Integral parameters of high-power femtosecond laser radiation at filamentation in air

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The main effective parameters of high-power femtosecond laser radiation (energy transfer coefficient, effective radius, effective duration, limiting angular divergence, and effective intensity) during its propagation along a horizontal atmospheric path under conditions of filamentation have been investigated theoretically. It is shown that the process of self-action of this radiation is characterized by formation of a nonlinearity layer, behind which the radiation propagates linearly with the limiting divergence lower than the initial diffraction-limited divergence of the beam. The effective pulse duration and the effective beam radius increase after the passage through the nonlinearity layer, and their values are mostly determined by the initial beam power and weakly depend on the initial spatial focusing of the beam. The coefficient of energy transmission for the femtosecond pulse is lower than in the linear medium and has a tendency to decrease with the increasing radiation power.

Introduction

The propagation of high-power laser radiation of femtosecond duration through gaseous and condensed media occurs in the nonlinear regime and leads to significant changes in the temporal, spatial, and spectral characteristics of the light beam.[1] As known, for the stationary self-action of laser radiation, it is convenient to study the transformation of its energy characteristics in terms of effective parameters, such as the power (energy) transfer coefficient, beam radius, angular divergence, and intensity, which characterize the global changes in the light beam. In some cases, for example, at stationary self-focusing in a cubic-nonlinearity medium[2] and thermal blooming of long laser pulses,[3] for some of the effective characteristics it is possible to write equations, which can be analyzed qualitatively or quantitatively.

The self-action of ultrashort laser pulses in the atmospheric air occurs with the participation of a large number of physical factors, which determine the process of radiation propagation and can be in dynamic balance with each other. These factors include the effects of diffraction and Kerr nonlinear refraction, as well as the effects occurring against their background. These effects are associated with the formation of an ionized channel inside the beam at multiphoton absorption in a strong light field (nonlinear refraction of radiation and absorption in plasma), as well as with the frequency dispersion of the air.

In the general case, to correctly describe self-focusing of femtosecond pulses, it is necessary to solve numerically the four-dimensional wave equation for the electric field vector, which is a difficult problem even for modern computers. The description of the phenomenon of self-focusing in terms of the effective characteristics of radiation is productive, because it allows one to follow the main transformations occurring with the beam in a nonlinear medium and to predict its propagation to distances, far exceeding the diffraction length of the initial beam.

In this paper, the numerical solution of the nonlinear Schrödinger equation for the complex envelope of the field of the spatially limited femtosecond laser pulse propagating in the atmospheric air is used to study the change of the effective parameters of the pulse, namely, the energy transfer coefficient, the effective radius, the effective duration, the limiting angular divergence, and the effective intensity. The dependences of these parameters on the initial power of radiation and the parameter of spatial focusing are established.

Integral parameters of a light pulse

One of the main integral parameters of a laser pulse is the coefficient of transfer of the light energy once the beam passes the distance z:

$$T_e(z) = E(z)/E_0, \quad (1)$$

where $E(z)$ and E_0 are the current and the initial values of the total energy of the light pulse.

Another integral parameter is the effective radius of the light beam R_e, whose square is determined as a normalized second-order moment of the transversal radiation energy density profile $w(\mathbf{r}_\perp, z)$:

$$R_e^2(z) = \frac{1}{E(z)} \iint\limits_{\mathbf{R}_\perp} d^2\mathbf{r}_\perp \left[w(\mathbf{r}_\perp, z) |\mathbf{r}_\perp|^2 \right] - R_g^2(z). \quad (2)$$

In Eq. (2)

$$R_g(z) = 1/E(z) \int\limits_{\mathbf{R}_\perp} d\mathbf{r}_\perp \left[\mathbf{r}_\perp w(\mathbf{r}_\perp, z) \right]$$

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is the coordinate of the beam centroid. It is obvious that for the beams with the symmetric spatial intensity profile the beam centroid is always located at the beam axis, if the medium is initially optically homogeneous. Therefore, below we assume $R_g = 0$.

