RECONSTRUCTION OF PULSE SHAPE AND CORRECTION OF COORDINATES OF AN ISOTROPICALLY EMITTING SOURCE OBSERVED THROUGH A CLOUD LAYER

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We discuss the reconstruction of the pulse width and correct coordinates of an isotropically emitting source from remote observations through homogeneous clouds. The approach suggested is based on the use of known solutions of the radiative transfer equation, and requires a high-resolution observation system enabling one to resolve the images of isolated portions of the beam spot at the cloud tops.

Certain remote-sensing problems require that one record optical signals from isotropically emitting sources through a cloud layer. If the layer is dense and strongly attenuates direct radiation, then the signal recorded by a receiver is formed by radiation coming from different points of the reemitting spot at the upper boundary of the layer (plane $z = 0$ in Fig. 1) due to scattering. The intensity distribution of the scattered radiation at the upper boundary of the layer can be written, in the thin-screen approximation, as:

$$U(r)=[1+(r/h)^2]^{-3/2}\exp\{[(1+(r/h)^2)^{1/2}-1]\times(\Lambda-1)\tau_0\}, \quad (1)$$

where h is the distance from the source to the upper boundary of the layer; r is the radial coordinate on the plane $z = 0$ with the reference point at the centre of the beam spot; τ_0 is the optical depth of the layer along the direction from the source to the spot's centre; Λ is the probability of photon survival in a single-scattering process (for water droplets at visual wavelengths, it is close to unity).

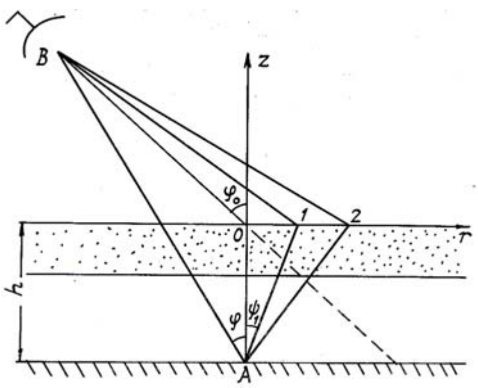

Fig. 1. The experimental arrangement for observing a nonstationary isotropic source of radiation through a cloud layer.

In traversing a cloud layer, information on pulse shape and source coordinates is partially lost. As a consequence, the distorting effects of the atmosphere must be corrected for.

In a real situation a receiver has a finite beam pattern, and the signals recorded are formed by superposition of large number of signals coming from different points of the spot with certain relative delays. This leads to a lengthening of the resulting pulse relative to that emitted by the source. Observed from large distances ($S = AB \gg h$), this lengthening can be described by a pulse function $\xi(t)$, which is obtained from the distribution (1) with the coordinate transformation (isochrone equation)

$$(1+x^2/h^2+y^2/h^2)^{1/2}+x\sin\varphi/h=t/t_0, \quad (2)$$

$$t_0=h/c, \quad t\geq t_0\cos\varphi,$$

where φ is the viewing angle, and c is the speed of light. The function $\xi(t)$ is obtained by integrating $U(r = (x^2 + y^2)^{1/2})$ over x and y within the limits

$$n_{2,1}(t,y)=\frac{h}{\cos\varphi}\left[\pm(t^2/t_0^2\cos^2\varphi-1-y^2/h^2)^{1/2}-(t/t_0)\,\mathrm{tg}\,\varphi\right], \quad m_{2,1}(t)=\pm h(t^2/t_0^2-1)^{1/2},$$

determined by Eq. (2). Thus, $\xi(t)$ is defined as

$$\xi(t)=A\frac{d}{dt}\int_{m_1}^{m_2}dy\int_{n_1}^{n_2}dx\,U(x,y), \quad t\geq t_0\cos\varphi, \quad (3)$$

where A is a normalizing factor. Keeping terms only to first order in $\sin^2\varphi$, one obtains from (2) and (3)

$$\xi(t)=\eta\,\frac{\Delta\omega\cos\varphi}{2\pi t_0}\left[\frac{t_0}{t}\right]^2\left[1-\frac{3}{2}\sin^2\varphi\left[\frac{t_0}{t}\right]^2\right], \quad (4)$$

where $\Delta\omega$ is the solid angle subtended by the receiver aperture at the source, and η is the transmission of the layer. The duration q of the response (at some level) is related to the dimensions l of the luminous spot at the same level by

$$q \simeq (1/c)[(1+h^2/l^2)^{1/2}+\sin\varphi-(h/l)\cos\varphi]. \qquad (5)$$

The original signal is lengthened to this value at the receiver, the output signal being the convolution of the input signal and the impulse response $\xi(t)$ of the system.

The original pulse shape for known impulse response function of the path, may be reconstructed by solving certain ill-posed problems associated with convolution integrals of the first kind[1].

For a narrow beam pattern, enabling one to resolve portions of the observed surface of size cAt, where At is the required temporal resolution of the system, the above effect vanishes. However, there remain delay effects due to multiple scattering of radiation propagating though the cloud layer. If the system resolves individual points on the spot and allows separate measurements to be made of the signals coming from them, then one can assess from the experimental data the signal distortions due to multiple scattering, using one or another model of light propagation through clouds, and, reconstruct the initial shape of the pulse and the source coordinates. Below we discuss one such procedure, based on the small-angle approximation of radiative transfer theory for isotropically scattering media[2–4].

