normal to the earth's surface from the point À is $\psi_0 = (\pi/2) - \varphi_0$. The change of the atmospheric refractive index with height $n(h)$ causes the change of the angle ψ between the tangent to the observation trajectory and the normal to the earth's surface, as well as the length of the projection of the observation trajectory onto the earth's surface $L_{\rm cur}$. Assuming the earth's surface to be spherical and considering the atmosphere as a set of thin layers with the refractive coefficient n constant within each layer and $n(0) = n_0$ in the layer including the viewing point À, we can easily show, by analogy with Refs. 3 and 4, that

$$\psi = \psi_0 + \sin\psi_0 \times$$
$$\times \int_0^h \frac{\left[n(h) - (R + h_0 - h)\dfrac{\mathrm{d}n(h)}{\mathrm{d}h}\right]\mathrm{d}h}{n(h)(R + h_0 - h)\sqrt{\left(\dfrac{n(h)}{n_0}\dfrac{R + h_0 - h}{R + h_0}\right)^2 - \sin^2\psi_0}};$$
$$L_{\rm cur} = R\sin\psi_0 \times$$
$$\times \int_0^h \frac{\mathrm{d}h}{(R + h_0 - h)\sqrt{\left(\dfrac{n(h)}{n_0}\dfrac{R + h_0 - h}{R + h_0}\right)^2 - \sin^2\psi_0}}. \quad (1)$$

At $h = h_0$, Eqs. (1) determine the angle of the tangent to the observation trajectory at the surface point and the range to the object viewed from the aircraft along the surface.

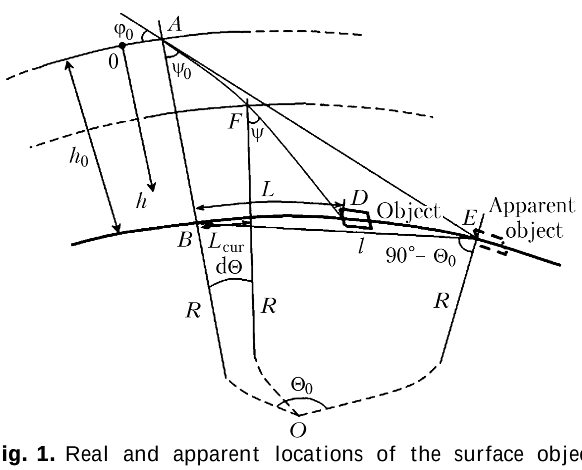

Fig. 1. Real and apparent locations of the surface object viewed from aboard an aircraft.

In retrieving the vertical profile of the atmospheric refractive index from the data of temperature and wind sensing, the dependence $n(h)$ within $h_{i-1} < h < h_i$ (h_{i-1} and h_i are the upper and lower boundaries of the ith layer, corresponding to the neighboring points at the sensing height) can be represented as

$$n(h) = n_{i-1} + \gamma_{i-1}(h - h_{i-1}),$$

where

$$n_{i-1} = n(h_{i-1}) \text{ and } n_i = n(h_i)$$

are the refractive indices at the boundaries of the considered layer;

$$\gamma_{i-1} = (n_i - n_{i-1}) / (h_i - h_{i-1})$$

is the vertical gradient of the refractive index inside the considered layer. Then, introducing parameters

$$R_i = R + h_0 - h_i,\ y_{i-1} = (h_i - h_{i-1}) / R_{i-1},\ \mu_i = \frac{\gamma_i}{n_i} R_i$$

and taking into account that $y_{i-1} \ll 1$, the equations for the range to the viewed object along the earth's surface with respect to the aircraft position, which is determined as a sum of the surface projections of the parts of the object observation trajectory through k atmospheric layers between the earth's surface and the aircraft, and for the angle between the tangent to the observation trajectory and the normal to the earth's surface at the boundary of the ith layer can be presented as

$$L = \sum_{i=1}^{k} L_{i\,\mathrm{cur}} = \sum_{i=1}^{k} R \sin\psi_{i-1} \times$$

$$\times \int_0^{y_{i-1}} \frac{\mathrm{d}x}{(1-x)\sqrt{(1+\mu_{i-1}x)^2 (1-x)^2 - \sin^2\psi_{i-1}}} =$$

$$= \sum_{i-1}^{k} R y_{i-1} \tan\psi_{i-1} \left[1 + \frac{1}{2}\left(1 + \frac{1-\mu_{i-1}}{\cos^2\psi_{i-1}}\right) y_{i-1}\right] + o(y_{i-1}); \quad (2)$$