Similarly, for the nonstationary self-action we can introduce the effective pulse duration t_{pe}, determined from the temporal profile of the total beam power $P(t; z)$:

$$t_{pe}^2(z) = \frac{1}{E(z)} \int_{-\infty}^{\infty} dt \left[t^2 P(t;z) \right] - t_{pg}^2(z), \qquad (3)$$

where

$$t_{pg} = 1/E(z) \int_{-\infty}^{\infty} dt \left[t\, P(t;z) \right]$$

is the temporal position of the power maximum at every point of the optical path.

One more important parameter is

$$\theta_e^2(z) = \frac{1}{2}\left(\frac{d^2 R_e^2}{dz^2} \right), \qquad (4)$$

which determines the square of the limiting divergence of the beam $\theta_{e\infty}^2$ at $z \to \infty$.

The use of integral parameters for description of the propagation of a light beam is essentially equivalent to the replacement of the real spatiotemporal intensity profile by the uniform distribution over a circle with the radius $R_e(z)$, rectangular in time with the duration $t_{pe}(z)$, and the effective intensity

$$I_e(z) = E(z) \Big/ \left(\pi R_e^2(z) t_{pe}(z) \right). \qquad (5)$$

With the introduced integral characteristics (1)–(5), let us analyze the main stages of nonstationary self-focusing of the femtosecond laser pulse.

Mathematical model of self-focusing of a femtosecond light pulse in air

The numerical calculations were performed based on the nonlinear Schrödinger equation, describing the propagation of an electromagnetic wave through a medium in the approximation of a slowly varying field amplitude. For the complex envelope of the electric field of the light wave $U(\mathbf{r}_\perp, z; t)$ in the coordinate system moving with the group velocity of the pulse, this equation has the form (see, for example, Ref. 4):

$$\left\{ \frac{\partial}{\partial z} - \frac{i}{2 n_0 k_0} \nabla_\perp^2 + i \frac{k''_\omega}{2} \frac{\partial^2}{\partial t^2} \right\} U(\mathbf{r}_\perp, z; t) -$$

$$-i k_0 n_2 \left\{ (1 - f_R) |U|^2 + f_R \int_{-\infty}^{\infty} dt' \Lambda(t - t') |U(t')|^2 \right\} U(\mathbf{r}_\perp, z; t) +$$

$$+ \frac{\eta_{cas}}{2} (1 + i\omega_0 \tau_c) \rho_e(t) U(\mathbf{r}_\perp, z; t) +$$

$$+ \frac{\eta_{MPA}^{(m)}}{2} |U|^{2m-1} U(\mathbf{r}_\perp, z; t) = 0, \qquad (6)$$

where ω_0 is the central frequency of the laser radiation; $k_0 = n_0 \omega_0 / c$ is the wave number; $k'' = \partial^2 k / \partial \omega^2$ is the dispersion of the group velocity; n_2 is the coefficient of the nonlinear addition to the refractive index of a gas n_0; f_R is the specific fraction of the delayed Kerr effect with the response function $\Lambda(t - t')$ [Ref. 5] in the total change of the nonlinear refractive index (usually $f_R = 0.5$ is taken); τ_c is the characteristic electron collision time; $\eta_{MPA}^{(m)}$ and η_{cas} is the rate of the m-photon and cascade ionization of a gas, respectively. Equation (6) accounts for the diffraction of the light wave in the presence of frequency dispersion of the air, as well as for the main physical mechanisms of the medium nonlinearity for the ultrashort radiation: instantaneous and delayed Kerr effect, absorption and refraction of radiation by plasma produced as a result of gas ionization.

The change in the concentration of free electrons ρ_e was calculated by the model of quasi-equilibrium plasma, neglecting the loss for recombination:

$$\frac{\partial \rho_e}{\partial t} = \frac{\eta_{MPA}^{(m)}}{m \hbar \omega_0} |U|^{2m} + \frac{\eta_{cas}}{\Delta E_i} \rho_e |U|^2,$$

where ΔE_i is the effective ionization potential of air molecules.