The dispersion D_1 (the second moment of the energy distribution in time) of a pulse with the plane wavefront that has traveled a distance L through a cloud, differs from the initial variance D_0 by the value

$$D_s=\frac{\alpha^4}{2c^2}\int_0^L (L-\xi)[g\tau_s(\xi)+\tau_s^2(\xi)]d\xi, \qquad (6)$$

where $\alpha^2 \equiv \langle\gamma^2\rangle$ is the variance of beam's deviation angle in a single scattering event. This variance is determined by the shape of scattering phase function[2] $P(\gamma)$; $\tau_s = \Lambda\tau \simeq \tau$; $g = \alpha^{-4}\langle\gamma^4\rangle$. For a homogeneous path where' the scattering phase function is approximately by Gaussian[3], i.e.,

$$P(\gamma)=(4/\alpha^2)\exp(-\gamma^2/\alpha^2),$$

D_s is given by

$$D_s=(\alpha^4/6)t_L^2(\tau_s+\tau_s^2/4), \qquad (7)$$

where

$$\alpha^2=[2.66(d/\lambda)^2]^{-1}, \qquad (8)$$

d is the mean diameter of cloud droplets; λ is the wavelength of the radiation; $t_L = L/c$. The dispersion D_s is related to the coherence bandwidth ω_k by[3]

$$D_s=2.8/\omega_k^2.$$

Assuming the wave front to be locally plane at large distances from the source (at least within the limits of a resolved element of the cloud layer), one can use these equations to determine the height h of the upper cloud boundary, the optical depth of the layer in the vertical direction $\tau_{s_0}\simeq\tau_0$, and the initial dispersion D_0 of the pulse from measurements of the shapes of pulses received from different points on the luminous spot. If the points are all in the same plane XZ, then

$$h=(x^2-\delta^2c^2)(\delta c-x\sin\varphi)^{-1}2^{-1}$$

$$D_0=D_{10}-D_{s0}$$

$$D_{s0}=\frac{(A_2^4-A_2^3)\Delta_1-(A_1^4-A_1^3)\Delta_2}{(A_2^4-1)(A_1^3-1)-(A_1^4-1)(A_2^3-1)} \qquad (9)$$

$$\tau_{s0}=4\frac{(A_2^3-1)\Delta_1-(A_1^3-1)\Delta_2}{(A_1^4-1)\Delta_2-(A_2^4-1)\Delta_1}.$$

In equations (9), 6 is the difference between the moments when the pulses appear from the central point of the spot and from one of the other three points selected, located or a distance x_1 from the center; D_{10} is the measured dispersion of the pulse coming from the central point; Δ_1 are the dispersion differences between the pulses from the i-th and central points of the spot;

$$A_i=(\cos\Psi_i)^{-1}=(1+x_i^2/h^2)^{1/2}, \quad i=1,2. \qquad (10)$$

From the known h and φ_0 (angular coordinate of the central point) and the distance s between the receiver and source, one can determine the angular coordinate of the source φ. For $s \gg h$ one obtains

$$\varphi=\varphi_0-h\sin\varphi_0/s. \qquad (11)$$

Using the calculated value D_{s0} and measured spectrum of the envelope $E_{10}(\omega)$ of a signal leaving the cloud layer, one can reconstruct (within a limited bandwidth $\omega < (\alpha^2\tau_s)^{-3/2}(H/c)^{-1}$, where H is the geometric thickness of the layer[2]) the spectrum of the input signal envelope:

$$E_0(\omega)=kE_{10}(\omega)\exp(D_{s0}\omega^2/2) \qquad (12)$$

One thereby determines at the same time the shape of the input pulse. Constant k in Eq. (12) depends on τ and Λ.

Representation of the scattering phase function $P(\gamma)$ by a Gaussian function is not always a good approximation[4,5]. In a more general case, the quantity D_s should be given by

$$D_s=a\alpha^4t_L^2(\tau_s+b\tau_s^2+e\tau_s^4), \qquad (13)$$

where the coefficients b and e can differ from 1/4 and zero respectively, in contrast to Eq. (6). In order to determine experimentally the value D_s in this case, one should measure the signals from more than three points on the luminous spot. Of course, this complicates the measurement procedure. For a polychromatic source of radiation, one can simplify the measurements by recording signals from one point but in different spectral regions. In doing so one takes account of the fact that the quantity α^4 is proportional to the fourth power of the radiation wavelength[6] λ.

When the instrumental resolution is high enough, and signal delays from different points on the observed surface can be neglected compared to the effects of multiple scattering, the main contribution to the uncertainty in D_s and τ_s using (9) comes from the inherent uncertainty in the small-angle approach. For moderate τ values, this uncertainty is given by[2]

$$\sigma_D = 150\alpha^2\tau_s. \qquad (14)$$

This value is close to $1.5\tau_s$% for $d/\lambda = 3$. It is obvious that in real experiments the contribution of measurement errors to the total uncertainty can also be large. These errors enter the expressions for D_s and τ_s in the form of differences multiplied by weighting coefficients whose values depend on the distance between the points selected for measurement. The contribution of these errors to the overall uncertainty in D and τ_s becomes small and comparable to the measurement error Δ_1 when the distance x_1 between points is comparable to h.

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