$$\psi_{i-1} = \psi_{i-2} + \sin\psi_{i-2} \times$$

$$\times \int_0^{y_{i-2}} \frac{\left[1 + \mu_{i-2}(2x-1)\right]\mathrm{d}x}{(1-x)(1+\mu_{i-2}x)\sqrt{(1-x)^2(1+\mu_{i-2}x)^2 - \sin^2\psi_{i-2}}} =$$

$$= \psi_{i-2} + y_{i-2}\tan\psi_{i-2} \times$$

$$\times \left\{1 - \mu_{i-2} + \frac{1}{2}\left[1 + \mu^2_{i-2} + \frac{(1-\mu_{i-2})^2}{\cos^2\psi_{i-2}}\right] y_{i-2}\right\} + o(y_{i-2}).$$

The equation for the error in the visual determination of the range to the object along the earth's surface can be written as $\Delta L = L_{\mathrm{app}} - L$, where $L_{\mathrm{app}} = R\theta_0$ is the range to the viewed object along the earth's surface at the straight-line observation trajectory. From the consideration of OBE and ABE triangles in Fig. 1, we can easily find the central angle Θ_0, at which L_{app} is seen from the center of the globe O and demonstrate that

$$L_{\mathrm{app}} = -R\psi_0 + R\arcsin\left[\left(1 + \frac{h_0}{R}\right)\sin\psi_0\right] = h_0\tan\psi_0 \times$$

$$\times \left[1 - \frac{1}{2}\frac{h_0}{R}\tan^2\psi_0 + \frac{1}{6}\frac{h_0^2}{R^2}\tan^2\psi_0\left(1 + 3\tan^2\psi_0\right) + o\left(\frac{h_0^2}{R^2}\right)\right]. \quad (3)$$

At $R \to \infty$, Eqs. (2) and (3) coincide with the equations derived in Ref. 3.

Equations (2) and (3) can be used to estimate the refraction errors in determination of the range to surface objects in the case of visual observation from aboard an aircraft from the vertical profiles of temperature and absolute humidity, as well as the profiles of the refractive index retrieved from them (Fig. 2). The temperature and humidity profiles shown in Fig. 2 were retrieved from the data of nighttime and morning temperature and wind sensing of the atmosphere on April 21 of 1990 in Voronezh (Russia).

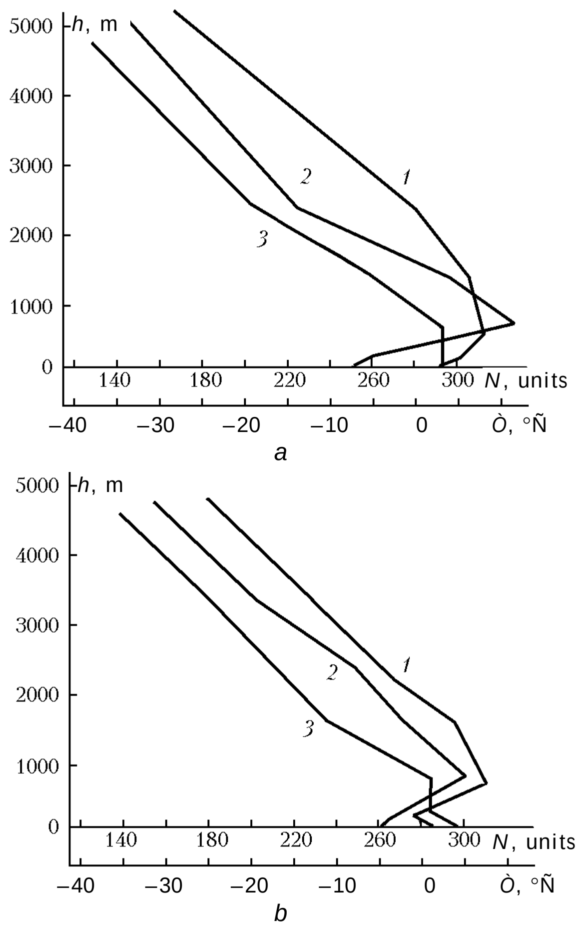

Fig. 2. Vertical profiles of temperature (*1*) and humidity (*2*) and the profiles of the atmospheric refractive index (*3*) retrieved from the data of the temperature and wind sensing of the atmosphere: nighttime (3 LT) (*a*) and morning (9 LT) (*b*) sensing.