Results and discussion

In the dissipation-free medium with nonlinearity of the Kerr type with no frequency dispersion and nonlinear aberrations of the beam, the parameters T_e, θ_e^2, t_{pe} are invariant with respect to z up to the nonlinear focus, what shows the square dependence of the effective radius along the longitudinal coordinate and the constancy of its temporal profile.[6] At self-focusing of the ultrashort radiation pulse plasma is produced, as a result of the multiphoton ionization of the medium, in the zones of maximum intensity. This plasma causes the nonlinear absorption of light wave and its defocusing. The joint manifestation of the effects of Kerr self-focusing and defocusing in plasma leads to a strong phase self-modulation of radiation and distorts the invariance of these parameters.

Figures 1 to 3 show the dependence (along the propagation path) of the normalized parameters

$$\bar{R}_e^2(z) = R_e^2(z) / R_0^2; \;\; \bar{\theta}_e^2 = \left(d^2 R_e^2 / dz^2 \right) \left(k_0^2 R_0^2 / 2 \right);$$

$$\bar{t}_{pe}(z) = t_{pe}(z) / t_p; \text{ and } \bar{I}_e(z) = I_e(z) / I_0,$$

obtained from the numerical simulation of atmospheric self-action of a laser pulse with the Gaussian spatiotemporal profile and the following parameters: wavelength $\lambda_0 = 810$ nm, duration $t_p = 80$ fs, radius $R_0 = 1$ mm, the radius of curvature of the initial wave front $F = 1.2 L_R$, peak power $P_0 = 15 P_c$ (critical self-focusing power $P_c = 3.2$ GW). These figures also show the geometric square size of the beam R_f, determined from the half-maximum of the beam energy density, as a function of the distance. All the parameters are normalized to their initial values at $z = 0$, while the variable z itself is normalized to the Rayleigh length of the beam $L_R = 1/2(k_0R_0^2)$.

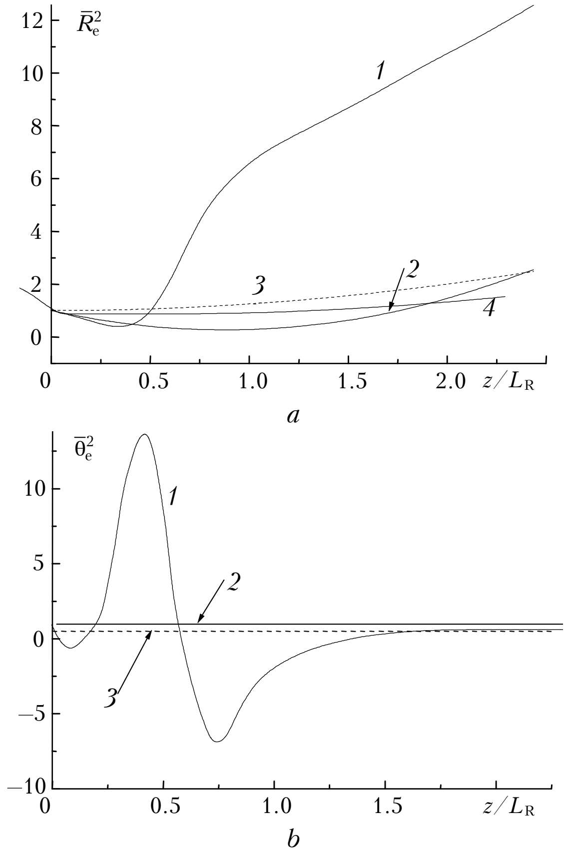

Fig. 1. Square normalized effective radius $\bar{R}_e^2$ of the femtosecond beam propagating in air: (*a*) complete model (*1*); linear diffraction of the focused (*2*) and collimated (*3*) beams; square radius $(R_f/R_0)^2$ of the filamented part of the beam (*4*); (*b*) dependence of the parameter $\bar{\theta}_e^2$ on the longitudinal coordinate during the propagation of the light beam in the air in the regime of self-action (*1*) and linear diffraction (*2*, *3*); curve *3* shows the linear propagation of the collimated beam (parameters of radiation are the same).

