Effect of phase fluctuations on propagation of the vortex beams

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We present some results obtained by numerical modeling of the propagation of vortex beams with a spiral phase through a randomly inhomogeneous medium being presented by a phase screen placed in the beginning of the propagation path. Such beams, if propagated under conditions of weak turbulence, also experience distortions, like Gaussian beams. However, the statistically averaged vortex beams conserve the central intensity dip with a nonzero intensity on the beam axis. The greater the beam vortex charge, the longer the beam propagation distance, at which the central dip is not smeared. The vortex beams being the Laguerre–Gaussian modes are found to have the same broadening properties while propagated through a randomly inhomogeneous medium as the Gaussian beams. The broadening of averaged vortex beams does not depend on the vortex charge and coincides with the broadening of a Gaussian beam.

Introduction

In recent years, interest has increased in the studies of non-Gaussian laser beam propagation through turbulent atmosphere. In particular, it is caused by the search for new types of laser beams for use in optical communication. The studies in Refs. 1 to 3 are analytical investigations into the properties of the *higher order annular Gaussian beams* (*HOAG beams* according to terminology in these papers) propagated through the turbulent atmosphere. It was noted that at their propagation through the turbulent atmosphere, the time-averaged intensity of such beams undergoes some stages of evolution. The averaged beam energy tends to concentrate near the axis at intermediate distances of propagation. Thus, the main averaged beam is formed. Finally, when propagated at a significant distance, the initially HOAG becomes purely Gaussian averaged beam.[3–5]

In Ref. 4, the propagation properties of tubular light beams of round, elliptical, and square-shaped cross sections were investigated analytically.

Recently, the light beams transferring the optical vortexes[6–8] have attracted a great interest. The lowest order Laguerre–Gaussian modes $LG_{0,l}$ of a laser resonator can serve as an example for such beams. Their intensity distribution forms a ring and their phase covers a spiral surface because of the optical vortex on the beam axis. The effect of a central dip and vortex on the Gaussian beam properties was investigated in Ref. 9 and it was shown that the vortex worsens the beam quality as compared with the quality of an ordinary Gaussian beam. Nevertheless, the unique capability of the beams that transfer the optical vortexes to transfer their orbital angular momentum had caused a large development in singular optics[10–12] and its various practical applications, including the so-called optical forcepses.[6–8] The beams can be obtained in several ways: by means of computer-synthesized holograms and diffraction optics,[10] at conversion of Hermit-Gaussian modes[11] and so on. Various ways of their formation are being developed and improved.

In recent years use of the orbital angular momentum (OAM)[12] transferred by the light beam with optical vortex has been actively investigated. It appears essential for information coding in lines of optical communication. It was shown that even weak turbulence is a serious problem for operation of optical communication systems on the basis of OAM transfer.

By means of numerical modeling, we have studied the propagation properties of vortex beams under conditions of slightly turbulent atmosphere by an example of a simple model with a single phase screen and the effect of optical vortex of the beam on these properties.

1. Vortex beams and their properties

Let us set the vortex beam with a topological charge l in the cylindrical coordinate system (r, θ, z) by the formula for the complex function of scalar field:

$$U_l(r,\theta,z) = \frac{A}{w(z)}\left(\frac{r\sqrt{2}}{w(z)}\right)^l \exp\left[\frac{-r^2}{w^2(z)}\right]\times$$
$$\times\exp\left[i\frac{kr^2}{2R(z)}\right]\exp(il\theta)\exp[i\varphi(z)], \quad (1)$$

where

$$A = \sqrt{2/(\pi l!)}$$

is the normalization factor;

$$w(z) = w_0\sqrt{1 + z^2/z_R^2}$$

is the beam half-width;

$$\varphi(z) = \arctan(z/z_R)$$

is the Gouy phase; $z_R = \pi w_0^2/\lambda$ is the diffraction beam distance (Rayleigh distance); $R(z)$ is the radius of curvature of the beam wave front.

