# Filamentation of a high-power frequency-modulated femtosecond laser pulse on a vertical atmospheric path

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A possibility of application of laser chirps to creation of an "altitude" light filament with accounting for altitude variation of nonlinear interaction constants in the case of high-power femtosecond radiation propagation along vertical atmospheric paths is under discussion. Based on the model of exponential decrease in air density and the corresponding increase in the self-focusing critical power, it is shown that position of the nonlinear beam focus can be under control with the use of chirping. At the same time, pulses free of frequency modulation propagate along the path linearly and the filament does not appear; while chirping such pulses leads to their provisional compression, and, as a result, to formation of a light filament of high peak intensity.

## Introduction

Knowledge of regularities in high-power femtosecond radiation propagation along atmospheric paths is of a great practical importance for a number of applications; among them are creation of an ionized atmospheric channel for thunderstorm electricity elimination,[1] detection of the atmospheric composition,[2] generation of high-power electromagnetic pulses in the terahertz range,[3] design of a source of wideband secondary radiation ("atmospheric lamp").[4]

The basis for all these applications is the effect of laser beam filamentation. The effect has been revealed experimentally in the beginning of 1990s for femtosecond pulses with the initial peak power greater than some critical value (in the ground air, the critical self-focusing power is ~3.2 GW in the near IR). It was shown, that the light filament can be formed and live inside a beam on sufficiently long paths (at a scale of fractions of the diffraction length).

Physically, the cause of the beam filamentation is a strong spatio-temporal self-modulation of the femtosecond pulse propagating in some medium. A light filament is a nonlinear waveguide channel occurring due to dynamic balance of the Kerr self-focusing, nonlinear multiphoton absorption, and radiation refraction in the medium of ionized plasma. An average filament diameter in atmospheric air is about 100 μm, peak intensity is $\sim 10^{14}$ W/cm$^2$, filament power has an order of the critical self-focusing power, and the plasma channel density (plasma thread) behind the filament can attain $10^{16}$–$10^{18}$ electron/cm$^3$; the filament length along a horizontal atmospheric path usually averages tens of meters.

At manifold overbalance of the critical self-focusing power, the light beam, propagating along a horizontal ground path, breaks into a few filaments. As a rule, such filaments are randomly distributed over the beam cross section and differ in length.

As is established, multiple filamentation occurs as a result of perturbation of a light wave amplitude and phase due to fluctuation of corresponding parameters in the laser beam profile, as well as the atmospheric turbulence effect. Once filaments break, they return the energy to the beam thus replenishing the total "photon reservoir" which can support further filamentation at distances exceeding the length of a single filament. Such a sequential multiple filamentation in air of high-power light beams with step-down filamenting in the cross section at a distance of hundreds of meters was detected experimentally.[5]

At the same time, the laser beam filamentation restricts the light energy transfer within the given solid angle in atmosphere to distances comparative with the diffraction beam length. After attaining the filamentation threshold, the angular divergence of the light beam rises sharply and, as a result, the beam diameter at the outlet from the filamentation zone can grow by an order of magnitude as compared to the initial one.[6]

One of the methods for solving the problem of ultrashort radiation propagation to distances comparable with the diffraction beam length consists in the use of sufficiently long laser pulses, which have the initial peak power less than the critical self-focusing power and are swept-frequency modulated (chirped). At a negative chirp, such modulation results in provisional pulse compression when radiation propagating in a medium with a standard dispersion. Thus, a long pulse linearly propagates along some initial path's part without self-focusing; then, time-compressed, it acquires a power greater than the critical self-focusing one and, in such a way, enters the mode of the filament structure formation. This method is sufficiently well tested and allows one

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to obtain filaments along horizontal atmospheric paths at distances up to 200 m.[7] Theoretical calculations of non-stationary self-focusing of a frequency-modulated femtosecond laser pulse are presented in Ref. 8.

