EFFICIENCY OF DIRECT AND HETERODYNE DETECTION OF THE OPTICAL PULSES PASSED THROUGH A TURBULENT CHANNEL

A.V. Hovhannisyan

Research Institute of Physics, ArmSSR Academy of Sciences, Ashtarak

For systems of direct and heterodyne detection the influence of atmospheric channel noise on the probability density of the photodetector output current is discussed.

Thermal noise as well as shot noise are taken into account. The efficiency of relevant optimal detectors is investigated as a function of the detector amplification and the signal-to-noise ratio.

In this paper the detection of binary optical pulses passed through the purely turbulent atmospheric channe1 characteristic of the open optical communication systems is considered. Binary systems Eire very simple to analyze, which is why they have been quite widely investigated[1–8]. At the same time, the derivation of explicit expressions describing the signal statistics at the photodetector output under rather general assumptions which are admissible for real systems enables one to consider problems of synthesis and analysis of receiving – transmitting systems operating with M-ital signals. These problems are of immediate practical interest, in addition to which the opportunity arises of considering more efficient systems of spaced reception[3]. Thus the solution of problems of synthesizing optimal systems for detecting optical pulses strongly depends, first of all, on the possibility of determining an explicit expression for the probability density of fluctuations of the detector output current, in other words, for the photocounts statistics.

In practice, when relatively strong optical pulses have to be recorded there can occur the situation in which the probability of more than one photon arriving at the photodetector during the resolution time of a recording system is too high. In this case it is advisable to record the pulses in analog form. Questions of efficiency and optimization in the case of direct detection systems whose operation is limited by thermal noise are analyzed in Refs. 3, 7, and 8. However, the problem of synthesis and analysis of an optimal detector with a high coefficient of internal amplification G when shot noise caused by the signal is of the same order of magnitude as the thermal shot noise or even higher has not yet been solved. Meanwhile the limited possibilities of increasing the optical transmitter power make it necessary to use photodetectors with high values of G in order to provide for proper operation of the systems for average-length and longer atmospheric paths. In the case of heterodyne detection, optimal signal processing has been investigated for the regime in which the detector operation is limited by shot noise of the local oscillator4–6. Such an analysis did not take into account the shot noise due to the interference signal as well as the thermal noise, which in fact can strongly effect the statistics of the photodetector output signal at small values of G and strong fluctuations of the optical signal. As a consequence, assesments of the efficiency of optical systems under such circumstances can not be quite correct.

Below we consider systems of direct and heterodyne detection whose operation is based on the use of the likelihood ratio and which provide for minimum possible probability of false response at known probabilities of transmitting an optical pulse P or pause $1 - P$. A conclusion on the presence (hypothesis $H1$) or absence (hypothesis $H0$) of input optical signal is drawn based on the analysis of the following relationship[3]

$$\frac{P_{s+n}(i)}{P_n(i)} \underset{H_0}{\overset{H_1}{\lessgtr}} \frac{1-P}{P} \qquad (1)$$

where $P_{s+n}(i)$ and $P_{s+n}(i)$ are the probability densities of the detector's output current fluctuations for the cases of presence and absence of the input optical signal, respectively. From the standpoint of practical applications the systems in which the duration of signal pulse τ is much shorter than the coherence time τ_s of the optical signal intensity fluctuations in a turbulent atmosphere are most useful. The expressions for $P_{s+n}(i)$ in this case can be written as follows

$$P_{s+n}(i) = \int_0^\infty \frac{\exp\left[-\frac{(i-zA_s-\bar{i}_{AD})^2}{2(\alpha(zA_s+\bar{i}_{AD})+\sigma_T^2} - \frac{(\ln z + DC)^2}{2\ D}\right]}{2\pi z(D(zA_s+\bar{i}_{AD})+\sigma_T^2))^{1/2}}\, dz \qquad (2)$$

where σ_T^2 is the variance of thermal noise, z is the power of an optical signal at the photodetector input normalized by its mean value, $\alpha = e\Delta FG$ (here e is the electron charge, F is bandpass of the postdetector circuitry). The constants A_s, i_{AD}, D, and C entering expression (2) are determined as follows:

