Influence of the outer scale of atmospheric turbulence on the quality of optical imaging

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In this paper I analyze the influence of the outer scale of the atmospheric turbulence on the quality of an optical imaging based on the method of moments applied to the optical transfer function of the "turbulent atmosphere – telescope" system at measuring an averaged image of a source of an incoherent optical radiation. Thus, the behavior is analyzed of the integral resolution and three types of limiting values of the spatial scales resolvable by an optical imaging system. The image scales are determined by the moments of the zero, second, fourth, and sixth orders, respectively. It is shown that the influence of the outer scale of atmospheric turbulence on the above-mentioned characteristics can manifest itself only at small values of the variance of fluctuations of a complex phase of a plane optical wave. Distortions of an image of a source of an incoherent radiation estimated by the criteria of quality based on the moments of the zero, second, fourth, and sixth orders in the helical atmospheric turbulence are less than in the Kolmogorov turbulent media, and the influence of the outer scale of the atmospheric turbulence is practically the same in both of the cases.

The turbulence is one of the main factors limiting the resolution of optical imaging systems operating in the atmosphere. In this connection, many attempts were initiated to develop specific methods or devices to improve the quality of optical imaging by elimination of the distortions due to atmospheric turbulence. To solve this problem, it is necessary to have specific criteria of estimating the image quality. The lack of universal criterion of the image quality has been noted repeatedly.[1–4] Random inhomogeneities of the medium are connected in a complex way with the image quality, but practically the image quality can commonly be characterized by a single number.[1–6] Such assessments have all the drawbacks of one-way description of a complex phenomenon. The problem lies in the fact that the single-number criteria of the image quality characterize only one aspect of much complicated process of optical imaging.

Within the framework of such concepts an investigation was made[1,2] of the influence of the outer scale of the atmospheric turbulence on the optical transfer function and the integral resolution of the "turbulent atmosphere – telescope" optical system for different methods of post-detector processing of images (recording of an averaged image, Labeyrie, Noks–Thompson methods, and triple correlation of the image intensity).

In few recent years a series of publications have appeared,[3,4] in which it was proved that the efficient outer scale of atmospheric turbulence on the vertical paths is not that large as it was assumed earlier. In any case, for the apertures of the existing telescopes the relation between the aperture diameter and the outer scale of the atmospheric turbulence cannot be considered infinitely small. The authors of Refs. 5 and 6 have proved that the outer scale of atmospheric turbulence is one of the main parameters determining the image quality of a source of an incoherent optical radiation observed through the turbulent atmosphere.

This paper presents the assessments of the influence of the outer scale of atmospheric turbulence on the quality of an optical image based on the method of moments applied to the optical transfer function of the "turbulent atmosphere–telescope" system when recording the averaged image of a source of an incoherent optical radiation. I have tried to analyze in a more detail the behavior of the integral resolution and three types of limiting values of spatial scales resolved by an optical imaging system that are determined by the moments of the zero, second, fourth, and sixth orders. The distorting effect is considered of not only Kolmogorov-type turbulence, as is commonly[1–6] done, but also of a helical turbulent atmosphere.[7,8]

To assess the quality of an optical image deteriorated by the atmospheric turbulence, let us use static moments of the optical transfer function of the "turbulent atmosphere–telescope" system. Consider now recording of the averaged image.[1,2] In this case the normalized optical transfer function of the "turbulent atmosphere–telescope" system is factored[1] and can be presented in the form

$$\tau(\mathbf{p}) = \tau_0(\mathbf{p})\tau_{\text{atm}}(\mathbf{p}), \quad (1)$$

where

$$\tau_0(\mathbf{p}) = \iint\limits_{-\infty}^{\infty} d\boldsymbol{\rho} K(\boldsymbol{\rho}) K^*(\boldsymbol{\rho}+\mathbf{p}) \Bigg/ \iint\limits_{-\infty}^{\infty} d\boldsymbol{\rho} K(\boldsymbol{\rho}) K^*(\boldsymbol{\rho})$$

is the normalized optical transfer function of the receiving optical system; $K(\boldsymbol{\rho})$ is the transmission function of the optical receiving system; $\boldsymbol{\rho}$ is the two-

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dimensional transverse coordinate at the input aperture of the telescope;

$$\tau_{\rm atm}(\mathbf{p}) = \exp\left[-\frac{1}{2}D(\mathbf{p})\right]$$

is the normalized optical transfer function of the turbulent atmosphere; $D(\mathbf{p})$ is the spatial structure function of the fluctuations of the complex phase of a plane optical wave[1,2]; $\mathbf{p}$ is the spatial scale.

Moments of the optical transfer function (1) can be determined as follows:

$$m_n = \int\int_{-\infty}^{\infty} \mathrm{d}\mathbf{p}\mathbf{p}^n \tau(\mathbf{p}), \qquad (2)$$

where $n \geq 0$ is the integer number. In what follows we consider the first seven moments: m_0, m_1, ..., m_6.