Operation range of the femtosecond laser sources increases on inclined and vertical paths. Air density decrease with altitude results in exponential increase of the critical self-focusing power as compared to the ground level. Application of the chirp technique makes it possible to control for the position of the beam filamentation zone on a path. Within the framework of the international project Teramobile, experiments with a terawatt Ti:Sapphire laser (beam diameter of 3 cm, radiation wavelength of 800 nm, pulse frequency of 10 Hz) were carried out,[2,9] in which the pulse duration varied from 100 to 600 fs owing to negative chirping. A supercontinuum signal reflected from the layer at a height of 13 km was detected; white-emitting image size at this altitude has shown the beam filamentation to occur at distances far exceeding the beam filamentation length on a horizontal path.

Thus, direct observation of femtosecond terawatt laser pulses propagating along vertical paths proved the possibility of nonlinear atmospheric propagation of high-power ultrashort laser pulses in the vertical direction. Field experiments demonstrated the controllability of parameters of the arising supercontinuum through spatial (telescope focal length varying) and temporal (setting of frequency modulation) pulse focusing. It has been also shown, that a vertical light beam, having formed after the nonlinear focus, has a less angular divergence as compared to horizontal one.

Another important moment is the fact that the beam propagating in the vertical direction kept its intensity irregularities. This shows that the turbulence does not cause there the multiple filamentation, which restricts a concentrated transport of laser energy along horizontal paths.

In this work, the peculiarities of light filament formation at chirped femtosecond pulse propagation along a vertical atmospheric path, i.e., at variation with altitude of nonlinear interaction constants, were theoretically investigated. Simulation results have shown a filament-free propagation of a non-phase-modulated pulse in linear regime. At the same time, the chirping of such pulses results in their temporal compression and, as a consequence, formation of a narrow light filament with a high peak intensity.

## Research technique

Numerical simulation of the femtosecond laser pulses propagation in atmosphere is based, in most cases, on solution of nonlinear Schrödinger equation for a slowly varying electric field envelope (NLSE) or its corrections and modifications (see, for example, Refs. 8, 10, and 11). For proper description of the femtosecond pulse filamentation, a numerical solution of wave equation in four-dimensional space is necessary. Today, a generalized computation of the kilometer-range propagation of femtosecond radiation seems to be problematic. A 50 μm grid pitch is required for spatial filament resolution; hence, covering of 5 initial radii of a 3-cm light beam demands a $3000 \times 3000$ transverse grid. Accounting for more or less correct time quantization step (~10 fs) and a corresponding step along the evolutionary variable (~ 3 μm) results in huge time and space resources required for computations.

Some ways for overcoming these limitations via simplification of the problem formulation are known. Among them are the time integration of the propagation equation resulting in reduction of the problem dimensionality,[12] radially-symmetric equation,[13] and a rough variational approach.[14] However, they result in a loss of important information on interaction of spatial and temporal constituents of a light wave, and the study of temporal focusing of a pulse and its multiple filamentation becomes impossible.

In Ref. 6, evolution of effective parameters of a femtosecond beam at its nonstationary self-acting along a horizontal path under conditions of a single filament formation is studied. Three space domains were differentiated according to stages of radiation self-focusing: a domain of beam transverse compression and filament formation, a domain of sharp extension of the effective beam area behind nonlinear focus, and a domain of quasi-linear diffraction of radiation. At that, new effective light beam parameters (size, length, and divergence) are formed to the end of the second domain, which, in fact, is a boundary of the nonlinearity layer in a medium. These effective parameters are mainly determined by the beam power and weakly depend on its initial focusing. In fact, a new beam propagates in a medium after the nonlinear layer, and the medium itself can be considered as linear for the beam. The results point out to similarity of effects of initial stage of the Gauss pulse nonstationary self-acting when comparing its relative integral parameters.

In this connection, it is interesting to consider the technique of scaling initial conditions for numerical computations, when initially centimeter in diameter beams are converted into millimeter ones and optical path dimensions converted from dozens to hundreds of meters at an invariable height profile of constants of nonlinear interaction. After direct conversion, self-action of a narrow beam with a preset initial profile along the shortened path is calculated, and then, a back scaling of the results is performed by the well-known algorithms. The conversion equivalence and its algorithms, in its turn, are established based on behavior of the light beam effective integral characteristics.

For numerical investigation of the femtosecond laser radiation propagation in a medium, the nonlinear Schrödinger equation for a slowly varying electric field complex amplitude is commonly used.