direct reception: $A_s = \bar{i}_s$; $\bar{i}_{AD} = \bar{i}_n$; $D = \sigma_s^2$; $C = 0.5$

heterodyne reception $A_s = 2\sqrt{\bar{i}_s \bar{i}_{10}}\cos(\theta_{10})$;

$$\bar{I}_{AD} = \bar{I}_n + \bar{I}_{10}; \quad D = \sigma_s^2/4; \quad C=1,$$

where σ_s^2 is the variance of In z ; i_s and i_{10}, are the average currents caused by the signal pulse, by the local oscillator, and by the additive noises (background radiation, dark current of the photodetector), θ_{10} is the phase difference of the local oscillator and signal fields in the reception plane.

In the case of heterodyne detection it is assumed that the optical frequencies of the local oscillator and informative signal coinside and the power of the local oscillator is much higher than that of the informative signal. Hence one can neglect the i_s value compared to that of i_{10}. It is supposed also that the coherence time τ_{bgr} of filtered background radiation is much shorter than τ. As the real filters have $\tau_{bgr} \sim 10^{-12}$ sec the condition $\tau > \tau_{bgr}$ practically always fulfilled. Besides, Eq. (2) is valid for heterodyne detection only if one can neglect the signal field phase fluctuations in the reception plane compared with amplitude fluctuations. According to Ref. 4 in the case of adaptive systems with small receiver apertures d, when $d/p_c \le 0.5$, (p_c is the coherency radius of the received optical field), the amplitude fluctuations of the optical signal dominate over the phase fluctuations so the latter can be neglected when forming the statistics of the photodetector signal current.

At the same time when the receiver aperture decreases, the curves of the probability density of the signal current fluctuations for an adaptive system which eliminates (in the linear approximation) the phase fluctuations tends to the analogous curves for nonadaptive (static) systems which take these fluctuations into account. This tendency shows that both systems are identical when $d/p_c \le 0^4$. This enables one to neglect the phase fluctuations in many applications if $d \ll p_c$. It should be noted that the good agreement which is obtained between the theoretical[4] and the experimental results[5] demonstrates the correctness of the initial assumptions[4] and of the conclusions drawn here. Thus when investigating the statistics of fluctuations of the photodetector signal current for adaptive systems which eliminate the phase fluctuations (in the linear approximation) of the optical signal received for $d/P_c < 0.5$ and also for static optical systems with $d \gg p_c$, one can practically neglect the spatiotemporal phase fluctuations compared to amplitude fluctuations.

In Refs. 9–11 one can find expressions for p_c which have been confirmed experimentally, on the basis of which one can estimate the diameter of a receiving aperture. In particular, for the vertical paths from a satellite to Earth the values of p_c are about 3.2 cm and 86 cm for the wavelength 0.69 mm and 10.6 mm, respectively, for levels of turbulence within realistic limits[12].

Thus it is entirely realistic that the condition $d \ll p_c$ for the static receiving systems with $\lambda > 10$ μm can be achieved.

Together with this, the decrease of the total received energy caused by the requirement $d \ll p_c$ can be compensated for by the use of spatially separated reception with a matrix receiver each element of which intercepts the optical field within a small area of radius $d \ll p_c$.

For the short-wavelength optical range one can apply the results obtained here to the systems with adaptation. Because of the great importance of adaptive systems much attention has been given to them in the literature (see, e.g. Refs. 11–14). And even if it is not feasible to eliminate the phase distortions completely in real systems, nonetheless the results obtained for experimental adaptive systems (in which an equivalent increase of the signal level up to 8 dB or the noise suppression up to 35 dB was obtained) demonstrate the high efficiency and the good prospects of such systems[13,14]. It can be hoped that real systems would be constructed in which the phase fluctuations make only insignificant contrbution to the formation of the photodetector signal current statistics compared with that of the amplitude fluctuations.