These moments have the following physical meanings. The zero-order moment ($n = 0$) is proportional to the integral resolution of the optical system:

$$m_0 = \int\int_{-\infty}^{\infty} \mathrm{d}\mathbf{p}\tau(\mathbf{p}),$$

that is

$$m_0 = \frac{F^2}{k^2}\Re,$$

where $\Re$ is the integral resolution of the optical system[2]; $k = 2\pi/\lambda$, λ is the optical radiation wavelength in vacuum; F is the focal length of the optical system. In a typical situation the optical transfer function of the "turbulent atmosphere–telescope" system is an even function,[1–6] therefore all odd moments ($2n + 1$) are equal to zero, i.e., in our case $m_1 = m_3 = m_5 = 0$. The even moments ($2n$) make it possible to determine the limiting values of spatial scales resolved by an optical system by use of the following formula:

$$p_{\rm lim,2n} = \sqrt[2n]{\frac{\int\int_{-\infty}^{\infty} \mathrm{d}\mathbf{p}(\mathbf{p}-\bar{\mathbf{p}})^{2n}\tau(\mathbf{p})}{\int\int_{-\infty}^{\infty} \mathrm{d}\mathbf{p}\tau(\mathbf{p})}},$$

where

$$\bar{\mathbf{p}} = \int\int_{-\infty}^{\infty} \mathrm{d}\mathbf{p}\mathbf{p}\,\tau(\mathbf{p}) = m_1$$

is the mean value of the spatial scale. As it is commonly supposed that $m_1 = 0$, then the final formula for determining the limiting values of spatial scales resolved by an optical system takes the form

$$p_{\rm lim,2n} = \sqrt[2n]{m_{2n}/m_0}. \qquad (3)$$

First we consider the moments of the optical transfer function of the telescope system in the absence of the atmospheric turbulence, i.e., at

$$\tau(\mathbf{p}) = \tau_0(\mathbf{p}).$$

For an ideal apodized system the transmission function of the optical receiving aperture is presented in the form of the Gaussian function:

$$\tau_0(\mathbf{p}) = \exp[-p^2/(4R^2)],$$

where R is the parameter of the Gaussian model having the meaning of the receiving aperture radius. In this case the even moments m_{2n} (2) take the following values: $m_0 = 4\pi R^2$, $m_2 = 16\pi R^4$, $m_4 = 128\pi R^6$, $m_6 = 1536\pi R^8$. Thus, using Eq. (3) one can obtain that

$$p_{\rm lim,2n} = \alpha_{2n}\, 2R, \qquad (4)$$

where

$$\alpha_2 = 1,\ \ \alpha_4 = \sqrt[4]{2} \cong 1.1892,\ \ \alpha_6 = \sqrt[6]{6} \cong 1.3480.$$

It turns out that the limiting values of the spatial scales resolved by an optical system even if defined in different ways are equal accurate to a factor α_{2n}. It should be noted that $p_{\rm lim,\,6} > p_{\rm lim,\,4} > p_{\rm lim,\,2}$, and in this case the increasing number of the moment, leads to $p_{\rm lim,\,2n}$ increase only because of the α_{2n} increase.

Similar results are also obtained for a circular aperture with a sharp edge. In this case, the normalized optical transfer function of the receiving optical system is as follows:

$$\tau_0(\mathbf{p}) = \begin{cases} \frac{2}{\pi}\left[\arccos\left(\frac{p}{2R_f}\right) - \frac{p}{2R_f}\sqrt{1-\left(\frac{p}{2R_f}\right)^2}\right], & \text{at } p \leq 2R_f, \\ 0, & \text{at } p > 2R_f. \end{cases} \qquad (5)$$

Note that the circular aperture radius with a sharp edge R_f is related to the model parameter of the Gaussian aperture R by the ratio:

$$R_f = \sqrt{2}\,R.$$

Under these conditions, the coefficients in the formula (4) take the following form:

$$\alpha_2 = 1,\ \ \alpha_4 = \sqrt[4]{5/3} \cong 1.1362,\ \ \alpha_6 = \sqrt[6]{7/2} \cong 1.2322.$$

Thus, the change of the aperture transmission at its edge results in a change of the coefficients α_{2n}. At the same time, it is easily seen that some general features of their behavior keep the same. Thus, in particular the increase with the increasing number of the moment is observed in this case too.

