# Polarization effects in lenses 

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#### Abstract

We consider the polarization effects occurring inside a focusing lens as well as due to formation by this lens of an asymmetric wave front of a spherical converging wave. We formulate the requirements to the experiment that would favor observations of the polarization effects occurring inside the lens that yield the deviation of a focused beam across the focal waist. Using numerical simulations, we have optimized parameters of the initial wave front. For this purpose, the inverse problem is considered on optimizing parameters of the converging wave front at a preset distribution of radiation across the focal waist. The parameters of the field on the spherical surface are calculated using the relation between the wave front in the focal waist and on the spherical surface obtained using direct and inverse Fourier transforms.


## Introduction

It was theoretically predicted ${ }^{1}$ that if circularly polarized radiation passes through an axially symmetric optical lens made of an isotropic substance, the lower half of which is opaque, the $z$ component of electromagnetic radiation deviates to the left or to the right from the symmetry axis depending on the sign of the polarization. The deviation should change the sign, if the upper half of the lens becomes opaque, while the lower becomes transparent. The cross-shift of the focal waist was experimentally confirmed in Ref. 2. The longitudinal component of electric field $\mathbf{E}$ was determined in Ref. 1 from the condition $\operatorname{div} \mathbf{E}=0$. For mathematical description of the situation when the intensity of radiation in the upper part of the lens is higher than in the lower one, solution of the Helmholtz equation was approximated by linear combination of the functions describing $1 s$ - and $2 p$-states.

The effect considered in Ref. 2 was quite weak. Therefore, to make this spin effect stronger, we suggest to change, a little bit, conditions of the experiment. For this we suggest to measure the ratio between the power of light incident on the left ( $\tilde{x} \leq 0$ ) and on the right ( $\tilde{x} \geq 0)$ half planes of the focal plane instead of measuring the center of the $z$-component. Here, the $\tilde{x}$ and $\tilde{y}$ axes are, respectively, parallel to the $x$ and $y$ axes of the spherical wave surface (at $\Delta \ll F^{2}$, where $F$ is the focal length of the lens, and $\Delta$ is the area of the segment). The spin effect considered can be made more strong if the parameters of the spherical wave front are optimized using, for example, "Linza" software product the description of which, including results of testing, are presented in Ref. 3. Thus obtained results are compared with the experimental results. ${ }^{2}$

The method for calculation of the intensity distribution in the image space was proposed and
numerically realized ${ }^{3}$ for a Gaussian lens model based on the Green's function method. ${ }^{4}$ The following problems are considered as tests: imaging of a squareshaped object area, diffraction on an infinitely long slit, ${ }^{5}$ focusing of a converging wave with a Gaussian profile of the amplitude in the area of the focal waist, ${ }^{6}$ imaging of a surface analogous to diffraction grating.

The disadvantage of the aforementioned approach is that it is quite difficult to choose the parameters of the initial wave front on the spherical surface, which would yield the wave with the prescribed properties in the focal waist area. To enhance the spin effect more, it is suggested in this paper to analytically determine the parameters of the wave front on the spherical surface (at the distance of the radius of the spherical surface) on the basis of direct and inverse Fourier transform, that would provide for achieving the prescribed properties of the wave in the focal waist area. This is the inverse problem compared to the aforementioned problem, when the parameters of the wave front in the image area are to be determined having known the parameters of the spherically convergent wave front. Besides, in this paper we consider another class of polarization effects in lenses. We have calculated the parameters of the wave front on the spherical surface that provide the transverse anisotropy of the intensity of the longitudinal component of radiation in the focal waist area, and the anisotropy changes its orientation at the change of the sign of circular polarization.