It can be seen from Fig. 1 that, at the initial stage of self-action, the quick transversal compression of the beam occurs due to the Kerr effect. The increase of the peak intensity of the radiation (see Fig. 2) and the related strong multiphoton absorption lead to the ionization of the medium and give rise to plasma. The defocusing effect of the plasma along with the radiation power inputs to the maintenance of plasma stop the collapse of the beam, leading to the formation of a stable waveguide channel (filament) at the beam axis. This filament has a quasi-Bessel spatial intensity distribution (Fig. 4) and weak angular divergence. The transverse dimension of the filament R_f in the air varies within ~ 75–150 μm at the peak intensity ~10^{14} W/cm^2 in the filament. The coordinate of the beginning of filamentation z_f (local nonlinear focus) depends on the initial peak power of the pulse P_0 and, within the accuracy satisfactory for the collimated radiation, it is determined by the known equation[6]:

$$z_f = \frac{2L_R}{2.725\sqrt{\left(\eta^{1/2} - 0.852\right)^2 - 0.022}}, \quad \eta = P_0/P_c.$$

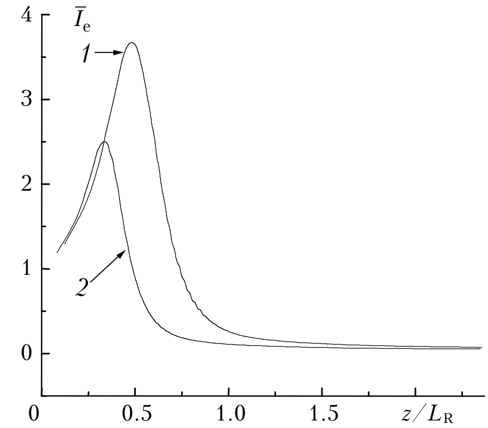

Fig. 2. Spatial variation of the effective radiation intensity $\bar{I}_e$ along the propagation path for beams with the initial peak power $P_0 = 6P_c$ (*1*) and $15P_c$ (*2*).

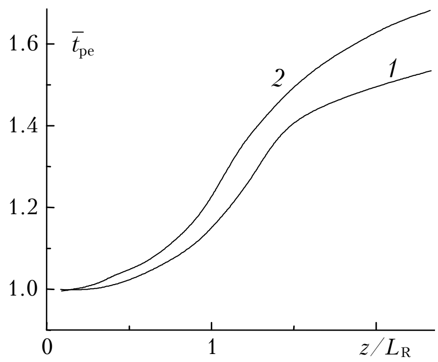

Fig. 3. Normalized effective duration of the laser pulse $\bar{t}_{pe}$ as a function of the evolution variable. The parameters of radiation are the same as in Fig. 2.

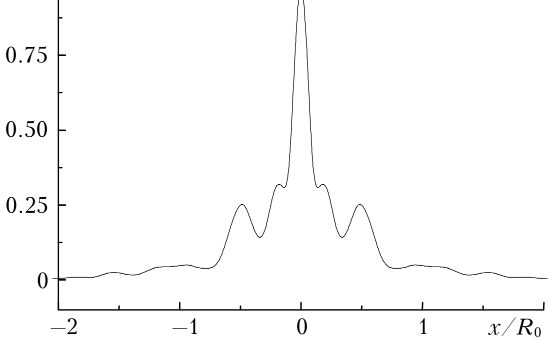

Fig. 4. Transverse profile (along the coordinate x) of the normalized light energy density of the laser beam ($t_p = 80$ fs, $F = 1.2L_R$, $P_0 = 15P_c$) at the distance $z = 0.5L_R$ from the path start point.