It is interesting to analyze the effect that can be expected from random turbulent inhomogeneities occurring in the atmosphere along the propagation path on these characteristics. Consider first the atmospheric turbulence to be of the pure Kolmogorov type, i.e., let us assume that the structure function of fluctuations of a complex phase of a plane optical wave equals

$$D(\mathbf{p}) = 2(p/\rho_0)^{5/3},$$

where

$$\rho_0 = \left[2^{-5/3}(18/5)0.033\pi^2\Gamma(7/6)/\Gamma(11/6)C_\varepsilon^2 k^2 h\right]^{-3/5}$$

is the radius of coherence of a plane optical wave in the Kolmogorov atmospheric turbulence; C_ε^2 is the structure parameter of the Kolmogorov atmospheric turbulence; h is the effective thickness of an optically active layer of the atmospheric turbulence.[1,2]

For the sake of simplicity of the provisional estimates we use the square-law approximation of the structure function of fluctuations of a complex phase of a plane optical wave in the turbulent atmosphere. Let us assume that the optical transfer function can be presented as follows:

$$\tau(\mathbf{p}) = \exp\left[-\frac{p^2}{4R^2} - \left(\frac{p}{\rho_0}\right)^{5/3}\right] \approx \exp\left[-\frac{p^2}{4R^2} - \left(\frac{p}{\rho_0}\right)^{2}\right],$$

then

$$m_0 \approx 4\pi R^2 \Big/ \left[1 + 4\left(\frac{R}{\rho_0}\right)^2\right], \qquad (6)$$

i.e., at $R \ll \rho_0$ in the regions of weak image distortions $m_0 \cong 4\pi R^2$, and at $R \gg \rho_0$ for the developed speckle-structure of the image $m_0 \cong \pi\rho_0^2$.

In a similar way, one can derive an expression for the limiting values of spatial scales resolved by the optical system:

$$p_{\mathrm{lim},2n} = \alpha_{2n}\, 2R \Big/ \sqrt{1 + 4(R/\rho_0)^2}. \qquad (7)$$

In this case at $R \ll \rho_0$ $p_{\mathrm{lim},\,2n} \cong 2R$, and at $R \gg \rho_0$ $p_{\mathrm{lim},\,2n} \cong \rho_0$, i.e., in the regions of weak image distortions the limiting value of spatial scales, resolved by the optical systems, is determined by the receiving aperture radius, and for the image with well developed speckle-structure by the coherence radius of an incident optical wave.

It is known[2] that the integral resolution $\Re$, as a standard of the optical quality of a system, determines the value of the minimum distance δl resolved by the system:

$$\delta l \cong 1/(2\sqrt{\Re}).$$

On the other hand, from Eqs. (6) and (7) the ratio can be obtained relating the value of the minimum distance δl resolved by the system and the limiting values of the spatial scales resolved by the optical system $p_{\mathrm{lim},\,2n}$:

$$\delta l \cong \alpha_{2n} \Big/ \left(2\sqrt{\pi}\frac{k}{F}p_{\mathrm{lim},2n}\right).$$

From this it follows that in the case considered the evaluations of resolution by moments of the zeroth and second order give the same result. Besides all the even moments $(2n)$ of the optical transfer function of the "turbulent atmosphere–telescope" system depend on the distorting effect of the atmospheric turbulence practically equally and, allegedly this would make it possible, in what follows, to limit the consideration only by two lower moments: the zeroth one (for investigating the integral resolution of the optical system $\Re$) and the second one (for analyzing the limiting value of spatial scales resolved by the optical system $p_{\mathrm{lim},2}$). However, this statement is correct only for a quadratic medium (i.e., for a medium with $D(\mathbf{p}) \sim p^2$) and only for it. This statement is incorrect already for Kolmogorov atmospheric turbulence (when $D(\mathbf{p}) \sim p^{5/3}$) and it becomes more incorrect for a Kolmogorov medium with the finite value of the outer scale of turbulence.

To estimate the influence of the finite value of the outer scale of the atmospheric turbulence,[3,4] the spatial structure function of the complex phase fluctuations of a plane optical wave is presented in the form:

$$D(\mathbf{p}) \cong 2\left(\frac{L_0}{\rho_0}\right)^{5/3}\left\{1 - \exp\left[-\left(\frac{p}{L_0}\right)^{5/3}\right]\right\}, \qquad (8)$$

where L_0 is the outer scale of the Kolmogorov atmospheric turbulence. In the region of small scales ($p < L_0$) this model of the spatial structure function of complex phase fluctuations of a plane optical wave (8) corresponds to the Kolmogorov model:

$$D(\mathbf{p}) \cong 2(p/\rho_0)^{5/3},$$

and for scales exceeding the outer scale of the atmospheric turbulence ($p > L_0$), the structure function (8) equals the doubled value of the variance σ^2 of the complex phase fluctuations of a plane optical wave:

$$D(\mathbf{p}) \cong 2(L_0/\rho_0)^{5/3} = 2\sigma^2.$$

The saturation to the level, which is equal to the double variance, for the model (8) occurs at $p \cong 2L_0$ accurate to 5%. A detailed comparison of the functional form of the structure function described by the model (8) with the known representations for the spatial structure functions of a plane optical wave[3,4] is given in the Appendix A.