The effect similar to the aforementioned one was considered in Ref.7. It was shown that if the conditions formulated in Refs. 1 and 2 are fulfilled, the beam displaces inside a focusing lens by the distance comparable with the wavelength of radiation. Unfortunately, the conditions of the experiment $^{2}$ do not allow one to observe the predicted effect, ${ }^{7}$ because in this case the parallel
beam displacements perpendicularly to the axis does not affect the position of the center of the focused beam spot. To experimentally study the predicted effect, ${ }^{7}$ it is necessary that the anisotropy of the beam be observed in the lens only on the side of the incident beam, while no anisotropy should occur along the cross coordinates in the back side of the lens. To do this, it is necessary to turn the system forming the anisotropy of the lens on the side of the incident beam by $180^{\circ}$. In this case, the center of the focused beam displaces by the length comparable with the wavelength of radiation. The effect is determined by the sign of circular polarization, and, in contrast to the effect described in Refs. 1 and 2, the cross components of the radiation take part in it. Let us consider the enumerated problems.

## 1. Polarization effects caused by anisotropy of intensity of the longitudinal component of radiation in the focal waist area

Let us show that one can observe another class of polarization effects by means of a lens. It is the cross anisotropy of the intensity of the longitudinal component of radiation in the focal waist area, which changes the orientation with the change of the sign of circular polarization. The longitudinal component of the electric field in the focal waist area can be determined from the condition ${ }^{1} \operatorname{div} \mathbf{E}=0$ :

$$
\begin{equation*}
E_{z}=\frac{i}{k} \nabla_{\perp} \mathbf{E}_{\perp}, \quad \mathbf{E}_{\perp} \sim\left(\mathbf{e}_{x}+i \mathbf{e}_{y}\right) \Psi . \tag{1}
\end{equation*}
$$

It is seen from expressions (1) that for determining the value of the cross deviation of the longitudinal component in the focal waist area, it is necessary to know the cross component of the field in the waist.

Let we know the amplitude of the field $f\left(\mathbf{r}_{\perp}\right)$ in the focal waist, where $r_{\perp}^{2}=(\tilde{x}, \tilde{y})$. Having known $f\left(\mathbf{r}_{\perp}\right)$, one can analytically determine the parameters of the wave front at the focal length. Let us derive the analytical solutions. Let us assume that propagation of radiation can be described by the scalar wave equation

$$
\begin{equation*}
2 i \beta \frac{\partial \Psi\left(z, \mathbf{r}_{\perp}\right)}{\partial z}+\Delta_{\perp} \Psi=0 \tag{2}
\end{equation*}
$$

where $\Psi\left(z=0, \mathbf{r}_{\perp}\right)=f\left(\mathbf{r}_{\perp}\right) ; \beta=2 \pi / \lambda, \lambda$ is the light wavelength in vacuum, $\Delta_{\perp}=\partial^{2} / \partial x^{2}+\partial^{2} / \partial y^{2}$.

Using direct and inverse Fourier transforms, we obtain from the Eq. (2) that

$$
\begin{gather*}
\Psi\left(z, \mathbf{k}_{\perp}\right)=f\left(\mathbf{k}_{\perp}\right) \exp \left(-i \frac{k_{\perp}^{2}}{2 \beta} z\right) \\
\Psi\left(z, \mathbf{r}_{\perp}\right)=\int f\left(\mathbf{k}_{\perp}\right) \exp \left(i \mathbf{k}_{\perp} \mathbf{r}_{\perp}-i \frac{k_{\perp}^{2}}{2 \beta} z\right) \mathrm{d} \mathbf{k}_{\perp}, \tag{3}
\end{gather*}
$$

where $f\left(\mathbf{k}_{\perp}\right)$ is the Fourier image of the function $f\left(\mathbf{r}_{\perp}\right)$.

It follows from Eq. (3) that the wave amplitude at the point ( $z, \mathbf{r}_{\perp}$ ) is unambiguously determined by the wave amplitude at the focal waist area $\Psi\left(z=0, \mathbf{r}_{\perp}\right)=f\left(\mathbf{r}_{\perp}\right)$.