Formula for the vortex beam intensity has the form

$$I_l(r,z) = |U_l(r,z,\theta)|^2 = \frac{A^2}{w^2(z)}\left(\frac{r\sqrt{2}}{w(z)}\right)^{2l} \exp\left[\frac{-2r^2}{w^2(z)}\right]. \quad (2)$$

Vortex beams (1) are the Laguerre–Gaussian modes $LG_{p,l}$ with $p = 0$. If $l = 0$, we shall obtain the Gaussian beam $LG_{0,0}$ as the lowest mode:

$$U_0(r,z) = \frac{A}{w(z)}\exp\left[\frac{-r^2}{w^2(z)}\right]\exp\left[i\frac{kr^2}{2R(z)}\right]\exp[i\varphi(z)]. \quad (3)$$

The effective radius r_{eff} of the vortex beam (1) appears proportional to $\sqrt{(l+1)}$, whereas the maximum radius of the intensity ring r_d is proportional to $\sqrt{l}$:

$$r_{\text{eff}}(z) = \sqrt{\frac{2\int r^2 I(r)\mathrm{d}S}{\int I(r)\mathrm{d}S}} = w(z)\sqrt{\frac{l+1}{2}},$$
$$r_d(z) = w(z)\sqrt{\frac{l}{2}}. \quad (4)$$

As known from the laser theory, all Laguerre–Gaussian modes under conditions of free propagation conserve its shape accurate within scaling.[13] Using formula (2), one can easily see that the diffraction distance for the vortex beams does not depend on l and coincides with the diffraction distance for the Gaussian beam $z_R^{\text{vortex}} = z_R^{\text{gauss}}$.

Let us consider a collimated vortex beam (1) in the source plane taken in the following form

$$U_l(r,\theta) = \frac{1}{\sqrt{\pi l!}} r^l \exp\left[\frac{-r^2}{2}\right]\exp[il\theta]. \quad (5)$$

The intensity distribution for this beam has the shape of a ring:

$$I_l(r) = \frac{1}{\pi l!} r^{2l} \mathrm{e}^{-r^2}. \quad (6)$$

Since formula (5) for the vortex beam with zero charge $l = 0$ agrees with the Gaussian beam formula:

$$U_0(r) = (1/\sqrt{\pi})\mathrm{e}^{-r^2/2}, \quad (7)$$

we shall consider the Gaussian beam as the vortex one with zero vortex charge $l = 0$, for a convenience.

The vortex beam (5)

$$U_l(r,\theta) = \frac{1}{\sqrt{\pi l!}} r^l \mathrm{e}^{-r^2/2}\mathrm{e}^{il\theta} \sim (x+iy)^l \mathrm{e}^{-r^2/2}, \; l = 1, 2, 3,$$

contains the point of zero intensity on the beam axis coinciding with the point of the phase singularity. The vortex beam wave front has the form of helicoidal (spiral) surface.

Note that the difference between the vortex beam (5) and the Gaussian beam (7) is in the vortex cofactor $r^l \mathrm{e}^{il\theta} = (x+iy)^l$. Such a cofactor in formula (5) denotes the optical vortex with the charge $l > 0$ on the beam axis. The Poynting vector characterizing the motion of light energy in the beam is directed along a spiral path around some line of zero field intensity. Trajectory of this vortex center coincides, in the 3-D space, with the beam axis. The optical vortex prevents the diffraction washout of a central dip of the vortex beam as compared with annular beams, without such a vortex cofactor.[9]

In Ref. 9 one can find an analytical description of the beam properties with and without a vortex at free propagation. Both types have identical amplitude factor and only differ by the phase factor $\mathrm{e}^{il\theta}$. At free propagation, the vortex beams conserve their shape in contrast to the annular beams without a vortex.

2. Description of the computer experiment

In the numerical experiment, we have investigated the effect of phase fluctuations on the vortex beam propagation. One phase screen is located at the beginning of the path. The calculations were made using known numerical model.[14,15] Within the limits of paraxial approximation, we have solved the parabolic equation in the dimensionless form

$$\frac{\partial W}{\partial z} = \frac{i}{2}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + T\right)W, \quad (8)$$

where $W(x, y; z)$ is the complex wave amplitude, T is the temperature field with the spectral density

