INFLUENCE OF SCATTERING ON LONG-WAVE RADIATION TRANSFER THROUGH BROKEN CLOUDS

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Radiation transfer through broken clouds is discussed. The system of equations is derived and solved for the case of long-wave radiation of moderate intensity, taking into account scattering effects. The influence of scattering effects on the brightness temperature of the cloud field-ground system was investigated. It is shown in this paper that at cloud optical depths of more than 15–20 *and viewing angles below* 60°–70° *one can neglect light scattering by clouds and consider the clouds as black emitters.*

Construction of realistic models of optical radiation transfer in the earth's atmosphere is urgently needed to solve the problems of general atmospheric circulation, weather forecasting, laser sensing of the atmosphere, etc. Recently, the problem of providing an adequate description of the radiation characteristics and brightness fields of cloud fields with stochastic geometry has become of currentinterest in radiation transfer theory. From our point of view the correct solution of this problem can be obtained only on the basis of the use of the stochastic radiation transfer equation. Averaging of this equation over the ensemble of cloud fields has made it possible, e.g., in Refs. 1 and 2, to solve for the mean value, variance, and correlation function of short-wave radiation intensity.

The equations for the mean value of the long-wave radiation intensity were derived in Ref. 3. In Ref. 3 the dependence of the brightness temperature on the cloud field parameters and on observation conditions, although the absorption of light by aerosol particles and atmospheric gases was not taken into account. It was also assumed in this reference that scattering of the long-wave radiation could be neglected when deriving the equations. In this connection there arises a question on the limits of applicability of the approximate expressions obtained in Ref. 3.

SOLUTION TECHNIQUE

Let OXYZ be a Cartesian coordinate system. Assume also that the atmosphere is in a state of local thermodynamic equilibrium at temperature $T(z)$, and that it is horizontally homogeneous except for the cloudy layer. Let $\alpha(z)$ be the total absorption coefficient due to both aerosol and gaseous constituents of the atmosphere. Let the underlying surface be a blackbody at the temperature $T_s = T(0)$. The cloud field occupies the layer Λ: $h \le z \le H$. Within this layer we will take into account only the interaction of the radiation with the cloud substance, i.e. we assume $\alpha(z) = 0$ for $z \in \Lambda$. The optical characteristics of clouds are given in the form of random scalar fields of the extinction coefficient $\sigma\kappa(r)$, as well as of the single scattering albedo $\lambda\kappa(r)$ and of the scattering phase function $g(\omega, \omega')\ \kappa(r)$, where $\omega = (a, b, c)$ is unit direction vector. Here $\kappa(r)$ is the random indicator field, which we will model based on the Poisson point processes on the coordinate axes OX and OY[1,4]. Such a cloud field is statistically homogeneous and anisotropic. The shapes of individual clouds are assumed to be parallelepipeds of the same height $\Delta H = H - h$. Size distributions of clouds along both coordinate axes are described by an exponential function. The first two moments of $\kappa(r)$ are expressed in terms of the unconditional ρ and conditional $V(r_1, r_2)$ probabilities of occurence of the cloud.

$$\langle \kappa(r) \rangle = p, \quad \langle \kappa(r_1)\ \kappa(r_2) \rangle = pV(r_1, r_2)$$

$$V(r_1, r_2) = (1-p)\exp\left[-A(\omega)\frac{|z_1 - z_2|}{|c|}\right] + p,$$

$$A(\omega) = A(|a| + |b|),$$

where $A = [1.65(N - 0.5)^2 + 1.04]/D$, $N=p$ is the cloud amount, D is the characteristic (average) cloud size.

Within the' limits of the layer Λ layer the random intensity of monochromatic radiation $I(r, \omega)$ satisfies the stochastic radiation transfer equation.

$$\omega \nabla I(r,\omega) + \sigma\kappa(r) I(r,\omega) = = \lambda\sigma \int_{4\pi} g(\omega,\omega')\ \kappa(r)\ I(r,\omega')\ d\omega' + + (1-\lambda)\ \sigma\ \kappa(r)\ B(z) \quad (1)$$

$$\overset{\uparrow}{I}(h,\omega) = 1/c \int_0^h B(\xi)\ \alpha(\xi)\ \beta(h,\xi,\omega)\ d\xi + + B(0)\ \beta(h,0,\omega), \quad c > 0 \quad (2)$$

$$I^{\downarrow}(H, \omega) = 1/|c| \int_{H}^{H_0} B(\xi)\alpha(\xi)B(H, \xi, \omega)d\xi, \ c < 0$$

where $B(z) = B(T(z))$ is the Planck function, H_0 is the height of the upper boundary of the atmosphere,

$$\beta(z, t, \omega) = \exp\left\{- 1/|c| \int^{z} \alpha(\eta)\, d\eta \right\}, \ z \ge t.$$

