Technique of ultra-short-term forecast of atmospheric parameters based on the Kalman filtering algorithm and 2D dynamic-stochastic model

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Technique of ultra-short-term forecast of atmospheric parameters based on the Kalman filtering algorithm and 2D dynamic-stochastic model of the regression type is considered, and the results of its qualitative estimate for the boundary layer (up to the altitude of 800 m) based on meteorological and aerological observations are discussed.

Introduction

Rapid development of industry, transportation, and energy production in recent years, as well as introduction of new production processes associated with weapon tests and recovery of ecologically hazardous technologies, brought about a considerable increase of pollutant emission to the atmosphere. In this regard, simulation and forecast of atmospheric pollution level on restricted territories (within big towns, industrial centers, and near the regions of testing and recycling of ecologically hazardous technologies) becomes an important problem of the environmental protection. Most urgent is the ultra-short-term forecast of atmospheric pollution necessary in taking timely measures against the increase of the pollutant concentration.

The time variations of pollutant concentration depend mostly on changes of meteorological situation and primarily on the temperature stratification (influencing the conditions of the turbulence development) and wind (influencing the pollutant transport)[1]; therefore, it is necessary to solve preliminary the problem of ultra-short-term forecast of just these parameters.

It is well known that presently the ultra-short-term (up to 12 h) forecast is based mainly on two approaches: hydrodynamic and physical-statistical. However, both have some disadvantages.

For instance, in case of application of the mesoscale hydrodynamic model,[2] we deal with quite a cumbersome algorithm of its implementation, with considerable (up to 18–25%) contribution of the input data uncertainty to errors of numerical forecast schemes,[3] and, finally, with a necessity to invoke the measurement data on quite large territories. The last disadvantage impedes the solution of the problem based on observational data from a single station.

At the same time, the physical-stochastic approach, based on the most widespread methods of the regression analysis, though usable in solving the problem of the ultra-short-term forecast by the data from a single station, nonetheless requires constructing (based on the data of many-year observations) of regression prognostic equations, whose parameters can not be revised in the process of time forecasting. In addition, the amount of the input data must substantially exceed the number of regression coefficients to be determined. Besides, the least square method, used in the calculations, introduces some error, arising due to smoothing the extreme values.

The above-mentioned disadvantages of the hydrodynamic and physical-stochastic approaches have motivated recent intensive studies into development of new methods of the ultra-short-term forecasting of atmospheric state parameters provided a minimum of initial experimental information. One of such methods, in particular, is the modified method of clustering of arguments (MMCA), developed at the Institute of Atmospheric Optics SB RAS.[4–6] This method, after use of the procedure of its integration with the method of optimal extrapolation of random process (the latter is used for forecasting only ground-based values of a meteorological variable) allows the ultra-short-term forecasting of the atmospheric state parameters given the data of a limited number of fast-speed measurements from a single station.

Despite significant advantages of the MMCA (base method) over traditional regression methods (its realizability with data of a limited number of experimental measurements; a multi-criterion choice of the prognostic model; the absence of the necessity to use the least squares method introducing extra errors in calculation results), it has some drawbacks. They include, in particular, the need to invoke (in solving the problem of the time forecast) the additional method of random process extrapolation required in forecasting for the near-ground level; the necessity in the advance receipt of a sample of real-time data of the volume no less than $N = k + 1$ ($k \geq 7$ is the number of used levels); and meeting the condition that the time interval of the forecast coincides with that of aerological measurements.

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On account of all the aforesaid, we propose a new methodical approach to the problem of the ultra-short-term forecast of the atmospheric state parameters, based on the Kalman filtering algorithm and two-dimensional dynamic-stochastic model, describing the altitudinal and temporal variations of these parameters. A distinctive feature of this approach is that these variations represent stochastic processes with prescribed correlation properties. Moreover, in contrast to the hydrodynamic approach, our procedure of the ultra-short-term forecast is accomplished using the data of aerological observations from a single station.