$$F(\kappa_x,\kappa_y) = C_T^2(\kappa_0^2 + \kappa_x^2 + \kappa_y^2)^{-11/6}\exp[-(\kappa_x^2 + \kappa_y^2)/\kappa_m^2],$$
$$\kappa_0 = 2\pi/L_0, \; \kappa_m = 2\pi/l_0, \quad (9)$$

where L_0 and l_0 are the outer and inner scales of turbulence normalized to the initial beam radius r_0, equal to 100 and 0.1 cm, respectively. The wave parameter $Z_0 = L\lambda/(2\pi r_0^2)$ was equal to 0.1 for $\lambda = 0.63$ μm. The values of the parameters correspond to the ground layer of the turbulent atmosphere for a horizontal path of 1 km long and the beam with $r_0 = 10$ cm.

The order of the grid matrix was equal to 512. We used the method of statistical tests, mean, over 200 realizations, were taken as estimates. The effect of turbulence on the beam was characterized by the scintillation index σ_I^2, being the normalized variance of fluctuations of the Gaussian beam intensity $I(z)$ calculated on the beam axis. Figure 1 presents the dependence of σ_I^2 on the structure characteristics of temperature fluctuations C_T^2.

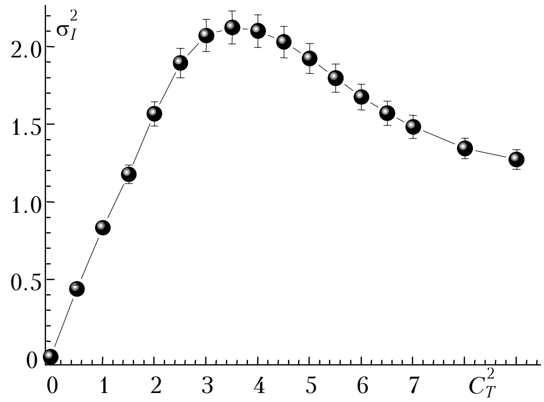

Fig. 1. Dependence of the scintillation index σ_I^2 on the dimensionless turbulence parameter C_T^2, obtained in the experiment. The root-mean-square deviations shown by vertical bars are taken as the confidence intervals.

Obviously, the vortex beams as well as the Gaussian ones, propagated through a randomly inhomogeneous medium undergo the effect of this medium. The random inhomogeneities of the medium refractive index along the propagation path cause beam fluctuations. These result in both random displacements of its axis and structure deformations. The point of zero intensity for the vortex beam initially being in the beam center is moved from the axis of the beam propagation but it does not disappear absolutely. The deformations of the initially symmetric intensity ring and distortions of the beam phase take place even under conditions of weak turbulence. For the beams with $l > 1$, the vortex core appears splitted into l elementary vortexes with $l = 1$, displaced from the axis of the beam propagation.

However, in practice the averaged parameters of beam realizations often are of higher interest instead of individual realizations. The averaged beam is the result of addition of all instantaneous realizations within a finite time interval. Single points with zero intensity in such a beam are absent, but there is a dip in its center. The type of beam intensity statistically averaged over a large number of realizations was assumed as radially symmetric (in the form of a ring with a nonzero dip in its center). It is convenient to describe this dip as follows

$$h = (I_{\max} - I_0)/I_{\max},$$

where I_0 is the intensity on the beam axis, $I_{\max}$ is the maximum intensity in the ring-maximum. The beam divergence is characterized by the normalized effective beam radius

$$\tilde{R}_{\text{eff}}(z) = r_{\text{eff}}(z)/r_{\text{eff}}(z = 0).$$

Figure 2 presents the intensity and phase of the initial collimated vortex beam with $l = 1$ and the intensity profiles. One can see the decrease of the dip depth in the center of the averaged beam due to the increasing distance from the phase screen.

We have also calculated the propagation of the collimated vortex beams with $l = 2$ and 3. Besides, we have found that at propagation of vortex beams passed through one phase screen at the beginning of the path that simulates the turbulence effect the gradual smearing of the central dip of the averaged beam takes place, and, the higher is the vortex beam charge, the longer is the propagation distance at which this dip disappears.

At higher values of C_T^2, the gap smears faster.