In contrast with the equations for the visible and near—IR regions, the source function in Eq. (1) has an additional term, which describes the self-radiation of the clouds. This circumstance does not produce any additional difficulties, and it is possible to use the same ideas and methods for deriving the equations for the mean intensity of long-wave radiation as those used for the case of short-wave radiation. A detailed description of these techniques and ideas can be found elsewhere in the 1 literature . (see, e.g., Ref. 1). Therefore we shall omit the intermediate calculations in our further discussion and give only final results.

Using the expression for correlation decoupling[1,4] one obtains, after averaging expression (1) over the $\kappa(r)$ ensemble, the following complete system of equations

$$\langle I(z,\omega)\rangle = p\,\sigma/|c| \int_{E_z} d\xi \left\{ \lambda \int_{4\pi} g(\omega,\omega')\, U(\xi,\omega')\, d\omega' + \right.$$

$$\left. - U(\xi,\omega) + (1-\lambda)\, B(\xi) \right\} + I_z(\omega) \tag{3}$$

$$U(z,\omega) = \sigma/|c| \int_{E_z} V(z,\xi,\omega)d\xi \left\{ \lambda \int_{4\pi} g(\omega,\omega')\, U(\xi,\omega')\, d\omega' - \right.$$

$$\left. - U(\xi,\omega) + (1-\lambda)\, B(\xi) \right\} + I_z(\omega) \tag{4}$$

$$E_z = \begin{cases} (h, z), & c > 0, \\ (z, H), & c < 0, \end{cases}$$

$$I_z(\omega) = \begin{cases} I^{\uparrow}(h,\omega), & c > 0, \\ I^{\downarrow}(H,\omega), & c < 0, \end{cases}$$

where $pU(z,\omega) = \langle \kappa(r) I(r,\omega) \rangle$, and brackets here denote the ensemble average. Since the boundary conditions are uniform and the model cloud field is assumed to be statistically homogeneous, the functions $\langle I \rangle$ and U are independent of X and Y. Analogous to the short-wave spectral region, the mean intensity of the long-wave radiation is also invariant with respect to the parameters $\tau = \sigma\Delta H$ and $\gamma = \Delta H/D$. A formal solution of Eq. (4), which can be obtained using the Laplace transform, is as follows

$$U(z,\omega) = \lambda/|c| \int_{E_z} d\xi \int_{4\pi} \sum_{i=1} D_i\, \lambda_i \exp\left[-\lambda_i \frac{|z-\xi|}{|c|} \right] \times$$

$$\times\, g(\omega,\omega')\, U(\xi,\omega')\, d\omega' + \Psi(z,\omega)\ ; \tag{5}$$

$$\Psi(z,\omega) = I_z(\omega) v(\tilde{z}) + \frac{(1-\lambda)}{|c|} \int_{E_z} \sum_{i=1}^{2} D_i\, \lambda_i \times$$

$$\times \exp\left[-\lambda_i \frac{|z-\xi|}{|c|} \right] B(\xi)\, d\xi\ ; \tag{6}$$

$$v(\tilde{z}) = \sum_{i=1}^{2} D_i \exp(-\lambda_i \tilde{z})\ ;$$

$$\tilde{z} = \begin{cases} (z-h)/c, & c > 0, \\ (h-z)/c, & c < 0, \end{cases} \tag{7}$$

$$\lambda_{1,2} = \frac{\sigma + A(\omega)}{2} \mp \frac{((\sigma + A(\omega))^2 - 4\, A(\omega)\, p\, \sigma)^{1/2}}{2},$$

$$D_1 = \frac{\lambda_2 - \sigma}{\lambda_2 - \lambda_1}, \qquad D_2 = 1 - D_1. \tag{8}$$

