

FEASIBILITY OF DETERMINING ATMOSPHERIC PARAMETERS BY MEASUREMENT OF THE ARRIVAL ANGLE OF SOUND WAVES

A.Ya. Bogushevich and N.P. Krasnenko

*Institute of Atmospheric Optics,
Siberian Branch USSR Academy of Sciences, Tomsk
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This paper analyzes in a geometrical acoustics approach some features of techniques for measuring arrival angles of sound waves caused by regular atmospheric inhomogeneity and by motion of the source. An expression is derived which relates the arrival angle to the phase difference of signals measured with an acoustic direction finder. Analytic expressions are obtained which relate the phase difference of the signals to the profiles of temperature and wind velocity. Based on the relations derived, a technique is proposed for sensing the lower atmospheric layer.

It is known¹ that atmospheric refraction makes the arrival angle γ of a sound wave depend on the profiles of temperature T and wind velocity v . This, in turn, allows for the determination of these meteorological parameters using measurements of acoustic wave arrival angles. Both active and passive remote sensing techniques can be used²⁻⁴. In active methods, sodar itself sends a narrow sound packet into the atmosphere where it is partially scattered by atmospheric inhomogeneities. In passive methods, the sodar works only as a receiver of sound signals from external sources, either natural or artificial.

The receiving antenna of the sodar used in such measurements is normally composed of two or more groups of phased microphones with a spacing between their centers d ^{2,4}. Using such a receiving system one measures only the phase difference $\Delta\Phi$ between the signals recorded by each of the groups, while the arrival angle γ is calculated according to the relationship

$$\Delta\Phi = k_0 d \sin\gamma, \quad (1)$$

where $k_0 = \omega/c_0$ is the wave number at the center of the baseline d , c_0 is the speed of sound in air, ω is the angular frequency of the sound oscillations. Here γ is the angle between the normal n to the phase front of the wave at the center of the antenna and the normal to the base line d .

This paper deals with two problems. First of all we should like to estimate the applicability of expression (1) to the case of an inhomogeneous moving medium (as is the case with the atmosphere) and a moving sound source. The second problem concerns the desired derivation of analytical expressions relating the measured phase difference $\Delta\Phi$ to the atmospheric temperature T and wind velocity V both for passive and active sodar techniques. The first of

these problems has never been considered before, while the second one was solved only in the simplest case of active sounding in the vertical direction^{4,5} using a monostatic sodar.

Let a point sound source move with respect to an observer at a constant subsonic speed V . In addition, let the wavelength of sound it produces be small compared to the regular atmospheric inhomogeneities. Assume then that the current phase of sound oscillations is described, in the coordinate system K' comoving with the sound source, by the function $\Phi_r(t')$. The position of the sound source in the coordinate system K , the origin of which coincides with the center of the receiving antenna, and z axis of which is directed vertically, we shall describe by the radius-vector $r(t)$.

Let us make use of the invariance of the phase of the sound wave in the two coordinate systems K and K' ^{6,7} related to each other by a Galilean transformation; this means that

$$\Phi(\rho, t) = \Phi'(\rho', t'), \quad (2)$$

where ρ, t and ρ', t' are the coordinates and time of the same event in coordinate systems K and K' , respectively. Since the wave disturbance produced by sound at the moment t'_i reaches the point ρ' in a finite time $\tau' = t' - t'_i \neq 0$, we have $\Phi(\rho', t') = \Phi'_i(t' - \tau')$. As a consequence one finds in the geometrical acoustics approach that

$$\Phi(\rho, t) = \Phi'_r(t) - k'_0 \Psi[r(t - \tau), \rho], \quad (3)$$

where $K'_0 = \omega'/c_0$, $\Psi[r(t_i), \rho]$ is the eikonal of sound propagating along the beam trajectory $L[r(t_i), \rho]$ from the point $r(t_i)$ to ρ .

One can get from Eq. (3) to the direction finder formula in the case of a homogeneous moving medium,

by assuming that the point ρ is within the receiving aperture ($\rho \leq d/2$), and expanding the right-hand side of (3) as a Taylor series in the small parameter $\rho/r \ll 1$. It is pertinent here to calculate to the accuracy with which the expression (1) was obtained for a homogeneous medium. For this reason we shall neglect terms of third and higher in ρ/r . Since the sound source is moving, the oscillations arriving at the receiving points 0 and ρ are emitted at different points separated by $\eta = r - r_0 = -v(t - \tau_0)$. Here $r_0 = r(t - \tau_0)$ and $r = r(t - \tau)$ (see Fig. 1). According to expression (3) the vector η is given by

$$\eta = v \Delta \Phi(\rho) / (c_0 k'_0), \tag{4}$$

where $\Delta \Phi(\rho) = \Phi(\rho, t) - \Phi(0, t)$.

Since $|\Psi(r_0, 0) - \Psi(r, \rho)| \leq \rho$ and $|v| < c$, $|\eta| < \rho$.