The formation of a filament does not terminate the compression of the beam as a whole, and therefore the position of the nonlinear focus z_{sf}, determined from the minimum of the effective radius R_e, appears to be to the right (along the coordinate z) from the local focus z_f. The position of the "global" focus z_{sf} corresponds to the value obtained within the framework of the aberration-free theory of self-focusing of the Gaussian beam[6] at the beam power equal to the peak one, when the parabolic dependence of the effective beam radius on the evolution variable is valid:

$$R_e^2(z) = R_e^2(0)\left[(1-\eta)\left(\frac{z}{2L_R}\right)^2 + \left(1-\frac{z}{F}\right)^2\right].$$

Hence, assuming $R_e^2(z_{sf}) = 0$, under the condition $\eta > 1$ we obtain the position of the beam self-focusing point:

$$z_{sf} = \frac{\bar{F} L_R}{\sqrt{\eta - 1}\left(\bar{F}/2\right) + 1}, \quad \bar{F} = F/L_R.$$

Just after the nonlinear focus, the effective area of the beam increases sharply, and the beam divergence near the point z_{sf} (see Fig. 1*b*) is close to that of the focused Gaussian beam in the linear medium with the initial radius of curvature of the phase front F equal to z_{sf}. During this, despite the strong diffraction widening of the power-intense part of the beam, the radius of the axial filament generally does not change. This is indicative of the spatial stability of the filament, resulting from the dynamic balance between the Kerr focusing, plasma defocusing, and multiphoton absorption near the beam axis.

In the same zone, one can see the increase of the effective duration of the laser pulse (see Fig. 3) due to dispersion of the group velocity of light in the air. Estimating the length L_{ds}, at which the effective pulse duration increases by $\sqrt{2}$ times, from Fig. 3, we find that L_{ds} is on the order of the Rayleigh length of the initial beam L_R. At the same time, $L_{ds}/L_R \sim 100$ if determined from the initial profile. That significant decrease in the dispersion length is connected with the fragmentation of the temporal profile of the initial pulse into a series of much shorter pulses ($\sim 0.1 t_p$), occurring against the background of the strong phase self-modulation. The pulse with the initially Gaussian temporal profile transforms into a series of time- and spatially separated individual pulses, experiencing much stronger dispersion blooming than the initial profile.

The following spatial zone $z/L_R > 0.6$–1.6 is characterized by the lower rates of growth of the effective beam radius and the decrease of the angular divergence of the beam (see Fig. 1). This is a consequence of sequential re-focusing of the periphery zone of the beam, having lower intensities, on concentric toroidal nonlinear lenses, formed at the previous stage around the filament. The transverse energy density distribution of the light beam, as can be seen from Fig. 4, is similar to the profile of the Gauss–Bessel beam, which is characterized by the lower angular divergence than in the Gaussian beam.

Beyond this "transient" zone, the dependence $R_e^2(z)$ becomes a square one, and the limiting divergence of the beam $\theta_{e\infty}$ is formed. Thus, the value of the spatial variable $z \equiv L_N > 1.6 L_R$ can be considered as some conditional boundary of the medium nonlinearity layer, behind which the evolution of the effective beam parameters obeys the linear laws of diffraction. The calculations show that as the initial power of the light beam increases, the right boundary of this nonlinearity layer shifts toward smaller z, and the thickness of the layer decreases.

From Figs. 1 and 3 we can conclude that the variations of the square effective beam radius $R_e^2(z)$ and the effective pulse duration $t_{pe}(z)$ behind the nonlinearity layer obey the following laws:

$$R_e^2(z) = R_{0N}^2 + \theta_{e\infty}^2 (z - L_N)^2;$$

$$t_{pe}(z) = t_{pe}(L_N)\sqrt{1 + \left[(z - L_N)/L_{dsN}\right]^2}, \quad z \ge L_N,$$

that is, coincide with the known dependences for the linear regime of beam propagation in a medium with dispersion, characterized by the new effective length L_{dsN}. For the radiation parameters considered in the calculations, $L_{dsN} > L_N$.