Results of numerical calculations of the moments of the zeroth and the second order of the optical transfer function of the "turbulent atmosphere – telescope" system (1) and (5) for the model of the spatial structure function of the complex phase fluctuations of a plane optical wave (8) are shown in Figs. 1 and 2.

These figures show the normalized values of the zeroth moment ($m_0/\lim m_0$), where $\lim m_0$ is the limiting value of the zeroth moment for the telescope with an infinite radius of the receiving aperture ($R \to \infty$) and at an infinite outer scale of the Kolmogorov atmospheric turbulence ($L_0 \to \infty$), and the second ($m_2/[(2R)^2 m_0]$) order moment of the optical transfer function of the "turbulent atmosphere–telescope" system, which are the functions of dimensionless parameters R/ρ_0 and L_0/ρ_0. The dependences of moments of the fourth and sixth order ($m_4/[(2R)^4 m_0]$ and $m_6/[(2R)^6 m_0]$) are qualitatively similar to the behavior of the second order moment ($m_2/[(2R)^2 m_0]$).

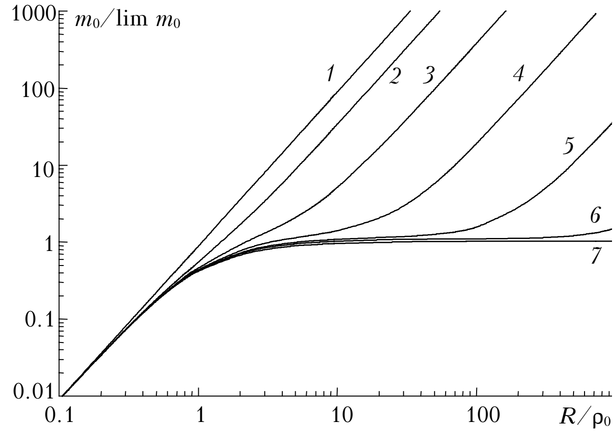

Fig. 1. Normalized resolution of the "turbulent atmosphere – telescope" optical system for different values of the outer scale of the Kolmogorov atmospheric turbulence: $L_0/\rho_0 = 0.1$ (*1*); 1 (*2*); 2 (*3*); 3 (*4*); 4 (*5*); 5 (*6*); ≥ 10 (*7*).

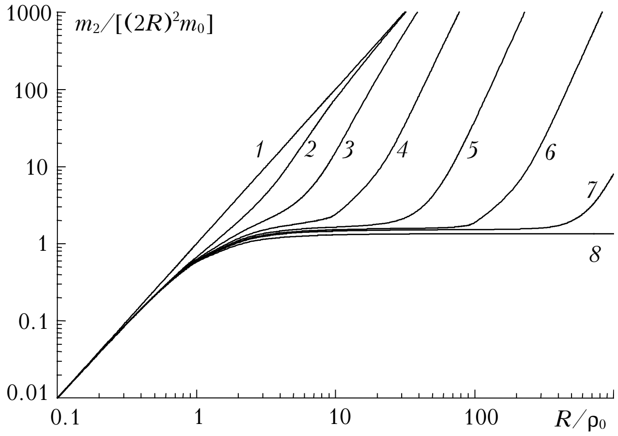

Fig. 2. Normalized moment of the second order of the optical transfer function of the "turbulent atmosphere – telescope" optical system for different values of the outer scale of the Kolmogorov atmospheric turbulence: $L_0/\rho_0 = 0.1$ (*1*); 2 (*2*); 3 (*3*); 4 (*4*); 5 (*5*); 6 (*6*); 7 (*7*), ≥ 10 (*8*).

Figures 1 and 2 reveal the peculiarity of the static moments (2) of the optical transfer function of the "turbulent atmosphere–telescope" system (1) and (5) that an essential dependence of those on the value of the outer scale of the Kolmogorov atmospheric turbulence manifests itself only if two conditions are fulfilled simultaneously: 1) the radius of the receiving aperture must be larger than the outer scale of the Kolmogorov atmospheric turbulence ($R > L_0$), and 2) the radius of the optical wave coherence must have the same order of magnitude as the outer scale of the Kolmogorov atmospheric turbulence ($\rho_0 \sim L_0$).

At simultaneous fulfillment of these two conditions the quality of imaging a source of incoherent radiation is found to be much higher than in the medium with an infinite value of the outer scale of the atmospheric turbulence and commensurable with the image quality in a homogeneous medium. In this case we can observe an increase in the integral resolution of the optical system with the growth of the receiving aperture radius by the same law as in the homogeneous medium ($\sim R^2$). An asymptotic expression can readily be obtained for the zeroth order moment at $R > L_0$ and $\rho_0 \sim L_0$:

$$\frac{m_0}{\lim m_0} \cong 4\left(\frac{R}{\rho_0}\right)^2 \exp\left[-\left(\frac{L_0}{\rho_0}\right)^{5/3}\right].$$

The estimates by the formula completely agree with the straight-line sections of the curves *1–5* in the upper part of Fig. 1.