By substituting Eq. (5) into Eq. (3) and changing the order of integration one obtains after integrating over one of the variables

$$\langle I(z,\omega)\rangle = \lambda\sigma p/|c| \int_{E_z} d\xi \int_{4\pi} \sum_{i=1}^{2} D_i \exp\left[-\lambda_i \frac{|z-\xi|}{|c|} \right] \times$$

$$\times\, g(\omega,\omega')\, U(\xi,\omega')\, d\omega' + \varphi(z,\omega), \tag{9}$$

$$\varphi(z,\omega) = I_z(\omega) j(\tilde{z}) + \sigma\, p\, \frac{(1-\lambda)}{|c|} \int_{E_z} \sum_{i=1}^{2} D_i \times$$

$$\times \exp\left[-\lambda_i \frac{|z-\xi|}{|c|} \right] B(\xi)\, d\xi, \tag{10}$$

$$j(\tilde{z}) = \sigma p \sum_{i=1}^{2} D_i/\lambda_i \exp(-\lambda_i \tilde{z}), \tag{11}$$

where $\varphi(z, \omega)$ has the meaning of the average intensity of direct radiation passed through the plane z along the ω direction. Note that in the limiting case of $\lambda \to 1$ the Eqs. (5) and (9) are identical to the corresponding equations obtained for solar radiation[5] in the visible region.

Transforming in Eq. (5) to the corresponding volume integral, we obtain

$$U(x) = \int_{X} k(x',x)\, U(x')\, dx' + \Psi(x), \tag{12}$$

$$k(x',x) = \frac{\lambda g(\omega,\omega') \sum_{i=1}^{2} D_i\, \lambda_i \exp(-\lambda_i |r - r'|}{|r - r'|^2} \times$$

$$\times\, \delta\left[\frac{r - r'}{|r - r'|} - \omega \right], \tag{13}$$

where X is the phase space of coordinates and directions, $x = (r, \omega)$, and $\Psi(x)$ is defined by Eq. (6).

Consider now the Monte-Carlo-based algorithm for estimating the mean intensity $\langle I(z_*, \omega_*)\rangle$ of radiation coming through the plane $z = z_*$ along the direction $\omega = \omega_*$. Since in this problem the receiver is located at asingle point while the radiation source is distributed over the entire space X, we shall use the method of conjugate trajectories[6] to calculate the mean intensity value $\langle I(z_*, \omega_*)\rangle$.

Within the framework of this method the trajectories are modeled starting from the point $r_*(o, o, z_*)$ with the initial direction ω_*. The initial and intermediate probability densities as well as the stochastic supplementary weights are determined by expressions analogous to these derived in Ref. 5. In order to estimate $\langle I(z_*, \omega_*)\rangle$ it is necessary to average the value $\Psi(x_c)$ over all points x_c at which collisions take place (after modeling the new direction). The contribution of radiation arriving at the receiver directly (not scattered by the clouds, the underlying surface, or the atmosphere above and below the clouds) can be calculated using expression (10).

CALCULATIONAL RESULTS

In our calculations we used the optical characteristics corresponding to the cloud C_1 [Ref. 7] at $\lambda = 10$ μm. For simplicity the influence of aerosols and gases was neglected ($\alpha(z) = 0$ at $0 < z < Ho$) and clouds were assumed to be isothermal at the temperature T_c. Under these assumptions one obtains from Eqs. (2) and (6)

$$\Psi(z,\omega) = I_z(\omega)\, v(\tilde{z}) + (1-\lambda)\, B_c[1 - v(\tilde{z})], \tag{14}$$

where $I_z(\omega) = B_s$ at $c > 0$ and $I_z(\omega) = 0$ at $c < 0$, $B_s = B(T_s)$, and $B_c = B(T_c)$. Let us write the function $\langle I(z, \omega)\rangle$ in the form $\langle I(z, \omega)\rangle = \varphi(z, \omega) + i(z, \omega)$, where i is the mean intensity of diffuse radiation. According to Eq. (10) one has

$$\varphi(z,\omega) = I_z(\omega)\, j(\tilde{z}) + (1-\lambda)\, B_c[1 - j(\tilde{z})], \tag{15}$$

and

$$i(z,\omega) = i_s(z,\omega) + i_c(z,\omega)$$

where i_c and i_s are the mean intensities of diffuse radiation from clouds and the underlying surface.