The equation has the following form in a system of coordinates moving with the pulse group velocity:

$$\left\{\frac{\partial}{\partial z}-\frac{i}{2n_0k_0}\nabla_\perp^2+i\frac{k''_\omega(z)}{2}\frac{\partial^2}{\partial t^2}\right\}U(\mathbf{r}_\perp,z;t)-ik_0n_2(z)\times$$

$$\times\left\{(1-f_\mathrm{R})|U|^2+f_\mathrm{R}\int_{-\infty}^{\infty}\mathrm{d}t'\Lambda(t-t')|U(t')|^2\right\}U(\mathbf{r}_\perp,z;t)+$$

$$+\frac{\eta_\mathrm{cas}}{2}\big(1+i\omega_0\tau_\mathrm{c}(z)\big)\rho_\mathrm{e}(t)U(\mathbf{r}_\perp,z;t)+$$

$$+\frac{\eta_\mathrm{MPA}^{(m)}(z)}{2}|U|^{2m-1}U(\mathbf{r}_\perp,z;t)=0, \quad (1)$$

where $U(\mathbf{r}_\perp,z;t)$ is the complex envelope; $\omega_0$ is the central frequency of laser radiation; $k_0=n_0\omega_0/c$ is the wave number at the central frequency; $k''_\omega=\partial^2k/\partial\omega^2$ is the group velocity dispersion; $k$ is the wave number at $\omega$-frequency; $n_2$ is the coefficient in a nonlinear addition to the gas refraction index $n_0$; $f_\mathrm{R}$ is the specific fraction of the delayed Kerr effect with the response function $\Lambda(t-t')$ in the total change of the nonlinear refractive index (usually, the damped oscillator model is used with the frequency $\Omega_\mathrm{R}$ = 20 THz and a time constant of ~70 fs (Ref. 15); $\tau_\mathrm{c}$ = 350 fs is the characteristic electron collision time; $\eta_\mathrm{MPA}^{(m)}$ and $\eta_\mathrm{cas}$ are the rates of the $m$-photon and cascade gas ionization, respectively. Equation (1) accounts for the light wave diffraction in the presence of frequency air dispersion, as well as the main physical mechanisms of the medium nonlinearity for ultrashort radiation: instantaneous and delayed Kerr effects, radiation absorption and refraction by plasma produced as a result of gas ionization.

The evolution of local concentration of free plasma electrons $\rho_\mathrm{e}$ was described by the rate equation chosen in accordance with the quasi-equilibrium plasma model neglecting losses for recombination:

$$\frac{\partial\rho_\mathrm{e}}{\partial t}=\frac{\eta_\mathrm{MPA}^{(m)}}{m\hbar\omega_0}|U|^{2m}+\frac{\eta_\mathrm{cas}}{\Delta E_\mathrm{i}}\rho_\mathrm{e}|U|^2, \quad (2)$$

where $\Delta E_\mathrm{i}$ is the effective ionization potential of air atoms.

Further, a chirped pulse with the Gaussian time profile will be considered:

$$U(\mathbf{r}_\perp,z=0;t)=\tilde{U}(\mathbf{r}_\perp,0)\exp\left\{-t^2/t_\mathrm{p}^2(1+ib)\right\}, \quad (3)$$

where

$$\tilde{U}(\mathbf{r}_\perp,0)=\exp\left\{-\frac{\mathbf{r}_\perp^2}{2R_0^2}\left(1+\frac{ik_0}{F}\right)\right\}$$

is the transversal beam profile, Gaussian on the space coordinate $\mathbf{r}_\perp$ with the initial curvature of phase front $F$; $b$ is the chirping parameter, and $t_\mathrm{p}$ is the initial pulse duration.