All the limitations of the heterodyne detection except for the condition $\tau \gg \tau_{bgr}$ are of no significance for the case of direct detection. By using the method of steepest descent for formula (2) one can obtain a final expression for $P_{s+n}(i)$ in the form

$$P_{s+n}(i) = \frac{\exp\left[-\gamma^2/2\beta \;(\ln z_0 + DC)^2/2D\right]}{\sqrt{2\pi}\; B} \qquad (3)$$

where

$$B = \left[\left(\frac{\gamma\alpha z_0 A_s}{\beta}\right)^2 D + \frac{\alpha z_0 A_s D(4\gamma z_0 A_s - \gamma^2 - \alpha z_0 A_s)}{2\beta} - \right.$$

$$\left. - D z_0 A_s(\gamma - z_0 A_s - \alpha/2) + \beta\right],$$

$$\gamma = i - z_0 A_s - \bar{I}_{AD}, \beta = \alpha(z_0 A_s + \bar{I}_{AD}) + \sigma_T^2,$$

and z_0 for i is defined by the following equation

$$\frac{\gamma z_0 A_s}{\beta} + \frac{\alpha z_0 A_s(\gamma^2 - \beta)}{2\beta^2} - \frac{\ln z_0 + DC}{D} = 0 \qquad (4)$$

In this case the expression for $P_n(i)$ can be represented in the form[3]

$$P_n(i) = \frac{1}{\left[2\pi(\alpha\bar{I}_{AD} + \sigma_T^2)\right]^{1/2}} \exp\left[-\frac{(i - \bar{I}_{AD})^2}{2(\alpha\bar{I}_{AD} + \sigma_T^2)}\right] \qquad (5)$$

Thus expressions (1), (3)–(5) define the logic of a signal processing with optimal receivers for both direct and heterodyne signal detection. According to expression (1) the optimal threshold current i_0 can be found from the equation

$$\frac{P_{s+n}(i_0)}{P_n(i_0)} = \frac{1-P}{P} \tag{6}$$

By substituting the expressions for $P_{s+n}(i)$ and $P_n(i)$ into expression (6) when $i = i_0$, one obtain the following system of equations for determining i_0:

$$-\gamma_0^2/2\beta-(\ln z_0+DC)^2/2D+\frac{(i_0-\bar{i}_{AD})^2}{2(\alpha\bar{i}_{AD}+\sigma_T)^2}+\frac{1}{2}\ln(\alpha\bar{i}_{AD}+\sigma_T^2)-$$

$$-\frac{1}{2}\ln\left[\left[\frac{\gamma_0\alpha z_0A_s}{\beta}\right]^2-D+\frac{\alpha Dz_0A_s\left[4\gamma_0z_0A_s-\gamma_0^2-\alpha z_0A_s\right]}{2\beta}-\right.$$

$$\left.-Dz_0A_s(\gamma_0-z_0A_s-\alpha/2)+\beta\right]-\ln\frac{1-P}{P}=0$$

$$\frac{\gamma_0z_0A_s}{\beta}+\frac{\alpha z_0A_0(\gamma_0^2-\beta)}{2\beta^2}-\frac{\ln_0+DC}{D}=0 \tag{7}$$

where $\gamma_0 = i_0 - z_0A_s - i_{AD}$. Since the left-hand sides of Eqs. (7) change their signs when approaching the boundaries (–∞, ∞) for i_0 and (0, ∞) for z_0, solutions i_0, z_0 always exist. Solution of this system of equations can be carried out on a computer using the secant method[15]. For known i one can make the choise of hypotheses H_1 or H_0 according to the rule:

$$i \underset{H_0}{\overset{H_1}{\gtrless}} i_0 \tag{8}$$

For systems without spacing when an optical signal is receiving by a single photodetector, predetermination of i_0 and the use of Eq. (8) can on the whole improve the throughput of a receiving system. However, this advantage is lost for spatially separated reception[3]. Moreover, the solution of this system of equations in the general case takes much more time than the solution of the single equation (4) does. This in turn requires juxtaposition of the time necessary for obtaining i_0 and the characteristic times of optical channel stationarity and of the processes which result in slow changes of the parameters $\bar{i}_s$, $\bar{i}_h$, $\bar{i}_n$, and σ_T^2. (In particular the time it takes to obtain only one i_0 value with the computer "Electronika D3-28" is equal approximately to 40-50 minutes if the left-hand sides of Eqs. (6) differ from zero by an error $|\varepsilon| \le 10^{-9}$.) Thus the problem of the choise of the signal processing structure based on expressions (1) or (2) depends on the specific conditions under which the system operates and on the requirements imposed on the system.