Let us take the following functions as the wave amplitudes in the focal waist area:

$$
\begin{gather*}
f_{1}\left(\mathbf{r}_{\perp}\right)=F_{1} \exp \left(-r_{\perp}^{2} /\left(2 a^{2}\right)+i \tilde{\kappa} x y\right)  \tag{4}\\
f_{2}\left(\mathbf{r}_{\perp}\right)=F_{2} r_{\perp}^{m} \exp \left(-r_{\perp}^{2} /\left(2 a^{2}\right)+i m \varphi\right)
\end{gather*}
$$

According to expressions (1), the following intensities of the longitudinal component of the field $E_{z}$ correspond to such amplitudes:

$$
\begin{gather*}
\left|E_{z}\left(\mathbf{r}_{\perp}\right)\right|^{2}=\left|F_{1}\right|^{2}\left\{x^{2}\left(1 / a^{2}+\sigma \tilde{\kappa}\right)^{2}+\right. \\
\left.+y^{2}\left(1 / a^{2}-\sigma \tilde{\kappa}\right)^{2}\right\} \exp \left(-r_{\perp}^{2} / a^{2}\right)  \tag{5}\\
\left|E_{z}\left(\mathbf{r}_{\perp}\right)\right|^{2}=\left|F_{2}\right|^{2}\left[\left(\partial / \partial r_{\perp}-\sigma m / r_{\perp}\right) f\left(\mathbf{r}_{\perp}\right)\right]^{2}
\end{gather*}
$$

It follows from Eq. (5) that in the case of $f_{1}\left(\mathbf{r}_{\perp}\right)$ some ellipsoids correspond to the longitudinal intensity $\left|E_{z}\right|^{2}$ with the ellipticity being the highest at $\tilde{\kappa}=a^{2}$ and determined by the sign of circular polarization $\sigma$. In the case of $f_{2}\left(\mathbf{r}_{\perp}\right)$ it is necessary that $\left|E_{z}\right|^{2}$ be equal to zero. If this condition holds then, according to Eq. (5), the longitudinal intensity either increases or decreases depending on the sign of $\sigma$.

Taking the inverse Fourier transform of $f_{1}\left(\mathbf{r}_{\perp}\right)$ and $f_{2}\left(\mathbf{r}_{\perp}\right)$, we obtain by formulas (4) with the account of Eq. (3) that
$\left|\Psi_{1}\left(z, \mathbf{r}_{\perp}\right)\right|^{2}=$ const $\left|\exp \left\{-x^{2} /\left[2\left(a^{2} /\left(1+\tilde{\kappa}^{2} a^{4}\right)+i z / \beta\right)\right]\right\}\right|^{2} \times$ $\times\left|\exp \frac{\left[i y+\frac{a^{4} \tilde{\kappa} x}{a^{2}+i z\left(1+\tilde{\kappa}^{2} a^{4}\right) / \beta}\right]}{2\left[\frac{a^{2}}{1+\tilde{\kappa}^{2} a^{4}}+i \frac{z}{\beta}+\frac{a^{8} \tilde{\kappa}^{2}}{\left(1+\tilde{\kappa}^{2} a^{4}\right) /\left[a^{2} /\left(1+\tilde{\kappa}^{2} a^{4}\right)+i z / \beta\right]}\right]}\right|^{2} ;$

$$
\begin{align*}
& \Psi_{2}=\operatorname{const}\left\{r_{\perp}^{m} /\left(a^{2}+i z / \beta\right)^{m+1}\right\} \times \\
& \times \exp \left\{-r_{\perp}^{2} /\left[2\left(a^{2}+i z / \beta\right)\right]+i m \varphi\right\} \tag{6}
\end{align*}
$$

The following relationship ${ }^{8}$ was used when deriving the second relationship of the system (6):

$$
\int_{0}^{\infty} z^{m+1} J_{m}(b z) \exp \left(-\Lambda^{2} z^{2}\right) \mathrm{d} z=\frac{b^{m} \exp \left[-b^{2} /\left(4 \Lambda^{2}\right)\right]}{\left(2 \Lambda^{2}\right)^{m+1}}
$$