Note that the ratio between the outer scale and the coherence radius of a plane optical wave in the Kolmogorov atmospheric turbulence has a clear physical meaning, i.e., and it is proportional to the value of the variance of the complex phase fluctuations of a plane optical wave along the path of length h:

$$L_0/\rho_0 = \left(\sigma^2\right)^{3/5}.$$

Thus, Fig. 1 shows that the integral resolution of the "turbulent atmosphere – telescope" optical system begins to exceed its limiting value for the telescope with an infinite radius of the receiving aperture ($R \to \infty$) and at an infinite outer scale of the Kolmogorov atmospheric turbulence ($L_0 \to \infty$) and $\sigma^2 \le (5)^{3/5} \cong 14.6201 \approx 15$ while at $\sigma^2 < 1$ the integral resolution coincides with the similar characteristic for the homogeneous medium. Consequently, only under conditions when the variance of the complex phase fluctuations of a plane optical wave is below a certain level, namely, when $\sigma^2 \le 15$, the image quality (by the criterion of the integral resolution) in the turbulent medium will be higher because of the effect of the limitedness of the outer scale of the Kolmogorov atmospheric turbulence than in the medium with an infinite value of the outer scale. At $\sigma^2 > 15$ the effect of the outer scale on the integral resolution will not manifest itself at $R \le 10^3\rho_0$. As to the moments of the second, fourth, and sixth orders, the boundary of this effect will be shifted toward the increase of the variance of the complex phase fluctuations of a plane optical wave:

$$\sigma^2 \le 7^{5/3} \cong 25.6151 \approx 25,$$

$$\sigma^2 \le 9^{5/3} \cong 38.9407 \approx 40,$$

and

$$\sigma^2 \le 11^{5/3} \cong 54.4070 \approx 55$$

(see, for example, Fig. 2).

The reason is that the scales of the distortion of the wave front of the optical radiation transmitted through the turbulent atmosphere and received by a telescope under conditions of $R > L_0$ and $\rho_0 \sim L_0$, are found to be less or comparable with the size of the receiving aperture, and in forming the image, a partial compensation for them by the optical system itself occurs (mainly at the cost of elimination of wave front slopes). It is for this reason, that the models of turbulence with the infinite outer scale take into account the scales of atmospheric turbulence, which do not exist in reality that leads to overestimating the distorting effect of the turbulence in estimating the image quality of sources of incoherent radiation viewed through the atmosphere.

It is a common knowledge that for a wide range of conditions, realized in the Earth's atmosphere, the atmospheric turbulence can be considered as the Kolmogorov one (–5/3 law). Correspondingly, in the majority of investigations of the influence of the atmospheric turbulence on the image quality, the Kolmogorov spectrum of the atmospheric turbulence is normally used.[1–6]

However, in the atmosphere the turbulence can exist with a chaotic helical motions.[7,8] To characterize the helical motions, the notion of helicity has been introduced.[7] The helicity has a considerable effect on the stability and evolution of both laminar and turbulent flows where the helical cascades can occur. In helical media the reverse energy transfer is possible into the long-wave range (because of the tendency toward confluence of the helical vortices – the Bernoulli effect). Under conditions of the Earth's atmosphere there exists a wide range of effects generating both the helicity itself and its intense fluctuations. In particular, the simultaneous presence of such factors as gradients of density and temperature, intense temperature fluctuations, the shear flows, and inhomogeneous rotation is sufficient.

In the atmosphere two limiting cases[7] can be realized: 1) parallel flows of energy and helicity by the spectrum, corresponding to the Kolmogorov cascade ("–5/3 law"), and 2) the helicity flow without the energy flow, which is a purely helical cascade ("–7/3 law"). The effect of an indented relief of the landscape can be a governing factor for a type of the generated turbulence. Thus, it should be expected that over the hilly or mountainous land the helical characteristics of the turbulence manifest themselves stronger than over a flat relief.

For the case of helical (vortex) atmospheric turbulence an analog of Eq. (8) has the form (see the Appendix B):

$$D(\mathbf{p}) \cong 2\left(\frac{L_{0h}}{\rho_{0h}}\right)^{4/3}\left\{1-\exp\left[-\left(\frac{p}{L_{0h}}\right)^{4/3}\right]\right\}, \qquad (9)$$

where L_{0h} is the outer scale of the helical atmospheric turbulence; ρ_{0h} is the coherence radius of a plane optical wave in the helical atmospheric turbulence. Because very little is known about the helical (vortex) atmospheric turbulence, we have no concrete data on the values of the parameters L_{0h} and ρ_{0h}. However, let us assume that the magnitudes of these parameters for the helical turbulence are of the same order as those for the Kolmogorov turbulence.