The probability of false response P_{fr}, which represents the efficiency of the receiving system is given, in the form:

$$P_{fr} = P\int_{-\infty}^{i_0} P_{s+n}(i)\,di + (1-P)\int_{-\infty}^{i_0} P_n(i)\,di \tag{9}$$

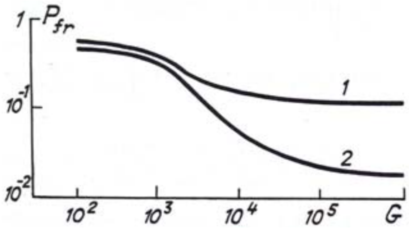

FIG. 1. Curves of the dependence of the probability of false response P_{fr} on the internal amplification coefficient G of a photodetector when p = 0.5. $\Delta F = 10^8$ Hz, $\sigma_T^2 = 0.8$ μA^2, $\bar{i}_S/G = 10^{-3}$ μA, $\bar{i}_N/G = 10^{-4}$ μA.

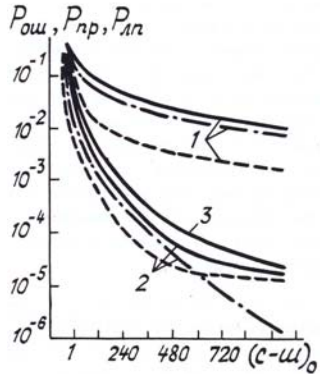

FIG. 2. Curves of the dependence of the probability of false response P_{fr} on the signal-to-noise ratio. Curve 1 — $\sigma_s^2 = 2.25$, $\bar{i}_m = 10$ μA; curve 2 — $\sigma_s^2 = 0.5$; $\bar{i}_N = 10$ μA; curve 3 — $\sigma_s^2 = 0.5$; $\bar{i}_N$ 0 μA; $G = 10^4$.

Figure 1 presents the probability of false response as the function of the internal amplification coefficient of a photodetector. One can see from Fig. 1 how the detection efficiency changes when passing from the conditions of limitation by thermal noise ($G < 10^3$) to the conditions of limitation by shot noises ($G > 10^5$). Comparison of curve 1 ($\sigma_s^2 = 2.25$) with curve 2 ($\sigma_s^2 = 0.5$) shows that the influence of atmospheric turbulence which determines the value of σ_s^2) on the detection efficiency increases with increase of the parameter G. It is also seen from Fig. 1 that further increase of G (in our case, beyond $G \simeq 10^6$) does not improve the detection efficiency, therefore it is not reasonble to make it greater than some critical value. The curves illustrating the dependence of the false response probability on the signal-to-noise ratio $(s/n)_0 = \bar{i}_s^2/)\alpha(\bar{i}_c + \bar{i}_n) + \sigma_T^2$ corresponding to an optical signal with no intensity fluctuations are presented in Fig. 2. Here solid lines represent the values of the probability of false response P_{fr}, dashed-dotted lines represent the probability of missing the signal P_m and dotted curves represent the probability of false detection P_{fd}. It is also seen from this figure that the contributions of P_m and P_{fd} to the probability P_{fr} are different in moderate and strong turbulence.

The dependences of P_{fr} and P_m on σ_s^2 and on (s/n_0) obtained here must be taken into account when selecting the value of P. Comparison of curves 2 and 3 for P_{fr} shows that, in contrast to the receiving systems where the shot noise is negligible and P_{fr} depends on (s/n_0) only[8] in a synthesized system for moderate levels of the optical signal, the parameter P_{fr} depends both on (s/n_0) and on the absolute values of the parameters $\bar{i}_s$, $\bar{i}_n$, and σ_T^2. At the same time, a tendency of the detection efficiency to increase to saturation is observed with growth of (s/n_0) (i.e., saturation of the decrease of P_{fr}). This is illustrated by the behavior of curves 2 and 3, which become closer to each other at large (s/n_0) values. For example, the relative difference between P_{fr} of curve 2 ($P_{fr,2}$) and that of curve ($P_{fr,3}$), defined as the ratio $(P_{fr,3} - P_{fr,2})/P_{fr,3}$, is approximately equal to 0.36 for (s/n_0) equal to 10^3, while for $(s/n_0) = 533$ it is five times greater.