## 2. Mathematical simulation

 of the effect of cross shift of the focal waist caused by the sign of circular polarizationTo optimize the experimental conditions for observation of the aforementioned polarization effect, we have made a series of calculations using "Linza"
software package. The final purpose was to optimize the parameters of spherically converging asymmetric wave front necessary for observation of the effect in the focal waist area. To do this, the amplitude of the cross component of the field was set, according to Ref. 1, on the rectangular spherical segment

$$
\begin{equation*}
\Psi_{3}(x, y)=M_{00}(x, y)+\alpha M_{01}(x, y), \tag{7}
\end{equation*}
$$

where $\alpha$ is the complex number. The functions $M_{00}(x, y)$ and $M_{01}(x, y)$ have the form

$$
\begin{gather*}
M_{00}=\exp \left\{-\left(x^{2}+y^{2}\right) /\left[2(\Delta a)^{2}\right]\right\},  \tag{8}\\
M_{01}=\frac{y}{\Delta a} \exp \left\{-\left(x^{2}+y^{2}\right) /\left[2(\Delta a)^{2}\right]\right\}
\end{gather*}
$$

Calculations were performed for radiation with the wavelength $\lambda=0.63 \mu \mathrm{~m}, \Delta a=0.25 \cdot 10^{4} \mu \mathrm{~m}$,

$$
\begin{gathered}
-0.5 \mathrm{~cm} \leq x \leq 0.5 \mathrm{~cm} ;-0.5 \mathrm{~cm} \leq y \leq 0.5 \mathrm{~cm} ; \\
\Delta x \times \Delta y=250 \times 250 \text { points; } \\
-0.001 \mathrm{~cm} \leq \tilde{x} \leq 0.001 \mathrm{~cm} ;-0.001 \mathrm{~cm} \leq \tilde{y} \leq 0.001 \mathrm{~cm} ; \\
\Delta \tilde{x} \times \tilde{y}=300 \times 300 \text { points. }
\end{gathered}
$$

The cross component of the filed was determined by "Linza" program in the image plane $\tilde{x}, \tilde{y}$. The longitudinal component ( $z$-component) was calculated by formula (1) (see Ref. 1)

$$
\begin{equation*}
E_{z}(\tilde{x}, \tilde{y}) \sim \frac{i}{k}\left(\frac{\partial}{\partial \tilde{x}}+i \sigma \frac{\partial}{\partial \tilde{y}}\right) \Psi(\tilde{x}, \tilde{y}, \tilde{z}=0) \tag{9}
\end{equation*}
$$

where $\sigma= \pm$ is the sign of circular polarization.
The intensities $I_{\sigma= \pm 1}=\left|E_{z}(\tilde{x}, \tilde{y}=0)\right|^{2} \quad$ of the longitudinal fields are shown in Fig. 1 as functions of $\tilde{x}$ at $\tilde{y}=0$ for $\alpha=1, \Delta a=0.25 \mathrm{~cm}$.


Fig. 1. Intensities $I_{\sigma= \pm 1}=\left|E_{z}(\tilde{x}, \tilde{y}=0)\right|^{2}$ as functions of $\tilde{x}$ at $\alpha=1: \sigma=1$ (curve 1); $\sigma=-1$ (2).