As a consequence, for $v \neq 0$ one has the additional small expansion parameter $\eta/r_0 \ll 1$, which simultaneously depends on ρ and v . Thus, one obtains a double series of the form

$$\begin{aligned} &(\Phi(\vec{\rho}, t) = \Phi(\vec{\rho}, r(t - \tau), t)): \\ &\Phi(\vec{\rho}, t) = \Phi(0, r_0) + (\vec{\rho} \nabla_\rho) \Phi(0, r_0) + (\eta \nabla_r) \Phi(0, r_0) \\ &+ 1/2 (\vec{\rho} \nabla_\rho)^2 \Phi(0, r_0) + 1/2 (\eta \nabla_r)^2 \Phi(0, r_0) \\ &+ (\eta \nabla_r) [(\vec{\rho} \nabla_\rho) \Phi(0, r_0)] + \dots, \end{aligned} \tag{5}$$

where ∇_ρ and ∇_r are the operators of differentiation with respect to ρ and $r = r_0 + \eta$, respectively.

Taking into account the fact that in angular measurements it is not the absolute value of the wave phase that is important but its variations within the receiving area, one can write down an equation for $\Delta \Phi(\rho)$ based on expressions (5) and (3). Using the eikonal equation¹ for a sound wave propagating through an inhomogeneous moving medium and Eq. (3) one can write

$$(\rho \nabla_\rho) \Phi(0, r_0) = -k'_0 \rho n_0(0) \mu(0), \tag{6}$$

where $\mu_0(\mathbf{r}) = c_0 / [c(\mathbf{r}) n_0(\mathbf{r})]$ is the atmospheric refractive index for sound waves. The index '0' denoting the wave parameters is used here to show that the wave propagates along the sound ray $L = L_0(r_0, 0)$ (see Fig. 1).

Analogously, one has

$$(\eta \nabla_r) \Phi(0, r_0) = k'_0 \eta n_0(r_0) \mu_0(r_0), \tag{7}$$

Expression (7) has the opposite sign right-hand side as compared to (6) because when $(\eta \cdot \nabla_r) \Psi(r_0, 0) > 0$ the scalar product $\eta \cdot \eta(r_0)$ is negative.

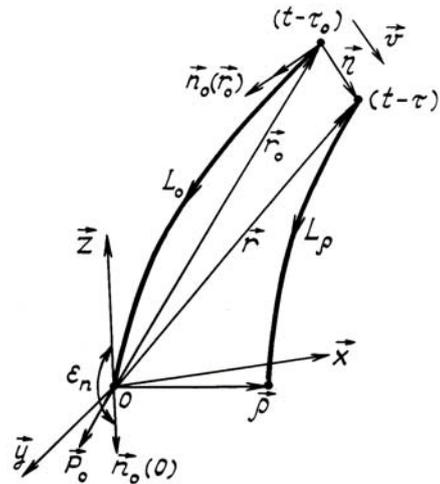


FIG. 1

In the atmospheric ground layer $|\mu - 1| \ll 1$; therefore, allowance for refraction of sound in the atmosphere can result only in small relative corrections⁸. Owing to this fact, the influence of sound refraction on the remaining second- and higher-order terms in the series (5) can be neglected. Thus the radius of phase front curvature of the wave at the point 0 can be assumed to be equal to r_0 , and using (6) and (7) one has

$$\begin{aligned} &(\rho \nabla_\rho)^2 \Phi(0, r_0) \approx -k'_0 \rho_{0\perp}^2 / r_0, \\ &(\eta \nabla_r)^2 \Phi(0, r_0) \approx -k'_0 \eta_{0\perp}^2 / r_0, \\ &(\eta \nabla_r)^2 [(\rho \nabla_\rho)^2 \Phi(0, r_0)] \approx k'_0 \eta_{0\perp}^2 \rho_{0\perp}^2 / r_0, \end{aligned} \tag{8}$$

where

$\rho_{0\perp}^2 = \mathbf{n}_0(0) \times [\rho \times \mathbf{n}_0(0)]$ and $\eta_{0\perp}^2 = \mathbf{n}_0(r_0) \times [\eta \times \mathbf{n}_0(r_0)]$ are the transverse components of ρ and η relative to the normal $n_0(r)$ at the point of origin of these vectors. Substituting (5) into (6) and (8) and taking (4) into account, one can obtain for $\Delta \Phi(\rho)$ the ordinary quadratic equation $A \Delta \Phi^2(\rho) + B \Delta \Phi(\rho) + C = 0$ with coefficients

$$\begin{aligned} &A(\mathbf{v}) = [v_{0\perp} / U_0(r_0)]^2 / (k'_0 r_0), \\ &B(\rho, \mathbf{v}) = -k'_0 [v_{0n} / U_0(r_0) - v_{0\perp} \rho_{0\perp}] / [r_0 U_0(r_0)], \\ &C(\rho) = k'_0 [\rho_{0n} \mu_0(0) + \rho_{0\perp}^2 / (2r_0)], \end{aligned}$$