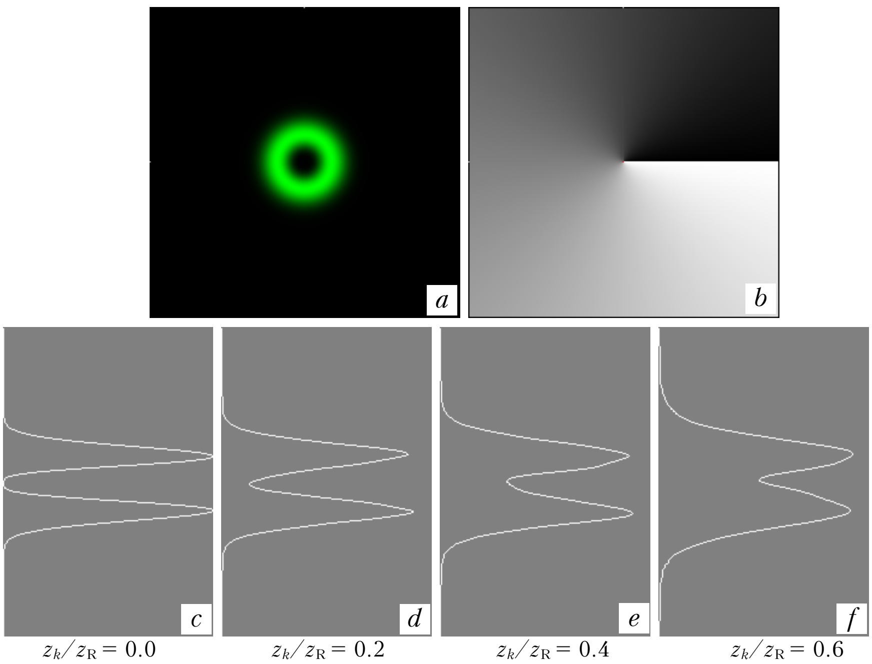

Fig. 2. Intensity (a) and the phase (b) of the initial collimated vortex beam with $l = 1$; profiles of the beam intensity calculated for some propagation distances $z_k/z_R = 0.0$; 0.2; 0.4; 0.6 (c–f) using the values of the dimensionless parameter $C_T^2 = 0.1$.

Figure 3 shows the dependence of squared normalized effective beam radius $\tilde{R}^2_{\rm eff}(C_T^2)$, that coincides for all considered vortex and Gaussian beams and is presented by a straight line. That is why the dependences of $r^2_{\rm eff}(C_T^2)$ on C_T^2 for the vortex beams with l = 1, 2, and 3 and the Gaussian beam are parallel straight lines with the identical slope angle. This means that the vortex beams considered as well as the Gaussian beams undergo identical broadening while propagated through the medium behind the phase screen.

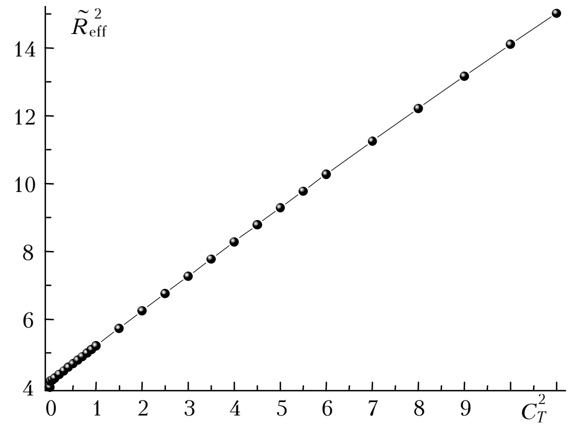

Fig. 3. Dependences of $\tilde{R}^2_{\rm eff}(C_T^2)$ for vortex beams at $z_k/z_{\rm R} = 0.2$.

At propagation of these vortex beams along the path, there occurs smearing of the central dip and the greater the vortex beam charge (and, therefore, the wider the vortex funnel), the longer the dip conserves.

Besides, we have revealed that the vortex beams, being the lowest Laguerre–Gaussian modes, have the same broadening properties as the Gaussian beam. The broadening of the averaged vortex beams does not depend on the vortex charge l and coincides with the Gaussian beam broadening.

Acknowledgments

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