It is seen from Fig. 1 that the maxima of the intensities are shifted relative each other by $\Delta \tilde{x} \approx 0.00063 \mathrm{~cm} \equiv 6.26 \mu \mathrm{~m}$. The curves overlap quite strongly. As it is supposed to experimentally determine not the shift of the maxima of intensities $I_{\sigma= \pm 1}$, but the value of power incident on the left $\tilde{x} \leq 0$ and right $\tilde{x} \geq 0$ half-planes, the power was also numerically determined:

$$
\begin{align*}
& W_{1, \sigma= \pm 1}=\int_{0}^{\infty} \mathrm{d} \tilde{x} \int_{-\infty}^{\infty}\left|E_{z, \sigma= \pm 1}(\tilde{x}, \tilde{y})\right|^{2} \mathrm{~d} y,  \tag{10}\\
& W_{2, \sigma= \pm 1}=\int_{-\infty}^{0} \mathrm{~d} \tilde{x} \int_{-\infty}^{\infty}\left|E_{z, \sigma= \pm 1}(\tilde{x}, \tilde{y})\right|^{2} \mathrm{~d} y,
\end{align*}
$$

where $W_{1, \sigma= \pm 1}+W_{2, \sigma= \pm 1}=1$. At $\alpha=1$ expression (7) yielded $W_{1, \sigma=1} / W_{2, \sigma=1}=W_{2, \sigma=-1} / W_{1, \sigma=-1} \approx 0.27$. As $\alpha$ increases, the distance $\Delta \tilde{x}$ between the maxima increases, as is seen in Table 1. This strengthens the effect under study. However, as $\alpha$ decreases, the ratios of the secondary maximum to the main one $K$ and the value $W_{1, \sigma=1} / W_{2, \sigma=1}=W_{2, \sigma=-1} / W_{1, \sigma=-1}$ increase, while the value $\left|E_{z}(\tilde{x}=0, \tilde{y}=0)\right|$ decreases (it is well seen in Fig. 2), what leads to weakening of the considered effect. As $\alpha$ increases, the values $K$ and $W_{1, \sigma=1} / W_{2, \sigma=1}=W_{2, \sigma=-1} / W_{1, \sigma=-1} \quad$ decrease, that strengthens the effect, but the effect is weakened due to the decrease of the distance $\Delta \tilde{x}$ between the maxima. Thus, the considered effect as the problem of optimization over several parameters has no extreme. So, one should choose the parameters depending on the conditions of experiment.

Table 1

| No. of <br> calculation | $\alpha$ | $\left\|E_{z}(\tilde{x}=0, \tilde{y}=0)\right\|^{2}$ | $\Delta \tilde{x}, \mu \mathrm{~m}$ | $K$ | $W_{1, \sigma=1} / W_{2, \sigma=1}$ |
| :---: | :--- | :--- | :--- | :--- | :--- |
| 1 | 0.1 | 0.0151 | 9.34 | 0.7514 | 0.8134 |
| 2 | 0.2 | 0.0526 | 8.94 | 0.5650 | 0.6654 |
| 3 | 0.3 | 0.1034 | 8.54 | 0.4257 | 0.5506 |
| 4 | 0.4 | 0.1609 | 8.26 | 0.3215 | 0.4631 |
| 5 | 0.6 | 0.2804 | 7.46 | 0.1854 | 0.3505 |
| 6 | 0.7 | 0.3377 | 7.2 | 0.1417 | 0.3170 |
| 7 | 0.8 | 0.3918 | 6.94 | 0.1090 | 0.2945 |
| 8 | 0.9 | 0.4421 | 6.54 | 0.0843 | 0.2805 |
| 9 | 1.0 | 0.4883 | 6.26 | 0.0656 | 0.2731 |
| 10 | 1.1 | 0.5307 | 6.0 | 0.0514 | 0.2707 |
| 11 | 1.2 | 0.5691 | 5.74 | 0.0406 | 0.2722 |
| 12 | 1.3 | 0.6040 | 5.6 | 0.0323 | 0.2767 |
| 13 | 1.4 | 0.6356 | 5.34 | 0.0259 | 0.2833 |
| 14 | 1.5 | 0.6642 | 5.2 | 0.0210 | 0.2917 |
| 15 | 1.6 | 0.6899 | 4.94 | 0.0171 | 0.3012 |
| 16 | 1.7 | 0.7133 | 4.8 | 0.0140 | 0.3116 |
| 17 | 1.8 | 0.7345 | 4.54 | 0.0116 | 0.3226 |
| 18 | 1.9 | 0.7536 | 4.4 | 0.0097 | 0.3340 |
| 19 | 2.0 | 0.7709 | 4.26 | 0.0082 | 0.3456 |



Fig. 2. Intensities $I_{\sigma= \pm 1}=\left|E_{z}(\tilde{x}, \tilde{y}=0)\right|^{2}$ as functions of $\tilde{x}$ at $\alpha=0.1: \sigma=1$ (curve 1); $\sigma=-1$ (2).