Figure 3 shows the compared magnitudes of the moments of the zeroth, second, fourth, and sixth orders of the optical transfer function of the "turbulent atmosphere – telescope" system for the case of helical atmospheric turbulence with the same characteristics for the Kolmogorov one.

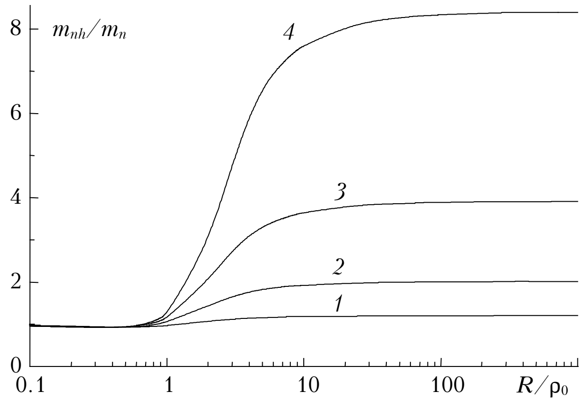

Fig. 3. The ratio between the moments of the zeroth (*1*), second (*2*), fourth (*3*), and sixth (*4*) orders of the "turbulent atmosphere – telescope" optical system for the helical atmospheric turbulence and similar value for the Kolmogorov atmospheric turbulence.

It turns out that in the helical atmospheric turbulence all the moments, at $R/\rho_0 < 1$, are smaller (although only slightly), than in the Kolmogorov atmospheric turbulence, and at $R/\rho_0 > 1$, on the contrary, are larger (in this case, with increasing n the difference grows). Because the random heterogeneous medium decreases the magnitude of a static moment, as compared with that in a homogeneous medium, a conclusion can be drawn that imaging of a source of an incoherent radiation through the layer of the helical atmospheric turbulence takes place with smaller distortions than that through the Kolmogorov atmospheric turbulence.

Figures 4 and 5 show calculated results on the same magnitudes that are shown in Figs. 1 and 2, but for the helical turbulent atmosphere. In both of these cases the behavior of these characteristics is practically the same. In particular, it turns out that the magnitude of the normalized moment of the zeroth order ($m_{0h}/\lim m_{0h}$, the definition being analogous to the case of the Kolmogorov atmospheric turbulence) is sensitive to the value of the outer scale of the helical atmospheric turbulence unless the variance of complex phase fluctuations of a plane optical wave satisfies the condition: $\sigma^2 \le 7.5^{4/3} \cong 14.6807 \approx 15$, for the moments of the second order ($m_2/[(2R)^2 m_0]$) $\sigma^2 \le 11^{4/3} \cong 24.4638 \approx 25$, the fourth order ($m_4/[(2R)^4\, m_0]$) $\sigma^2 \le 16^{4/3} \cong 40.3175 \approx 40$, and the sixth order ($m_6/[(2R)^6\, m_0]$) $\sigma^2 \le 20^{4/3} \cong 54.2884 \approx 55$. A comparison with analogous criteria for the Kolmogorov atmospheric turbulence enables us to conclude that the effects of the outer scale are much the same both for the helical turbulence and Kolmogorov atmospheric turbulence.

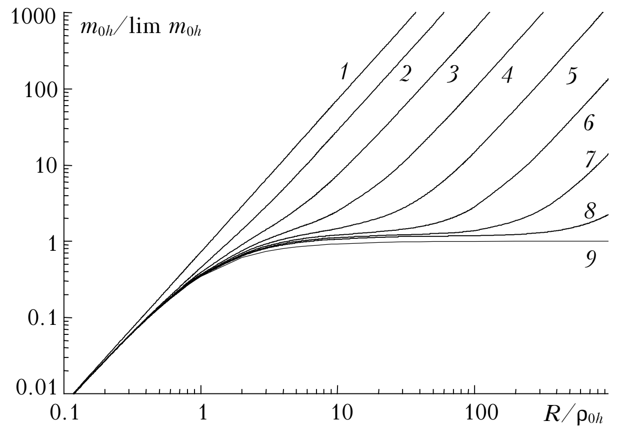

Fig. 4. Normalized resolution of the "turbulent atmosphere – telescope" optical system at different values of the outer scale of the helical atmospheric turbulence: L_{0h}/ρ_{0h} = 0.1 (*1*); 1 (*2*); 2 (*3*); 3 (*4*); 4 (*5*); 5 (*6*); 6 (*7*); 7 (*8*); ≥ 10 (*9*).