Figure 5 depicts the energy transfer coefficient of the light beam T_e, achieved at the distance $z = 2L_R$ with different initial radiation power. It can be seen that beams of higher power are characterized by the higher nonlinear losses along the path. The calculations show that the main channel of dissipation of the light energy is the multiphoton absorption in a gas, which forms a plasma filament on the beam axis. As a result, beams with sharper focusing (and, consequently, higher intensity in the zone of the "global" focus) are characterized by relatively high nonlinear losses as compared with the long-focus beams and by the lower energy transfer coefficient at the end of the path. However, the influence of the spatial focusing of the beam on T_e is normally low.

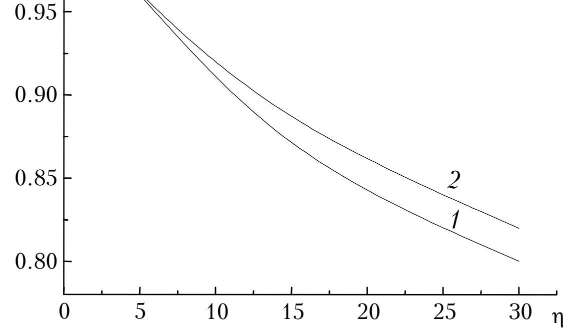

Fig. 5. Energy transfer coefficient T_e as a function of the relative initial power of the light beam with $F = 0.6L_R$ (*1*) and $1.2L_R$ (*2*) at $z = 2L_R$.

The dependence of the normalized limiting angular divergence $\bar{\theta}_{e\infty} = \theta_{e\infty}/\theta_D$, where $\theta_D = (k_0R_0)^{-1}$ is the diffraction-limited divergence of the initial beam, on the relative beam power η at the different initial curvature of its phase front is shown in Fig. 6. The parameter $\bar{\theta}_{e\infty}$ was estimated from its value at the point $z = 2L_R$. We can see the general tendency toward a decrease in the limiting divergence with the increase of the beam power, and at $\eta > 20$ the angular divergence becomes even lower than in the collimated beam ($F \to \infty$) of the same initial radius in the linear medium. Moreover, the limiting divergence of high-power beams appears to be almost independent of their initial spatial focusing. The interpolation analysis of the level, the value $\bar{\theta}_{e\infty}$ tends to at large η, has shown that it roughly corresponds to the value of the linear diffraction-limited divergence of the collimated Gaussian beam with the initial radius $R_{0N} = R_e(L_N)$; in this case, $R_{0N} = 3R_0$ (Fig. 6, curve *6*).

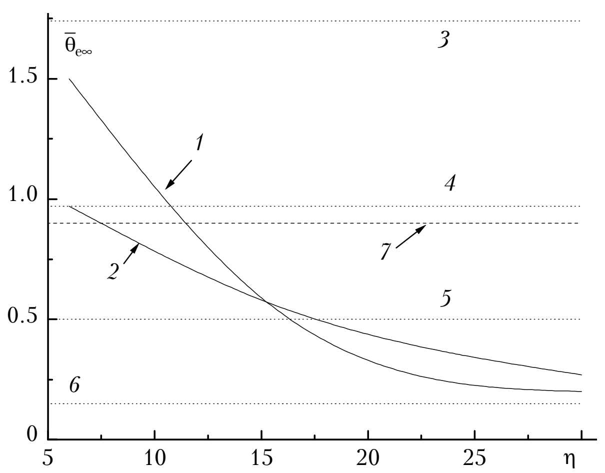

Fig. 6. Normalized limiting effective angular divergence of the laser beam $\bar{\theta}_{e\infty}$ as a function of the initial peak power at nonstationary self-action (*1*, *2*) and at linear diffraction in the air (*3–6*). The conditions of the beam focusing correspond to: $F = 0.6L_R$ (*1*, *3*); $1.2L_R$ (*2*, *4*). Lines *5* and *6* show the parameter $\bar{\theta}_e$ of the collimated radiation with $R_0/R_{0N} = 1$ and 3, respectively. Line *7* shows the angular divergence of the central part of the beam after the collapse of the filament.