The dependence of $\Delta \tilde{x}$ on $\Delta a$ at $\alpha=1$ is given in Table 2 for the initial signal (7).

Table 2

| $\Delta a, \mathrm{~cm}$ | 0.1 | 0.15 | 0.17 | 0.21 | 0.23 | 0.25 | 0.27 | 0.31 | 0.33 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\Delta \tilde{x}, \mu \mathrm{~m}$ | 12.4 | 8.4 | 7.6 | 6.7 | 6.4 | 6.26 | 6.26 | 6.13 | 6.13 |
| $\Delta a, \mathrm{~cm}$ | 0.35 | 0.41 | 0.43 | 0.45 | 0.47 | 0.51 | 0.53 | 0.55 | 0.63 |
| $\Delta \tilde{x}, \mu \mathrm{~m}$ | 6.13 | 6.26 | 6.4 | 6.4 | 6.4 | 6.54 | 6.54 | 6.54 | 6.8 |

The amplitude of the cross component of the field was set on the rectangular spherical segment and at the same parameters of the lens and radiation

$$
\begin{equation*}
\Psi_{4}(x, y)=\exp \left(-\left[x^{2}+(y-a)^{2}\right] /\left[2(\Delta a)^{2}\right]\right) \tag{11}
\end{equation*}
$$

where

$$
\begin{gathered}
0.5 \mathrm{~cm} \leq x \leq 0.5 \mathrm{~cm} ;-0.5 \mathrm{~cm} \leq x \leq 0.5 \mathrm{~cm} \\
\Delta x \times \Delta y=250 \times 250 \text { points } \\
-0.001 \mathrm{~cm} \leq \tilde{x} \leq 0.001 \mathrm{~cm} \\
-0.001 \mathrm{~cm} \leq \tilde{y} \leq 0.001 \mathrm{~cm}
\end{gathered}
$$

$$
\Delta \tilde{x} \times \Delta \tilde{y}=300 \times 300 \text { points, } \Delta a=0.25 \mathrm{~cm}
$$

The field (11) is the Gaussian beam of the width $a$ with the center at the point $x=0, y=a$. It was taken in calculations that $a=0.1,0.2$, and 0.4 cm .

As is seen from relations (7) and (11), at $\alpha \ll 1$ and $a /(\Delta a) \ll 1$, the function $\Psi_{4}$ is transformed to $\Psi_{3}$ at $\alpha=a /(\Delta a)$. Therefore, the qualitative dependences of $\left|E_{z}(\tilde{x}=0, \tilde{y}=0)\right|^{2}, \quad \Delta \tilde{x}, \quad K, \quad$ and $W_{1, \sigma=1} / W_{2, \sigma=1}$ on $a$ for $\Psi_{4}$ coincide with the dependences of these values on $\alpha$ for $\Psi_{3}$ (Table 3).

Table 3

| No. of <br> calcu- <br> lation | $a$ | $\left\|E_{z}(\tilde{x}=0, \tilde{y}=0)\right\|^{2}$ | $\Delta \tilde{x}$, <br> $\mu \mathrm{m}$ | $K$ | $W_{1, \sigma=1} / W_{2, \sigma=1}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $0.5 \Delta a$ | 0.2149 | 7.86 | 0.2503 |  |
| 2 | $\Delta a$ | 0.4549 | 6.54 | 0.070 |  |
| 3 | $2 \Delta a$ | 0.6784 | 5.06 | 0.0188 | 0.27 |

Table 3 presents the dependences of $\left|E_{z}(\tilde{x}=0, \tilde{y}=0)\right|^{2}, \quad \Delta \tilde{x}, \quad K$, and $W_{1, \sigma=1} / W_{2, \sigma=1}$ on $a$ for the Gaussian profile of the cross component of the field (11) where $\max \left|E_{z}(\tilde{x}, \tilde{y})\right|=1$ and $\Delta a=0.25 \mathrm{~cm}$.

