## EXTINCTION COEFFICIENT MEASUREMENT ERRORS IN A FINITE BANDWIDTH LIDAR CHANNEL

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Lidar facilities facilitate measurements of various atmospheric parameters, including such an important optical parameter as the extinction coefficient. Moreover, lidars have not just become purely scientific research instruments; they are also used more and more frequently routine observations in many industrial applications. One such application is to meteorological measurements in aviation, where measurements of the mean extinction coefficient over a slant path can provide information on visibility in the vicinity of an airport from aboard a plane. The use of lidars in such important applications assumes necessary metrological tests of lidars. In this regard, a thorough analysis of all measurement error components is necessary, and can be considered a first step to a full analysis of measurements error.

This paper is devoted to an analysis of systematic errors in atmospheric extinction coefficient measurements made with a band-limited photodetector system.

The analysis has been carried out for a homogeneous model atmosphere. Making the usual assumptions<sup>1</sup>, the lidar equation can be written as

$$P(z) = G(z) \exp(-2\mu z)/z^2$$
, (1)

where  $\mu$  is the extinction coefficient, *z* is the range and G(z) is a geometric function of the lidar.

In calculations we used the following model of G(z):

$$G(z) = \begin{cases} 0 & , z < z_{1} \\ 1/2 & (1 + \cos \left(\frac{z_{k} - z}{z_{k} - z_{1}} \pi\right), z_{1} < z < z_{k} \\ 1 & , z > z_{k} \end{cases}$$
(2)

where  $z_1$  is the distance at which the sounding beam first comes into the lidar field of view,  $z_k$  is the distance at which sounding beam has been fully intercepted by the lidar field of view.

This model of the geometric function is close to that of the real lidar facility used in our investigations.

The analysis of errors introduced by the photodetector used the signal described by Eq. (1). Such a treatment measurement errors is valid if no correction of lidar returns for  $r^2$  dependence is performed either in the PMT or in the electronics.

In the Elektronika- $03^2$  lidar unit, where such a correction is made in the PMT, the error analysis used a signal of the form

$$S(z) = G(z) \exp(-2 \mu Z)$$
 (3)

The effect of hand-limiting of the detection channel on the shape of lidar returns was investigated using fast Fourier transforms  $(FFT)^{3,6}$ .

The simplest FFT algorithm requires that the signal be sampled at  $2^{N}$  points uniformly spaced in time. Taking into account the capabilities of the computer used, we took N = 11. The sampling time for signals of the form (1) was 10 ns ( $\Delta z = 1.5$  m) and it was 20 ns ( $\Delta z = 3$  m) for signals of the form (3), giving a Nyquist frequency of 25 MHz for signal (3) and 50 MHz for signal (1).

The discrete spectrum obtained via the FFT was then multiplied by the frequency response of the signal-processing electronics. We then took the inverse Fourier transform, using the same FFT algorithm. As a result, a distorted signal was obtained.



*Fig.* 1. *Experimental*(1), *Lorentzian* (2), *and Gaussian*(3) *amplitude*(*a*) *and phase*(*b*) *response.* 

The frequency response of the Elektronika-03 was determined experimentally by measuring transmission

coefficients for sinusoids at different frequencies. The amplitude of these signals was 2U peak-to-peak with constant component U, since the lidar electronics were designed to transmitting unipolar signals. In these measurements, we therefore obtained both the amplitude response and phase response (see Fig. 1).

The frequency response of the electronic circuitry is described as follows

$$K(if) = A(f) \cos \Phi(f) + i A(f) \sin \Phi(f)$$

It should be noted that measurements of frequency response become difficult at frequencies above 10 to 12 MHz due to nonlinear distortions of the input sinusoidal signal, and above 15 MHz they are altogether impossible. Since in our model we did not take account of nonlinear distortions, we assumed the following form of the function A(f)

$$A(f) \mid_{f \ge 15 \text{MHz}} \equiv 0$$

Subsequent calculations used this function together with the Lorentzian ones described by the function

$$K(f)) = f / (f + if),$$

where  $f_0$  is the half-power point (Fig. 1). This model was preferred, ascompared with the Gaussian model in Ref. 7

$$K(if) = exp(-ln \sqrt{2} f^2/f_0^2),$$

because the Gaussian phase response is identically zero, in contrast to the response of the lidar used. It is of particular importance to account for the phase response in studying signals like (1), because after being distorted in the electronic circuit it is corrected for  $r^2$  time lag of an uncorrected noncorrected signal (for a linear phase response) results in distortions of the corrected signal shape.

In the same way, we studied the amplitude response of a constant-gain photodetector. It was found that reducing the load resistance increases the bandwidth considerably. The response is well approximated by a Lorentzian profile.