In addition to the description in a detail of the new method, we also present the results of its qualitative estimate based on the data of the temperature–wind sensing.

1. Formulation of the problem and method of solution

The problem of ultra-short-term forecast of the field of some atmospheric parameter (i.e., meteorological variable) at a given spatial point (x_0, y_0, z_0) of aerological observations is in estimation of the field's value at the moment $t_0 + 1$ according to measurements at the moment t and the earlier moments being the predictors. The estimates are obtained using the given mathematical model describing variations of the field in altitude and time.

As such a model, the following model can be used

$$\xi_h(k) = \sum_{m=h-i}^{h+i} \sum_{j=1}^{K} d_{j,m}\xi_m(k-j), \tag{1}$$

where $\xi_h(k)$ is the value of the meteorological variable field at the height h at the moment k; m is the number of the altitudinal level, where the forecast is made, ranging from $h - i$ to $h + i$ (here $i = 1, 2, \ldots, n$ is the maximal number of altitudinal information levels, data from which are taken into account in forecasting the field ξ at the level h); j is the current value of the discrete time, varying from 1 to k (which, thereby, defines the window depth of the autoregression); $d_{j,m}$ are the unknown coefficients, which are evaluated. They determine the interrelation between the value of the field $\xi_h(k)$ to be estimated and its values at preceding moments at a given altitude and adjacent altitude levels, i.e., $\xi_m(k - j)$.

Thus, in accordance with Eq. (1), the estimate of the field of the given meteorological variable $\xi_h(k)$ at a fixed altitude level h and at a fixed moment k is a linear combination of the vector of unknown parameters $d_{j,m} = D$ and values of this field at preceding moments to the depth k, at the same altitude level and at ith above-lying and below-lying levels.

Further, dealing with the method of ultra-short-term forecast and in numerical experiments, we will assume $i = 1$, i.e., one level above and below the fixed one.

Note that in Eq. (1), the measured values of the meteorological field (not converted) are used as input data. This, naturally, imposes some limitations on the field.

To obtain estimates of the forecasted variable with acceptable quality, it is necessary that the function describing time variations of meteorological field be smooth and free of inflections.

Let us consider in detail the method of ultra-short-term forecast (for one and several steps forth), based on the Kalman filter and the model (1).

The problem of forecasting the ξ field at some observation point is split into two stages. At the first stage, model coefficients $d_{j,m}$ are estimated using values of meteorological field at the point (x_0, y_0), at the moment k and preceding moments at a fixed altitude level h and neighboring levels.

At the second stage, values of the field at a given point at the moment $k + 1$ and height h are reconstructed, using the estimated coefficients and mathematical model (1) on the assumption of the field's homogeneity and isotropy.

Thus, the forecast equation for one step forth is

$$\hat{\xi}_h(k+1) = \sum_{m=h-i}^{h+i} \sum_{j=0}^{K-1} \hat{d}_{j+1,m}\xi_m(k-j). \tag{2}$$

Here $\hat{\xi}_h(k+1)$ is the estimate of the meteorological field at the moment $k + 1$; $\hat{d}_{j+1,m}$ are unknown model parameters, estimated at the kth time step; $\xi_m(k - j)$ are measured values of meteorological field at the point of forecast at altitude levels from $h - 1$ to $h + 1$ at the moments from k to 1.

Following Ref. 7, to estimate the unknown parameters of the model (1), i.e., $d_{j,m}$, it is necessary to specify the system of difference equations in the matrix form

$$\mathbf{X}(k+1) = \mathbf{F}(k)\cdot\mathbf{X}(k) + \mathbf{\Omega}(k), \tag{3}$$

where

$$\mathbf{X}(k+1) = |d_{1,0}(k+1),\ d_{2,0}(k+1),\ d_{3,0}(k+1),\ d_{2i+1,0}(k+1),\ d_{1,1}(k+1),\ \ldots,\ d_{2i+1,k}(k+1)|^{\mathrm{T}}$$

is the state vector with dimension $(2i + 1)k$ (here T denotes the matrix transpose operator), containing the unknown parameters of the dynamic system state and parameters to be estimated; $\mathbf{F}(k)$ is the transition matrix for discrete system of dimension $(2i + 1)k \times (2i + 1)k$; $\mathbf{\Omega}(k) = |\omega_1, \omega_2, \ldots, \omega_{(2i+1)k}|^{\mathrm{T}}$ is vector of random perturbations of the system (state noise vector).