## 3. Cross shift of the beam due to the effects occurring inside the focusing lens

Let us show that in the case of a lens the cross shift of the beam occurs due to spherically converging wave front, which is formed both after the lens (in this case the spherical front is formed by means of the lens, i.e., the lens is the passive element) and inside the lens. In Ref. 7 the equation was obtained of the beam trajectory in the presence of the gradient of intensity along the direction perpendicular to the trajectory:
$\frac{\partial \mathbf{S}}{\partial s}=\mathbf{1} \times \nabla \ln n \times \mathbf{S}-\frac{\sigma}{4 k}\left[(\nabla \times \mathbf{S}) \frac{\partial \ln n}{\partial s}+(\nabla \ln \rho \times \mathbf{S}) \frac{\partial \ln \rho}{\partial s}\right]$,
where $\rho=(\mu / \varepsilon)^{1 / 2}$ is the impedance. The first term in square brackets in Eq. (12) describes the cross shift of the beam in an inhomogeneous medium in the case when a functional dependence of the UmovPoynting amplitude on the cross coordinates exists. Indeed, let us assume that

$$
\begin{equation*}
\nabla \times \mathbf{S}=\mathbf{F} \times \mathbf{S} \neq 0 \tag{13}
\end{equation*}
$$

then, according to Eq. (12), the vector $\mathbf{S}$ at $\partial \ln n / \partial s=0$ turns towards the direction parallel to the vector $\mathbf{l} \times \mathbf{F}$, and the value of the deviation is determined by the sign of circular polarization. Moreover, the cross shift can exist even in the case of a linear path of the beam.

In should be noted that formula (12) was obtained assuming that the value $\partial \ln n / \partial s$ has no breaks. Therefore, one cannot quantitatively determine the cross shift of the beam at a stepwise change of the refractive index $n$. In this case one can proceed to limit assuming that $n$ changes from $n_{1}$ to $n_{2}$ within a narrow interval $\Delta s$. Then, according to Eq. (12), the angle of the cross deviation of the beam is as follows

$$
\begin{equation*}
\varphi=[\sigma /(4 k)]|\mathbf{l} \times \mathbf{F}| \ln n_{2} / n_{1} \tag{14}
\end{equation*}
$$

where $|\mathbf{F}|=$ const. One should expect that in the case of a stepwise change of $n$ the cross shift is comparable with the value given by Eq. (14) (Fedorov effect ${ }^{8,9}$ is not taken into account).

It is interesting to note that in the case of $n_{1} \rightarrow n_{2} \rightarrow n_{1}$ the direction $\mathbf{S}$ does not change, but the beam shifts across the axis by the distance $d$. In the case of a transparent plate, one should expect that

$$
\begin{equation*}
d \approx \frac{\sigma}{4 k}|\mathbf{l} \times \mathbf{F}| h \ln \frac{n_{2}}{n_{1}}, \quad n_{2}>n_{1} \tag{15}
\end{equation*}
$$

where $h$ is the plate thickness.