The mean intensity $\langle I_0(z, \omega)\rangle$ of radiation, transfered in the nonscattering clouds ($\lambda = 0$) is given by[3]

$$\langle I_0(z,\omega)\rangle = I_z(\omega)\, j(\tilde{z}) + B_c[1 - j(\tilde{z})] \tag{16}$$

In the case of nonscattering clouds the underlying surface contributes only to the upwelling radiation ($c > 0$), which is entirely accounted for by the function φ. If scattering effects are taken into consideration, the radiation from the underlying surface can make an appreciable contribution to the downwelling scattered radiation ($c < 0$) as well as to the upwelling scattered radiation, which can be one of the main reasons for the difference between $\langle I_0\rangle$ and $\langle I\rangle$. It can be seen from expressions (11) and (16) that under conditions of no temperature inversion $\langle I_0(z, \omega)\rangle$ is a decreasing function of the zenith look angle $\xi = \arccos|c|$ for upwelling radiation and an increasing function of the look angle for the case of downwelling long-wave radiation. One can obtain from Eqs. (15) and (16) that

$$\Delta I(z,\omega) = \langle I_0(z,\omega)\rangle - \langle I(z,\omega)\rangle = \lambda\, B_c[1 - j(\tilde{z})] - i(z,\omega), \tag{17}$$

where the first term describes the decrease of the direct radiation from clouds due to scattering.

Let T_0 and T be the brightness temperatures corresponding to $\langle I_0\rangle$ and $\langle I\rangle$. The value $\Delta T = T_0 - T$ is the error in the determination of the brightness temperature of the system "cloud field – underlying surface", due to ignoring of the cloud scattering properties in the infrared. Let us now investigate the dependence of ΔT on the zenith look angle, and on the geometrical and optical parameters of the clouds for upwelling ($\uparrow$) and downwelling ($\downarrow$) radiation at the cloud field boundaries. The calculations were made for temperatures $T_S = 290$ K and $T_C = 255$ K and height interval $\Delta H = 1$ km. The azimuth angle measured from the plane XOZ was taken to be zero, the other parameters are given in figure captions. It should be noted that at these temperatures the intensities of radiation emitted by the ideally black underlying surface and by a unit volume of clouds differ almost by a factor of 2.

The value ΔT is graphed in Fig. 1 as a function of ξ and N. In the case of downwelling radiation, an increase in N results in an increase of the cloud contribution to the direct radiation recorded by the receiver and an increase in $\lambda B_C[1 - j(\tilde{z})]$. On the other hand, for larger N values the intensity of scattered radiation on average increases, and the cloud-radiation interaction begins to play a significant role at large N values[1]. This, in turn, increases, the portion of radiation emitted by the warmer surface which is reflected by the cloud layer. The contributions due to these factors have opposite signs (expression 17), for which reason the intensity $i^{\downarrow}(h, \omega)$ increases with the increase of N more rapidly than the value $\lambda B_C[1 - j(\tilde{z})]$. The more rapid increase of $i^{\downarrow}(h, \omega)$ takes place first of all as a result of the contribution of the surface radiation reflected back by the cloudy layer. Therefore $\Delta T^{\downarrow}$ becomes smaller at larger N for all viewing angles. As $N \to 0$, and in the case of optically thick clouds also as $N \to 1$ the contribution from the surface radiation scattered by the clouds $i_S^{\uparrow}(h,\ \omega)$ to the total intensity $i^{\uparrow}(h,\ \omega)$ vanishes. This means that $i_S^{\uparrow}(h,\ \omega)$ is a nonmonotonic function of N, thus explaining the more complicated behavior of $\Delta T^{\uparrow}$ as a function of N.

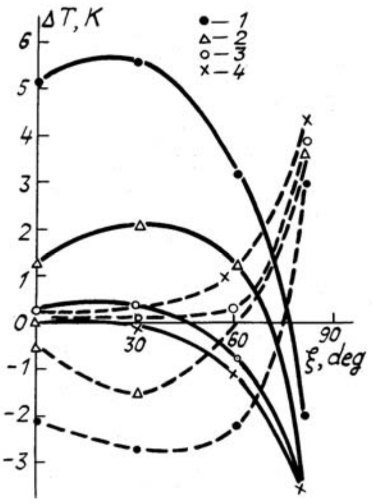

FIG. 1. Results illustrating the effect influence of zenith look angle ξ; and the cloud amount N on the value of ΔT for ψ = 1, τ = 10, and N = 0.1 (curve 1), 0.5 (curve 2), and 0.9 (curve 3). Here and below in Figs. 2 and 3 solid curves represent the results for downwelling radiation (c < 0, z = h), while the dashed curves represent the results for upwelling radiation (c > 0, z = H).