If to assume that the considered meteorological field is isotropic and stationary, and the unknown parameters and parameters to be estimated $\mathbf{X}(k)$ do not change on average on the given time interval, then

$$\mathbf{X}(k+1) = \mathbf{X}(k) \tag{4}$$

and the transition matrix $\mathbf{F}(k)$ is a unit matrix.

The mathematical model of measurements, which are used in the Kalman filtering algorithm to estimate the system state, in the general case is described by an additive mixture of the useful information and the measurement error, i.e.,

$$\mathbf{Y}(k) = \xi(k) = \mathbf{H}(k) \cdot \mathbf{X}(k) + \mathbf{E}(k), \quad (5)$$

where $\mathbf{Y}(k)$ is vector of actual measurements. In our case, $\mathbf{Y}(k)$ is a scalar (number), representing a measurement at a fixed altitude at instant k; $\mathbf{H}(k)$ is the observation matrix of dimension $((2i + 1)k \times 1)$, i.e., it is a row vector whose elements are the predictors chosen with a certain weight and following one after another; and $\mathbf{E}(k)$ is vector of measurement errors (measurement noise).

Let us consider in more detail the procedure of filling the observation matrix $\mathbf{H}(k)$, because each element of this matrix includes weighting coefficients. Introduction of these coefficients makes it possible to take into account the time and vertical correlations between individual values of the meteorological field, obtained at preceding moments at different altitude levels located below and above the forecast level. Assuming the time and vertical correlations for the given field dependent, we can, in conformity with Ref. 8, introduce the relation

$$\mu_\xi(\tau_k, \Delta h_m) = \exp\left(-\sqrt{\left(\frac{\tau_k}{\tau_0}\right)^2 + \left(\frac{\Delta h_m}{h_0}\right)^2}\right), \quad (6)$$

where $\mu_\xi(\tau_k, \Delta h_m)$ is the weighting factor taking into account the correlations between measurements of meteorological field at the kth moment at mth altitude (here $\tau_k = \Delta t, 2\Delta t, 3\Delta t, \ldots, k\Delta t$ is time delay, Δt is the discretization interval in hours; k is the depth of the delay window; $\Delta h_m = h_m - h_i$ is the thickness of altitudinal layer (in km) for m levels, with $i = m + 1$ or $i = m - 1$ being the maximal number of altitudinal information layers whose data are taken into account in forecasting the ξ field at the level m); τ_0 and h_0 are radii of time and vertical correlations, respectively.

According to our studies, the radii of time (τ_0) and vertical (h_0) correlations in formula (6) are as follows: $\tau_0 = 30$ h and $h_0 = 4500$ m (for temperature); $\tau_0 = 24$ h and $h_0 = 1500$ m (for orthogonal components of wind velocity).

Thus, $\mathbf{H}(k)$ can be written as

$$\mathbf{H}(k) = \left| \frac{y_0(k-1)}{\mu_\xi(T,0)}, \frac{y_0(k-2)}{\mu_\xi(2T,0)}, \right.$$

$$\left. \frac{y_0(k-K)}{\mu_\xi(KT,0)}, \frac{y_1(k-1)}{\mu_\xi(T,\Delta h)}, \frac{y_{2i+1}(k-K)}{\mu_\xi(KT,\Delta h_{2i+1})} \right|, \quad (7)$$

that is, in such a way we introduce forgetful factors, ensuring the condition that the data (altitudinally and temporally) close to the forecast point contribute more than more distant data.