## Altitude model of parameters and the estimates

Consider the altitude model of parameters in Eq. (1). The model of altitude behavior of nonlinear interaction constants $\eta_\mathrm{MPA}^{(m)}$, $\eta_\mathrm{cas}$, $k''_\omega$, $n_2$, $\tau_\mathrm{c}$ is determined by corresponding models of atmospheric pressure and temperature. As is known, the barometric dependence of air molecule concentration $N$ on the altitude $h$ holds for the Earth atmosphere:

$$N(h)=N(h=0\ \mathrm{km})\exp\left(-\frac{gh}{R_\mathrm{g}T(h)}\right)\simeq N_0\exp\left(-\frac{h}{h^*}\right), \quad (4)$$

where $g$ is the gravitation acceleration; $R_\mathrm{g}$ is the specific gas constant; $T$ is temperature; $h^*\approx 6.8$ km is the atmospheric heterogeneity layer. Correspondingly, in the first approximation, suppose a similar behavior of the altitude dependence of the constants under consideration:

$$\zeta(h)=\zeta(0)\exp(-h/h^*)\equiv\zeta(0)G(h),$$

where $\zeta$ is any coefficient of the above listed. At that, the critical self-focusing power of a laser beam $P_\mathrm{c}=\lambda_0^2/2\pi n_0n_2$ rises exponentially with altitude.

Let us find conditions for the parameter $b$, under which the initially “precritical” pulse

$$\eta_0=P_0/P_\mathrm{c}(h=0)\le 1,$$

where $P_0$ is the initial beam power, becomes self-focusing ($\eta(z)>1$) along a vertical path in the region of the beam linear focus $F\approx L_\mathrm{R}$, where $L_\mathrm{R}$ is the Rayleigh length being half as large as the diffraction length $L_\mathrm{dif}=k_0R_0^2$.

From Eq. (1), integrating over the transverse beam profile and neglecting the plasma generation and the Kerr effect in the linear section, we obtain:

$$\eta^2(z)=\left[\frac{P_0(z)}{P_\mathrm{c}(z)}\right]^2=G^{-2}(z)\left\{D_0^2\bar{G}^2+\left(1-D_0b\bar{G}\right)^2\right\}\ge 1,$$

where

$$D_0=L_\mathrm{R}\left.\frac{k''_\omega}{t_\mathrm{p}^2}\right|_{z=0};\quad \bar{G}=1/L_\mathrm{R}\int_0^{L_\mathrm{R}}G(z)\mathrm{d}z.$$

Solving the equation for $b$, we obtain a sufficient condition for “time self-focusing” along a vertical path:

$$|b|\ge\frac{1}{D_0\bar{G}}\left(1-\sqrt{\eta_0^2G^2(z)-D_0^2\bar{G}^2}\right);\quad \mathrm{sign}(b)<0. \quad (5)$$

The possibility itself to operate under precritical power ($\eta>1$) due to time pulse compression in the $z\approx L_\mathrm{R}$ zone follows from nonnegativity of the root in Eq. (5) and is defined from the sufficient condition for “time self-focusing”:

$$\frac{P_0(0)t_\mathrm{p}^2}{R_0^2}\ge P_\mathrm{c}(0)k_0k''_\omega(0)\bar{G}. \quad (6)$$

Equation (6) connects the laser pulse initial parameters with physical characteristics of the propagation medium. Thus, for example, a Ti:Sapphire-laser pulse of 800 fs with a beam of a 4 cm radius ($L_R \simeq 6$ km) and the precritical power $P_0 = 0.94\ P_c$ becomes self-focusing along a vertical path in the region $z \approx L_R$ at an initial negative chirp $|b| \geq 3$.

## Calculation results

To study dynamics of the altitude filament formation, Eqs. (1) and (2) were solved through splitting the initial problem into physical factors. Self-action in atmospheric air of a chirp laser pulse with the Gaussian spatiotemporal profile (3) was simulated at the following parameters: $\lambda_0 = 810$ nm, $t_p = 800$ fs, $R_0 = 1$ cm, initial curvature radius of the phase front $F = 2L_R$, peak power $P_0 = 0.9P_c$. Decrease of air density with altitude according to Eq. (4) was taken into account. The laser pulse propagation was simulated along equivalent shortened vertical and horizontal paths of 400 m in length that almost agrees with the Rayleigh beam length with a chosen initial radius of 1 cm. For definiteness, the height of the atmosphere inhomogeneity layer $h^* = L$ was taken.

Figure 1 shows variations of relative radius of the laser beam propagating along different air paths.