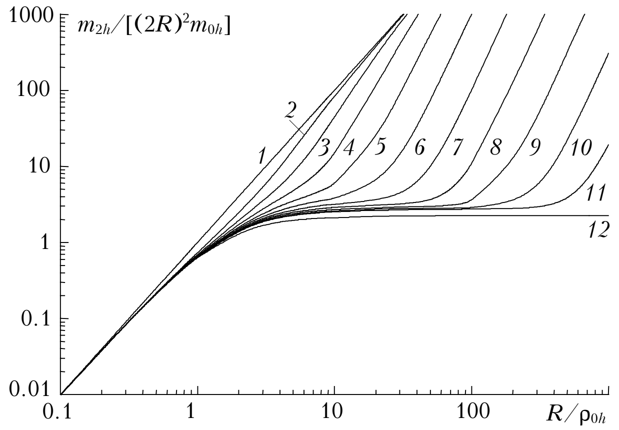

Fig. 5. Normalized moment of the second order of the optical transfer function of the "turbulent atmosphere – telescope" optical system at different values of the outer scale of the helical atmospheric turbulence: L_{0h}/ρ_{0h} = 0.1 (*1*); 2 (*2*); 3 (*3*); 4 (*4*); 5 (*5*); 6 (*6*); 7 (*7*); 8 (*8*); 9 (*9*); 10 (*10*); 11 (*11*); ≥ 13 (*12*).

The consideration of static moments of the optical transfer function of the "turbulent atmosphere–telescope" system has shown that in the general case one can equally use, as a measure of the optical system resolution, either the integral of the normalized optical transfer function (proportional to the zeroth order moment) or any of the three types of limiting spatial scales resolvable by the optical imaging system being determined by the moments of the second, fourth, and sixth order, since all the moments of the even orders ($2n$) have similar structure.

Appendix A

Let us compare the model (8), used in this paper for describing the spatial structure function of the complex phase fluctuations of a plane optical wave propagating through the Kolmogorov turbulent atmosphere with the finite value of the outer scale of the atmospheric turbulence, with the models known from the literature.[3,4] The most widely used models are the following:

– the exponential model[3,4]

$$\Phi_\varepsilon(\kappa) = 0.033\, C_\varepsilon^2\, \kappa^{-11/3}\left[1-\exp\left(-\kappa^2/\kappa_{0e}^2\right)\right], \quad (A.1)$$

where $\kappa_{0e} = 2\pi/L_{0e}$, L_{0e} is the value of the outer scale of the Kolmogorov atmospheric turbulence for the exponential model; $\kappa = 2\pi/l$, l is the spatial scale of the inhomogeneity of the turbulent medium;

– the Karman model[3,4]

$$\Phi_\varepsilon(\kappa) = 0.033 C_\varepsilon^2\left(\kappa_{0k}^2+\kappa^2\right)^{-11/6}, \quad (A.2)$$

where $\kappa_{0k} = 2\pi/L_{0k}$, L_{0k} is the value of the outer scale of the Kolmogorov atmospheric turbulence for the Karman model,

– the Greenwood–Tarasano model[3,4]:

$$\Phi_\varepsilon(\kappa) = 0.033 C_\varepsilon^2 L_{0t}^{11/3}\left(\kappa L_{0t}+\kappa^2 L_{0t}^2\right)^{-11/6}, \quad (A.3)$$

where L_{0t} is the value of the outer scale of the Kolmogorov atmospheric turbulence in the Greenwood–Tarasano model.

Below are written the relationships between the outer scale of the Kolmogorov atmospheric turbulence from different models of spectral density of fluctuations of the dielectric constant of the atmosphere obtained assuming the variance of complex phase fluctuations of a plane optical wave calculated by these models to be of the same magnitude:

$$L_{0e} = \pi L_0 \cong 3.1416\, L_0,$$

$$L_{0k} = \pi\left[6\Gamma(7/6)/\Gamma(11/6)\right]^{3/5} L_0 \cong 9.1292 L_0,$$

$$L_{0t} = \frac{1}{2}\left[\Gamma(8/3)\right]^{-3/5} L_0 \cong 0.3913 L_0.$$

Figure 6 shows a comparison of the results calculated using Eq. (8) on the normalized spatial structure function of the complex phase fluctuations of a plane optical wave ($D(\mathbf{p})/(2\sigma^2)$) with the data of numerical integration made using the spectral densities of fluctuations of the dielectric constant of the atmosphere (A.1), (A.2), and (A.3). It should be noted that the model (8), used in this paper, better agrees with the asymptotic for the inertial interval of wave numbers ($p < L_0$):

$$\frac{D(\mathbf{p})}{2\sigma^2} \cong \left(\frac{p}{L_0}\right)^{5/3},$$

than other models. Besides, the model (8) faster yields the limiting value for the spatial structure function, which is equal to a doubled value of the variance of the complex phase fluctuations of a plane optical wave.