Since expressions (1), (3)–(5) are similar for both direct and heterodyne detection, all the qualitative conclusions drawn above for the case of direct detection systems hold also for heterodyne systems. However it should be noted that there will be certain quantitative differences between heterodyne systems and direct detection systems because of the characteristic peculiarities of heterodyne systems which have an effect on the values of the parameters A_s, $\bar{i}_{AD}$, and D.

In conclusion we would like to not that the results obtained here can allow one to synthesize versatile detectors enabling one to perform optimal signal processing both in cases of direct and heterodyne detection and to carry out their analysis. In this case the limitations imposed on the choise of a photodetector and the conditions of its operation are eliminated for direct detection systems, which was not previously the case. As for the heterodyne systems, if the wavefront fluctuations are eliminated, the additional consideration of the signal shot noise and of the thermal noise makes possible a more accurate description of the signal statistics at the output of the photoreceiver and, therefore, an increase in the reception efficiency. It should be noted that use of the results of Ref. 4 should enable us to consider heterodyne systems (both static ($d \sim p_c$) and adaptive ($d < 0,5p_c$)) if the phase fluctuations are significant. However the structure of systems of optimal processing of the photoreceiver signal would become more complicated. Along with this, from a practical point of view systems whic are relatively simple to use and have a rather high speed of operation are preferable. This makes the approach adopted here for the synthesis of optimal systems of signal processing reasonable for heterodyne detection, the structure of the systems being close to that for direct detection.

The expressions obtained in this paper for the probability density of the current fluctuations at the photodetector output allow one in principle to solve more complicated problems of analysis and synthesis of the optimal systems of spaced and M-ital detection of optical signals with the pulse-code, position and polarization modulations.

REFERENCES

1. A.G. Sheremet'ev and R.G. Tolparev, *Laser Communications* (Svyaz', Moscow, 1974)
2. R.M. Gagliardi and S. Karp, *Optical Communications* (Wiley, New York, 1976)
3. R.A. Kazaryan, A.V. Oganesyan, K.P. Pogosyan, et al., *Optical Communication Systems in the Atmospheric Channel* (Radio i svyaz', Moscow, 1985)
4. J.H. Chunside and C.M. McIntyre Appl. Opt. **17**, 2141 (1978)
5. J.H. Chunside and C.M. McIntyre Appl. Opt. **17**, 2148 (1978)
6. J.H. Chunside and C.M. McIntyre Appl. Opt. **19**, 582 (1980)
7. R.A. Kazaryan and A.V. Oganesyan Izvestiya Akad. Nauk ArSSR, Ser. Fiz., **13**, 491 (1978)
8. R.A. Kazaryan and A.V. Oganesyan Kvant. Electron. **10**, 272 (1983)
9. J.W. Strohbehn (ed.), *Laser Beam Propagation in the Atmosphere. Topics in Applied Physics,* Vol. **25** (Springer Verlag, Berlin, 1978)
10. A.S. Gurvich, A.I. Kon, V.L. Mironov, et al., *Laser Radiation in a Turbulent Atmosphere* (Nauka, Moscow, 1976).
11. V.E. Zuev *Propagation of Laser radiation in the Atmosphere* (Radio i svyaz', Moscow, 1985)
12. N.D. Ustinov, I.N. Matveev, and V.V. Protopopov *Methods of Analysis of Optical Fields in Laser Radar Location* (Nauka, Moscow, 1983)
13. R. Fante, Proc. IEEE **68**, 75 (1980)
14. R.T. Compton, Jr., Proc. IEEE **66**, 23 (1978)
15. A.M. Ostrovsky, *Solution of Equations and Systems of Equations (Resheniye uravnenii i sistem uravnenii)* (Inostrannaya Literature, Moscow, 1963)