Let we have a plano-convex lens, and the light is incident onto the plane side of the lens, and the back side is convex. Let the wave front ${ }^{1}$ be formed at the plane side:

$$
\begin{equation*}
\mathbf{S}=S \mathbf{1} ; \quad W(x, y)=W_{0}[1+\alpha y / a(s)], \tag{16}
\end{equation*}
$$

where $W_{0}(x, y)$ is a symmetric function of the variables $x$ and $y$. At $\alpha \ll 1$ we obtain, from Eq. (16) and taking into account the expression (13), that

$$
\begin{equation*}
\mathbf{1} \times \mathbf{F}=-\mathbf{e}_{x} \alpha / a(s=0) . \tag{17}
\end{equation*}
$$

Let $W(x, y)$ on the back side of the lens be a symmetric function of the variables $x$ and $y$ (to achieve this, one should turn the system forming the wave front (16) by $180^{\circ}$ relative to the symmetry axis). Let also the wave vector of radiation incident on the lens be perpendicular to the plane surface of the lens. In this case, according to expression (15), the wave front inside the lens shifts by the distance $d$. As the value $W$ at the back side of the lens is a symmetric function of $y$, no cross deviation due to the considered effect occurs. The wave front on the back side deviates from the linear path only according to the Snell's law. ${ }^{10}$ As the wave front inside the lens is plane, the radiation is focused on the focal plane, and the focal waist is at the distance $\Delta x=n f d / h$ from the symmetry axis, where $f$ is the focal length of the lens. Then we obtain from expressions (15) and (17) that

$$
\begin{equation*}
\Delta x=\left(\sigma \alpha n_{2} f\right) /[4 k a(s=0)] \ln n_{2} / n_{1} . \tag{18}
\end{equation*}
$$

Let $\alpha=0.1, \quad \lambda=0.63 \mu \mathrm{~m}, \quad f=30 \mathrm{~cm}, \quad n_{1}=1$, $n_{2}=1.46$, and $a(s=0)=2 \mathrm{~cm}$. Then the cross shift $2 \Delta x$ of the focal waist is equal to $0.44 \mu \mathrm{~m}$ at the change of sign $\sigma$. It is seen that the cross shift of the beam due to the polarization effect in the lens is comparable with the radiation wavelength.

## Conclusion

Possible polarization effects in lenses are theoretically studied and numerically simulated in the paper. The parameters of the spherically converging wave front necessary for observation of the effect are calculated in order to determine optimal conditions for observation of the effect. It follows from the numerical results that for the wave front (7) corresponding to the case of a lens with an opaque half, both the distance $\Delta \tilde{x}$ between maxima of the intensity of the $z$-component of radiation (that leads to intensification of the effect) and the ratio of the secondary maximum to the main one $K$ and the value $W_{1, \sigma=1} / W_{2, \sigma=1}=W_{2, \sigma=-1} / W_{1, \sigma=-1}$ (that leads to weakening of the considered effect) increase at decreasing $\alpha$. Thus, the considered problem is reduced to the problem of optimization over several parameters. For the wave front (8), the dependence of intensity of the longitudinal component of radiation on the cross coordinates is the same at for
the wave front (7). The parameter $a$ in Eq. (8) plays the same role in the effect under study as $\alpha$ in expression (7). The problem for the field amplitude (8) is also reduced to the problem of optimization over several parameters.

Thus, the polarization effects are theoretically considered in this paper, which can be observed for the longitudinal component of radiation in the focal waist area, the polarization effects in the waist are observed due to anisotropy along the transverse direction of the longitudinal component of radiation. The cross distribution of the radiation field in the focus is considered as described within this approach, and the field parameters on the spherical surface are calculated using the relationship between the wave front in the focal waist area and on the spherical surface obtained by means of direct and inverse Fourier transforms.

It is shown in this paper based on the equation obtained in Ref. 7 describing the effect of cross shift of the beam in the presence of the anisotropy of the intensity of radiation across the beam, that a transformation of the wave front on the front and rear sides of a lens is possible such that the cross components of the radiation in the focus displace across the beam axis, and the direction of the displacement is determined by the sign of circular polarization of the incident radiation and the linear magnitude of the deviation is comparable with the wavelength of radiation. As applied to the planeparallel plate made from a transparent substance, the effect leads to the cross shift of the beam, and the beam trajectories before and after the plate remain parallel.

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