As follows from the initial formulation of the problem and from the above-discussed assumptions, not taking into account the radiation emitted $i^{\downarrow}(h,\ -\omega) = i^{\uparrow}(H,\ \omega)$, and according to (17), $\Delta T^{\downarrow}=\Delta T^{\uparrow}$. In fact, $\Delta T^{\downarrow} < 0$ for directions near the horizon and vice versa for $\Delta T^{\uparrow}$ (see Fig. 1). These sign changes and differences between $\Delta T^{\uparrow}$ and $\Delta T^{\downarrow}$ are due to the effect of surface radiation. Due to the large asymmetry of the scattering phase function of the cloud particles in the forward direction, the radiation of the underlying surface exiting the cloud tops and sides via a process of multiple scattering can strongly increase the intensity of the upwelling scattered radiation. The large role of the cloud sides in forming $i_S^{\uparrow}(H, \omega)$ in the presence of absorption ($\lambda = 0.638$) is also caused by the fact that radiation coming out through them undergoes fewer scattering events than the radiation coming out of clouds through their tops. The value of the mean intensity $i_S^{\uparrow}(H, \omega)$ reaches its maximum at looking angles $\xi \le 50$–60 when the received radiation can come from both cloud tops and sides. At larger ξ and N the intensity of the upwelling radiation $i^{\uparrow}(H,\ \omega)$ rapidly decreases because of the fast growth of the mean optical depth of such a viewing direction, which, as a result, significantly decreases the probability (due to absorption and scattering) of surface radiation coming through the clouds. This explains the angular dependence of $\Delta T^{\uparrow}$. For the case of downwelling radiation, the intensity $i_S^{\downarrow}(H, \omega)$ reaches its maximum in the vicinity of the horizon, which results in a decrease in $\Delta T^{\downarrow}$ at large ξ values.

The influence of variation of horizontal cloud dimensions on the formation of the long-wave radiation field is illustrated by Fig. 2.

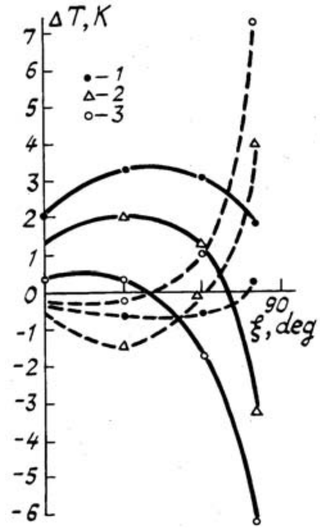

FIG. 2. The quantity ΔT as a function of zenith look angle ξ and the parameter γ = ΔH/D when N = 0.5, τ = 10, γ = 0(curve 1), 1 (curve 2), and 2 (curve 3).

For fixed geometrical cloud thickness the number of clouds forming the cloud field decreases, on the average, with decrease of $\gamma = \Delta H/D$. Therefore the relative role of the cloud sides becomes less important in forming the brightness field of the radiation modulated by broken clouds. For $\gamma = 0$, as in the case with optically thick clouds ($\tau \sim 10$ or more), one can see that $i_S^{\uparrow}(z, \omega) \ll i_c(z,\ \omega)$ and the upwelling diffuse radiation is formed only by the radiation from clouds, while the radiation emitted by the underlying surface is practically entirely! absorbed and reflected back by the cloudy layer. In the asymptotic case $\gamma \to 0$, the parameter $A(\vec{\omega})\sim 1/D$ also vanishes and the conditional probability $V(z, \xi, \vec{\omega}) \to 1$. Then, as follows from expressions (3) and (4),

$$<I(z,\omega) = N\ \tilde{I}(z,\omega\ ) + (1-N)\ I_z(\omega), \qquad (18)$$

where $\tilde{I}(z, \vec{\omega})$ is the intensity of long-wave radiation in a solid cloud cover with extinction coefficient σ. Maximum values of γ for stratiform clouds are[8] 10^{-2} to 10^{-3} and expression (18) well describes the brightness field of stratus clouds partially covering the spatial region of interest.

Neglecting the boundedness of the horizontal dimensions of cumulus clouds ($\gamma \sim 0.5$ to 2.0)[8] when estimating the contribution of scattering to the process of thermal radiation transfer leads to errors in excess of 1 K for the above parameters of the problem (see Fig. 2, curves 1, 2, and 3).