If we consider separately the spatial dynamics of the central part of the beam, then we can see the zones, in which the light filament is formed, and, as follows from Fig. 6 (curve *7*), after the collapse of the filament, the angular divergence of the central part remains approximately constant regardless of the variation of η. In this case, the value of the derivative (dR_f/dz) is much smaller than the angle, at which the light beam of the radius R_f would propagate in a linear medium.

Figure 7 shows R_{0N} in relative units as a function of the power parameter η. Along with the increase of the "boundary" effective beam radius and the increase of its power, we can also see saturation of this dependence, and the curves with different values of the focusing parameter F, as in the case with $\bar{\theta}_{e\infty}$, tend to the same level.

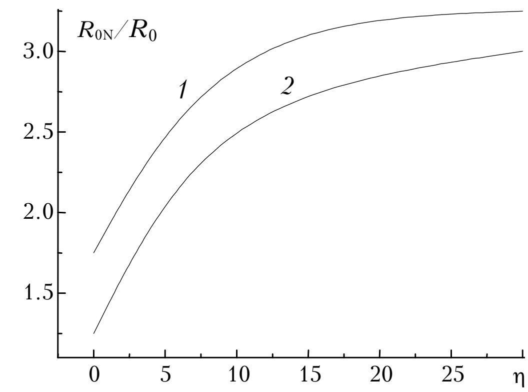

Fig. 7. Normalized effective radius of the light beam R_{0N} at the exit from the nonlinear layer as a function of the initial radiation power at $F = 0.6L_R$ (*1*) and $1.2L_R$ (*2*).

It should be noted that the numerical calculations, whose results are presented in this paper, were carried out using an ideal spatial profile of the light beam and in the absence of turbulence in the medium. In this case, at self-action of pulses, the maximum of the radiation intensity lies on the beam axis and only one axial filament is produced. Perturbations of the amplitude and phase of the light wave, arising due to imperfections in the optical system forming the beam and the effect of turbulence in the medium of propagation, in some cases lead to multiple filamentation over the whole cross section of the femtosecond beam, and the higher the beam power, the more pronounced is the multiple filamentation.

This self-action regime will change the particular values of the effective characteristics presented here and increase the medium nonlinearity layer and the limiting angular divergence of the beam. However, the general regularities of evolution of the effective laser pulse parameters (existence of three spatial zones: focusing zone, transient zone, linear zone) will keep the same, in our opinion.

Conclusions

Based on the analysis of the integral characteristics of the light beam (energy transfer coefficient, effective radius, effective duration, limiting angular divergence, effective intensity), the process of nonstationary self-focusing of the femtosecond laser radiation in the atmospheric air has been studied.

The effective parameters of the beam have been calculated based on the numerical solution of the nonlinear Schrödinger equation with allowance for the effects of diffraction, dispersion of the group velocity of the light pulse, Kerr self-focusing, multiphoton absorption, and plasma defocusing of radiation.

It has been found that, on the basis of the evolution of effective parameters along the optical path, it is possible to isolate three spatial zones, corresponding to the different stages of nonstationary self-focusing of the radiation: the zone of transversal compression of the beam and filament formation, the zone of sharp increase of the effective area of the beam after the "global" nonlinear focus, and the zone of linear diffraction of the radiation.

The analysis made shows that, by the end of the second zone, which serves a boundary of the medium nonlinearity layer L_N, the light beam "forgets" about its initial scale parameters and propagates in the linear regime.

The effective parameters of the radiation T_e, R_{0N}, $t_{pe}(L_N)$, and $\theta_{e\infty}$ are mostly determined by the beam power and weakly depend on the initial spatial focusing of the beam. With the increase of the initial power of the light beam, its effective size and the effective duration at the boundary of the medium nonlinearity layer increase, while the limiting divergence decreases. At the same time, the thickness of the nonlinearity layer itself decreases.

Acknowledgments

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