Lidar return signals, which were distorted by the band-limited electronics, were used to calculate the errors in the extinction coefficient measurements. The actual value of the extinction coefficient was calculated from the undistorted signal, due to the fact that we investigated only one type of uncertainty. The errors due to the geometrical function, for example, were therefore not taken into account.

The analysis was mainly concerened with the Elektronika-03 lidar, and was intended to assess its capabilities in measurements of the extinction coefficient, which ranged from 6 to 15 km<sup>-1</sup>, corresponding to visual ranges from 0.5 to 0.2 km, respectively. The range of these values is most important for aviation meteorology, being at the same time below the lowest possible visual ranges for this lidar. The average extinction coefficients were calculated using sig-

nal-averaging with this same lidar. The measurement baseline lay between  $z_0$  and  $z_1$ , the distance at which a 10-fold signal reduction takes place compared with its value at  $z_0$ .

Figure 2 illustrates the distortion of the signal due to band-limiting of the electronics. It is seen from this figure that there is less distortion of S(z) than P(z) because the former has a lower dynamic range.



Fig. 2. Backscattered signals. Curve 1 is the undistorted signal, curve 2 is the signal distorted by a frequency response with  $f_0 = 8$  MHz, curve 3 the same as curve 2 but for  $f_0 = 2$  MHz. a) is for signal S(z), b) is for distorted signal P(z) corrected for  $r^2$  dependence.

It is characteristic of S(z) distortions that they prolong the leading edge of the signal S(z), and reduce its peak value, extend the tail, which results in underestimating the extinction coefficient.

An analysis of this error dependence on  $z_0$  (Fig. 3) shows that the errors are always negative, their absolute value strongly dependent on the bandwidth of the amplitude response. These errors become quite acceptable at  $f_0 = 4$  MHz, even for the most rapidly varying signals at  $\mu = 15$  km<sup>-1</sup>. Similar results have been obtained for lower values of  $\mu$ showing lower levels of errors. A decrease in  $z_k$  shifts the most distorted part of the signal to the leading edge and significantly reduces the measurement errors at fixed  $z_0$ .

The results obtained are of entirely practical use since in a series of field tests of the Electronika-03 lidar with parameters close to those used to calculate the data presented in Figs. 2 and 3b, we obtained agreement to within 10 % between the lidar measurements and reference data from an RDV-2 transmissometer, althouth the uncertainties considered above were believed to be most important.

Consider now the case when the signal correction for  $r^2$  dependence is performed on the band-limited signal P(z), Fig. 2b. In that event relatively small distortions are strongly amplified upon multiplication by  $z^2$ . As a result, the signal peak is significantly displaced, the area below the signal curve increases, and



Fig. 3. The dependence of relative error  $\delta\mu$  of the extinction coefficient on  $z_0$  for S(z) processing at  $\mu = 15 \text{ km}^{-1}$ . The width  $f_0$  of the Lorentzian frequency response used is 2 MHz, 4 MHz, and 8 MHz for a, b and c respectively. Curves 1 are calculated for  $z_k = 25 \text{ m}$ , while curves 2 are for z = 100 m;  $z_1 = 0$  in all cases.

The errors calculated for this case are presented in Fig. 4. This figure shows that in comparison with the results of Fig. 3, there is a significant (3 to 10 times) increase in the errors, which also become positive, given the same model parameters. The error level strongly depends on the parameter  $z_k$ . An increase in  $z_k$  leads to smaller errors, due to the smaller distortion of the signal with longer and slower fall off. This is important when the correction for r dependence is made.

## CONCLUSIONS

1. Systematic errors in lidar measurements of the mean extinction coefficient due to bandlimited the electronics can be reduced by correcting for  $r^2$  dependence in the photodetector. The error also decreases as  $z_0$  increases.

most importantly, the rate of signal fall off after the peak increases, leading to an overestimate of the extinction coefficient.



Fig. 4. The error  $\delta\mu$  of the extinction coefficient measurements as a function of  $z_0$  calculated by processing signal P(z). Widths  $f_0$  of the Lorentzian frequency response used is 2 MHz, 4 MHz, 16 MHz for a, b and c respectively;  $z_1 = 0$ . Curves 1 are for  $\mu = 3 \text{ km}^{-1}, \quad z_k = 25 \text{ m},$  $\mu = 6 \text{ km}^{-1}, \quad z_k = 25 \text{ m},$  $\mu = 15 \text{ km}^{-1}, \quad z_k = 25 \text{ m},$  $\mu = 15 \text{ km}^{-1}, \quad z_k = 100 \text{ m}.$ curves 2 for are curves3 arefor curves 4 are for

2. In the Electronika-03 lidar, the 4 MHz bandwidth of the electronics is quite sufficient to measure a meteorological visual range down to 0.2 km.

3. In the case of no correction for  $r^2$  in the photodetector or in the electronics, the bandwidth of the amplitude response must be no narrower than 8 to 16 MHz, depending on  $z_k$  and  $z_0$ .

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