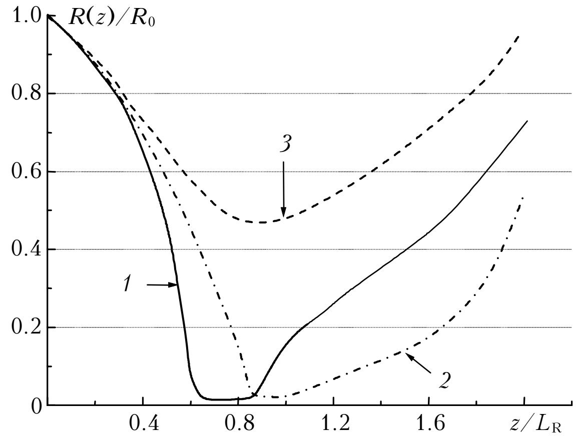


**Fig. 1**. Relative radius of a chirped beam propagating along the ground horizontal (*1*) and vertical (*2*) air paths at the frequency modulation parameter $b = -50$; (*3*) is the unchirped pulse.

As is seen, in the absence of frequency modulation, the pulse with precritical power does not form a filament (curve *3* in Fig. 1) but propagates under conditions close to linear diffraction. At the same time, the pulse chirping results in its temporal compression (Fig. 2), sharp rise of the peak intensity (Fig. 3), and, as a consequence, self-focusing and filamentation (horizontal parts of curves *1* and *2* in Fig. 1).

The beam filamentation begins earlier and the filament is more extent at a horizontal path under stable nonlinearity parameters than at zenith operation of the laser source.

Thus, it is quite possible to control a nonlinear focus position on vertical paths and move it to distances of the order of the atmospheric inhomogeneity layer $h^*$ with the help of chirping. Radiation filamentation at high altitudes $L \gg h^*$ requires extremely large values of frequency modulation, that can present a serious technical problem.

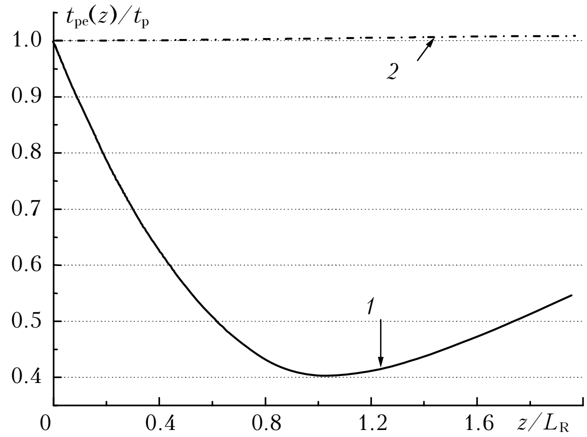


**Fig. 2.** Relative effective duration of the pulse $t_{pe}(z) = \sqrt{\int_{-\infty}^{\infty} dt\left[t^2 P(t;z)\right] \Big/ E(z)}$ ($E$ is the total pulse power) propagating along a vertical air path at the frequency modulation parameter $b = -50$ (*1*) and 0 (*2*).

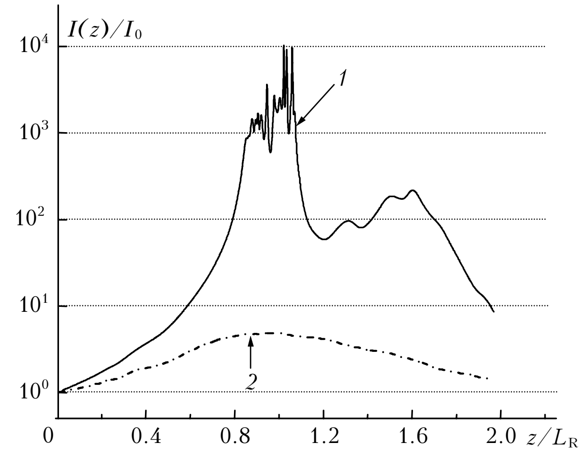


**Fig. 3.** Peak radiation intensity, normalized to the initial one, propagating along a vertical air path for a pulse with frequency modulation ($b = -50$) (*1*) and without modulation (*2*).

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