The curves in Fig. 2 indicate the presence of surface temperature inversions characteristic of nighttime conditions and elevated inversions typical of morning conditions. These inversions affect the vertical distribution of the refractive index. As can be judged from the figures, the thickness of the inversion layer decreased from 500 to 400 m for 6 hours between the time of the night and morning sensing events. Figure 3 depicts the calculated errors in the visual determination of the range to surface objects from aboard an aircraft for the vertical profiles of the refractive index shown in Fig. 2. The errors are presented as functions of the aircraft altitude. The calculations were performed, as in Ref. 3, for the viewing angle of 2°40′ (curves *1*) and 3°15′ (curves *2*) with respect to the plane parallel to the earth surface at the point of aircraft location.

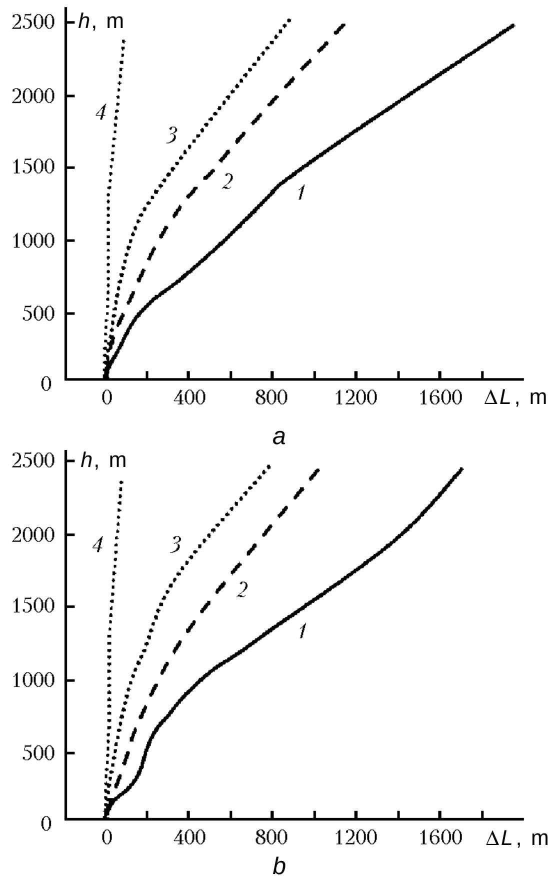

Fig. 3. Errors in the visual determination of the range to the viewed object along the earth's surface as estimated from the data of the temperature–wind sensing: nighttime sensing (3 LT) (*a*), morning sensing (9 LT) (*b*). Curves *1*, *2* are obtained with the allowance for the earth's surface curvature for the viewing angles of 2°40′ and 3°15′; curves *3*, *4* are obtained neglecting the earth's surface curvature for the viewing angles of 2°40′ and 3°15′.

To estimate the influence of the earth's surface curvature on the refraction errors, Fig. 3 also depicts the errors calculated based on the equations derived in Ref. 3 on the assumption of the plane earth surface (curves *3* for the angle of 2°40′ and curves *4* for the angle of 3°15′).

It can be seen from Fig. 3 that, as in Ref. 3, if the areas with temperature inversions are present in the surface atmospheric layer, then the refraction errors in visual determination of the range to surface objects along the earth's surface with respect to the aircraft location increase with the increasing height and decreasing viewing angle. Thus, the error in visual determination of the range to surface objects viewed at the angle of 2°40′ from aboard an aircraft flying at the altitude of 250 m in the presence of the near-surface inversion layer (3 LT) is 100 m, while in the case of the elevated inversion (9 LT) it is 120 m. If objects are viewed at the angle of 3°15′, these errors, as follows from curves *2*, decrease, respectively, to 50 and 60 m.

From the refraction errors in visual ranging of surface objects calculated based on the equations derived in Ref. 3 (curves *3*, *4*), all other conditions being the same, we can see that at the viewing altitude of 250 m the errors do not exceed 10–15 m. Consequently, the neglect of the earth's surface curvature in the considered case leads to the almost tenfold underestimation of the refraction errors in ranging of surface objects.

With the further increase of the viewing altitude, for example, for $h > 500$ m in the presence of the near-surface inversion layer (Fig. 3*a*) and for $h > 800$ m in the presence of the elevated inversion layer (Fig. 3*b*), the refraction errors in ranging of surface objects increase, also significantly exceeding the analogous calculated data obtained in Ref. 3.

Thus, the neglect of the earth's surface curvature when calculating the refraction errors in ranging of surface objects from the data of temperature and wind sensing leads to significant underestimation of this errors, and the extent of this underestimation increases with increasing height of flight.

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