One can see that starting at some value, further increase of the optical depth due to absorption by cloud particles results in a weakening of theinfluence of cloud size on the brightness field since there occurs in such a situation a decrease of the dimensions of the region where the radiation field exiting through the cloud sides is formed. In this particular sense an increase in τ is equivalent to a growth of D, which is confirmed by the calculated results presented in Fig. 3. If 1 K taken as the accuracy criterion, then for viewing directions near the horizon $\xi > 60°–70°$ one must take into account scattering effects at any optical depth of cumulus clouds while at viewing angles $\xi < 60°–70°$ the scattering properties of clouds can be neglected up to τ values of 15 to 20.

In this case, according to Eq. (16), the mean intensity is determined by the function $j(\tilde{z})$, which at such large optical deptho is a linear function of the probability that the viewing direction is occluded by clouds. As a consequence, at $\tau > 15–20$ and $\xi < 60°–70°$ one can neglect the scattering of radiation by clouds and, moreover, it is possible to treat the clouds as blackbody radiators, which essentially simplifies the investigation of the statistics of long-wave atmospheric radiation intensity in the presence of cumulus.

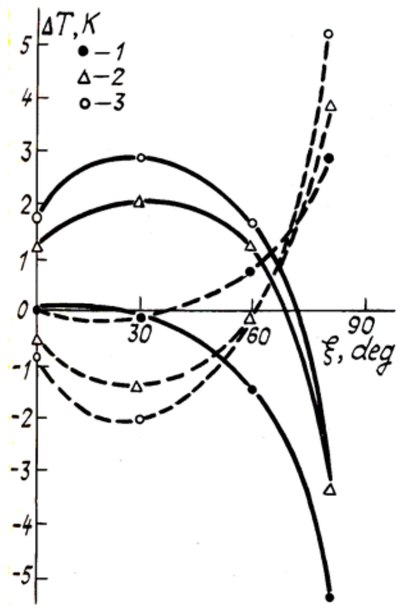

FIG. 3. Results illustrating the dependence of ΔT on zenith look angle ξ and optical depth τ at $N = 0.5$ and $\psi = 1$ for $\tau = 5$ (curve 1), 10 (curve 2), 40 (curve 3, and 100 (curve 4).

In Ref. 9 the cloud field is modeled by a regular three-dimensional array of clouds of parallelepiped shapes with the same dimensions. The radiation transfer equation is solved in this case using the modified two-stream approximation[9,10] determination of the boundary conditions, which must take into account, at least approximately, the multiple scattering of radiation between clouds, utilizes the assumption of the homogeneity and isotropy of radiation exiting the cloud sides. In order to assess the influence of cloud field stochasticity on the mean intensity of upwelling radiation, we have calculated the brightness temperatures for two models of cumulus clouds, the parameters of the problem being the same, on the average, in both cases. The results of these calculations are presented in Fig. 4. It is seen from this figure that at small and large N values the differences between the brightness temperatures are too small, while in the range $0.3 \le N \le 0.7$ they can amount to 5 K. It should be noted that maximum deviations are observed at smaller zenith angles for larger N values.

Thus, in conclusion, it can be stated that the contribution of scattering effects to the formation of the brightness field of broken clouds in the long-wave spectral region has been assessed. Also the limits of applicability of the approximate method for calculating the mean intensity have been estimated. The method treats the clouds as blackbody emitters. The results obtained here can be useful in estimating the distortions due to a cloud field in the problem of remote sensing of the ocean surface temperature from space.

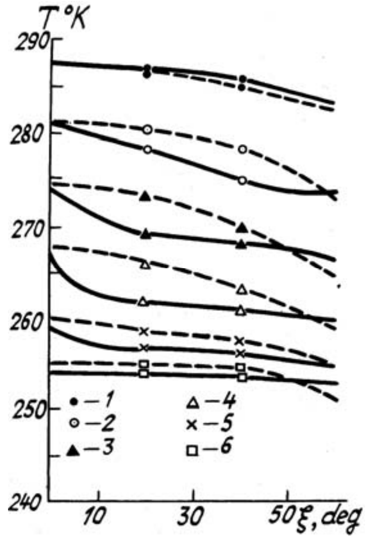

FIG. 4. The brightness temperature of scattering clouds as a function of zenith look angle ξ and cloud amount N, for two models of the cloud field at $\psi = 1$, $\tau = 10$. Solid curves represent results from Ref. 9, dashed curves ---- our calculations for broken clouds. Here $N = 0.1$ (curve 1), 0.3 (curve 2), 0.5 (curve 3), 0.7 (curve 4), 0.9 (curve 5) and 1 (curve 6).

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