The disadvantages of the model (8) are: 1) the lack of a simple analytical representation for $\Phi_\varepsilon(\kappa)$, which is the spectral density of dielectric constant fluctuations of the atmosphere; 2) it is impossible to allow for the inner scale of the atmospheric turbulence in this model.

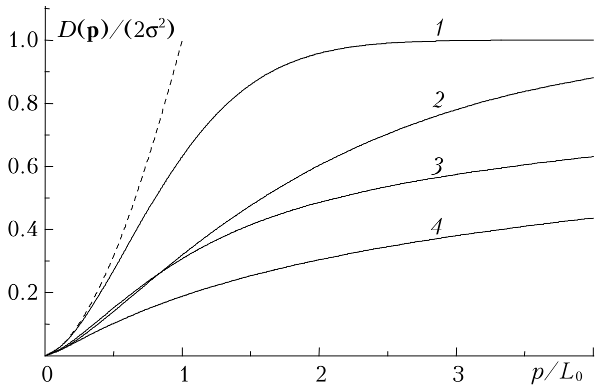

Fig. 6. Normalized spatial structure function of the complex phase fluctuations of a plane optical wave for different models of the Kolmogorov turbulence: the author's model (8) (*1*); the Karman model (*2*); the exponential model (*3*); the Greenwood–Tarasano model (*4*). The asymptotic for the inertial interval of wave numbers $D(\mathbf{p})/(2\sigma^2) \cong (p/L_0)^{5/3}$ is shown by the dashed curve.

Appendix B

It is known[7] that in a stratified atmosphere a region is formed in the turbulent flux with a cascade transfer of the helicity through the spectrum – the so-called region of helical scaling. For this cascade interval the helicity flow is the constant value independent of the scale. For locally isotropic field the structure function of the atmospheric dielectric constant fluctuations depends only on the distance between the observation points r, in the case of helical (vortex) turbulence the structure function can be presented in the form

$$D_{\varepsilon h}(r) = C_{\varepsilon h}^2 r^{1/3}$$

of the "1/3 law" for the inertial interval of the helical turbulence. Here $C_{\varepsilon h}^2$ is the structure parameter of the helical atmospheric turbulence in $\mathrm{m}^{-1/3}$.

In the case of locally isotropic field one can introduce a one-dimensional spectrum of the atmospheric dielectric constant fluctuations $V_{\varepsilon h}(\kappa)$:

$$V_{\varepsilon h}(\kappa) = \frac{1}{2\pi\kappa}\int_0^\infty d\rho\, \sin(\kappa r)\, D'_{\varepsilon h}(r) =$$

$$= \frac{\Gamma(1/3)\sin(\pi/6)}{6\pi} C_{\varepsilon h}^2 \kappa^{-4/3} \cong 0.0711 C_{\varepsilon h}^2 \kappa^{-4/3}.$$

The three-dimensional spectral density of the atmospheric dielectric constant fluctuations for the helical turbulence can be written as follows:

$$\Phi_{\varepsilon h}(\kappa) = -\frac{1}{2\pi\kappa}\frac{dV_{\varepsilon h}(\kappa)}{d\kappa} =$$

$$= \frac{\Gamma(1/3)\sin(\pi/6)}{9\pi^2} C_{\varepsilon h}^2 \kappa^{-10/3} \cong 0.0151 C_{\varepsilon h}^2 \kappa^{-10/3}.$$

The spatial structure function of the complex phase fluctuations of a plane optical wave for the inertial interval of the helical turbulence equals:

$$D_h(\mathbf{p}) = 2\pi^2 k^2 h \int_0^\infty d\kappa\, \kappa \Phi_{\varepsilon h}(\kappa)\left[1 - J_0(p\kappa)\right] =$$

$$\cong 2^{-7/3}\cdot 9\cdot 0.0151\pi^2 \frac{\Gamma(4/3)}{\Gamma(5/3)} C_{\varepsilon h}^2 k^2 h p^{4/3} \cong$$

$$\cong 0.2633\, C_{\varepsilon h}^2\, k^2 h p^{4/3} = 2\left(p/\rho_{0h}\right)^{4/3},$$

where J_0 is the zeroth-order Bessel function of the first kind;

$$\rho_{0h} = \left[2^{-10/3}\cdot 9\cdot 0.0151\pi^2 \Gamma(4/3)/\Gamma(5/3) C_{\varepsilon h}^2 k^2 h\right]^{-3/4} =$$

$$= 4.5759\left(C_{\varepsilon h}^2 k^2 h\right)^{-3/4}$$

is the coherence radius of a plane optical wave in the helical atmospheric turbulence.

Thus, the spatial structure function of the complex phase fluctuations of a plane optical wave is described by the "4/3 law" for the helical (vortex) turbulence, instead of the "5/3 law" for the Kolmogorov turbulence.

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