If the time and vertical (interlevel) correlations are weak, equation (6) can be considered as a product of two exponents, namely:

$$\mu_\xi(\tau_k, \Delta h_m) = \exp\left(\frac{\tau_k}{\tau_0}\right)\exp\left(\frac{\Delta h_m}{h_0}\right). \quad (8)$$

After determination of all elements entering equations (3) and (5), the estimation problem can be solved with the linear Kalman filter,[8] allowing us to estimate components of the state vector with minimal root-mean-square errors. A detailed description of the filter is given in Ref. 9.

The above-mentioned method, based on the Kalman filtering algorithm and two-dimensional dynamical-stochastic model, was estimated from the viewpoint of its applicability to solution of the problem of ultra-short-term (up to 6 h) forecast of temperature and orthogonal wind velocity components in the atmospheric boundary layer.

To estimate the quality of the method, we used data of two-term (00.00 and 12.00 GMT) aerological and eight-term (00.00, 03.00, 06.00, 09.00, 12.00, 15.00, 18.00, and 21.00 GMT) meteorological observations, obtained in January 2004 for two stations: Vienna station of temperature–wind sensing (48°16′N, 16°22′E) and synoptic station with the same name but somewhat different geographic coordinates (48°16′N, 16°21′E). Since the data from altitudinal levels for synoptic-scale terms (03.00, 06.00, 09.00, 15.00, 18.00, and 21.00 GMT) are usually absent, they were reconstructed from the ground-based meteorological measurements using the multiple regression model.[10] The model parameters were estimated using statistical characteristics (mathematical expectations, variances, and coefficients of interlevel correlation), preliminarily calculated from two-term aerological measurements. Naturally, the reconstructed values of temperature and wind have certain uncertainties, which, however, in the preliminary assessment of the developed method quality can be neglected.

The root-mean-square (δ) and relative (θ) errors of the ultra-short-term forecast of temperature and zonal and meridional wind velocity components are presented in Table; the forecast is performed by the dynamic-stochastic method for different altitudes.

Analysis of Table shows that:

– the suggested technique, based on the Kalman filtering algorithm and two-dimensional dynamic-stochastic model, can be used in practical applications, especially in forecasting for $\tau = 3$ h, with satisfactory results. Indeed, the relative error θ varies within the range 24–36%, independently of the meteorological variable and altitude level (except for the near-ground level, where the forecast using only data of time series, has larger errors);

– the quality of the ultra-short-term forecast, based on the suggested method, is markedly worse at $\tau = 6$ h; but even in this case the errors are within 34–66%, not exceeding a maximal permissible value of 66%.

Root-mean-square (δ) and relative (θ,%) errors of ultra-short-term (with lead time τ = 3 and 6 h) forecast of temperature and zonal and meridional wind velocity components

Height, m	τ = 3 h		τ = 6 h	
	δ	θ	δ	θ
	Temperature, °C			
0	1.3	35	2.1	54
100	1.0	30	1.8	54
200	0.9	29	1.5	50
300	0.9	28	1.5	50
400	0.9	29	1.5	50
600	0.8	28	1.3	45
800	0.7	24	1.0	34
	Zonal wind velocity component, m/s			
0	1.5	50	2.0	64
100	1.1	34	1.9	59
200	1.0	31	2.1	58
300	1.2	29	2.6	61
400	1.4	29	3.0	63
600	1.6	29	3.2	58
800	1.7	28	3.6	59
	Meridional wind velocity component, m/s			
0	1.2	52	1.6	66
100	0.9	36	1.4	56
200	1.0	33	1.8	60
300	1.2	33	2.3	62
400	1.3	31	2.6	62
600	1.7	31	3.4	60
800	1.9	29	3.7	56

Thus, our preliminarily conclusion is that the developed technique can be used in solution of the problem of ultra-short-term forecasting, especially for $\tau \le 3$ h. However, the obtained results of numerical experiments need further testing and refinement on the basis of aerological data of high temporal resolution (for example, radiometric and sodar measurements). This will be the subject of our further researches.

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