where $U_0(r_0) = c(r) + \mathbf{v}(r) \cdot \mathbf{n}_0(r)$ is the phase speed of sound, $v_{0\perp} = \mathbf{n}_0(r_0) \times [\mathbf{v} \times \mathbf{n}_0(r_0)]$, $\rho_{0n} = \rho n_0(0)$, and $v_{0n} = \mathbf{v} n_0(r_0)$. The physically meaningful root of this equation gives the following solution to the problem:

$$\Delta\Phi(\rho) \approx -\frac{\omega(\rho)}{c_0} \left\{ \rho_{0n} \mu_0(0) + \frac{\rho_{01}^2}{2r_0} + \frac{\rho_{0n}^2}{2r_0} \left[\frac{v_{01}}{B(\rho, \mathbf{v}) u_0(\mathbf{r}_0)} \right]^2 \right\}, \tag{9}$$

where $\omega(\rho) = \omega'/B(\rho, \mathbf{v})$ is the angular frequency of sound oscillations received at the point ρ , with the Doppler effect taken into account.

Note that if $v \ll c$, then the third term in the expression for $B(\rho, \mathbf{v})$ can be neglected, which means that the difference in the Doppler frequency shift of sound received at different points of the receiving antenna does not affect the value of $\Delta\Phi(\rho)$. However, in the more general case of $v < c$, these differences in the Doppler shift can be quite large, broadening the received signal spectrum by $\delta\omega = \omega(0) |\mathbf{v} \cdot \mathbf{n}_0(\mathbf{r}_0)| / d / (2r_0)$ as compared with the initial spectrum emitted by the source. Using Eq. (9) for the definition of the phase difference $\Delta\Phi = \Delta\Phi(\rho_1 = \mathbf{d}/2) - \Delta\Phi(\rho_2 = -\mathbf{d}/2)$ one has

$$\Delta\Phi = k' \mu_0(0) \mathbf{d} \cdot \mathbf{n}_0(0) / [1 - \mathbf{v} \cdot \mathbf{n}_0(\mathbf{r}_0) / u_0(\mathbf{r}_0)] \tag{10}$$

In contrast to Eq. (1), this expression contains a Doppler correction to $\Delta\Phi$ as given by the denominator in Eq. (10). The values of $n_0(r_0)$ and $u_0(r_0)$ entering into this equation depend on the profiles of T and \mathbf{v} , and therefore must be experimentally determined. One can therefore correctly determine the angle of arrival of sound from a moving source, $\gamma = \arcsin \{ \mathbf{d} \cdot \mathbf{n}_0(0) / d \}$, only if the central frequency $\omega(0)$ of the received signal spectrum is measured in addition to $\Delta\Phi$. At the same time, the spectral broadening $\delta\omega$ has essentially no influence on $\Delta\Phi$, even when $v \ll c$. The new factor $\mu_0(0)$ in Eq. (10) characterizes the effect of wind on the speed at which sound approaches the receiving antenna, and corresponds to replacing c_0 in Eq. (1) with $u_0(0) = c_0 + v_0 n_0(0)$, where $v_0 = v(0)$. In the form presented here Eq. (10) is not suitable for remote sounding of the atmosphere, and it is necessary to rewrite it in a form showing explicitly the dependence of $\Delta\Phi$ on $T(z)$ and $V(z)$. In doing so, it is sufficient to obtain a relationship linearized with respect to the small ratios v/c_0 and $|\Delta T| / (2T_0)$, where $\Delta T = T - T_0$ and $T_0 = T(0)$.

In all of the following calculations, the direction of the normal $n_0(0)$ in Eq. (10) is described by the angles α_n and β_n between the projections of $n_0(0)$ onto the planes $y = 0$ and $x = 0$ respectively and the z axis. The zenith angle ϵ_n between $\mathbf{n}_0(0)$ and \mathbf{z} (see Fig. 1) is also used. Angular parameters characterizing the directions of other unit vectors (e.g. ρ_N) are introduced in a similar manner, using the subscript 'N'.

In sodars, the signal phase difference can be measured simultaneously in both the $y = 0$ and $x = 0$ planes^{1,3}. For this purpose, two measuring channels are normally used, the orientation of whose baselines \mathbf{d}_x and \mathbf{d}_y is given by expressions $\mathbf{d}_x/d_x = \mathbf{i} \cos\alpha_R - \mathbf{k} \sin\alpha_R$ when $\Delta\Phi = \Delta\Phi_x$ is measured in the plane $y = 0$ (x channel), and $\mathbf{d}_y/d_y = \mathbf{j} \cos\beta_R - \mathbf{k} \sin\beta_R$ when

$\Delta\Phi = \Delta\Phi_y$ is measured in the plane $x = 0$ (y channel). Here $\mathbf{i}, \mathbf{j}, \mathbf{k}$ are the basis vectors of the coordinate system K . These formulas take into account the fact that the axis of the receiving antenna $\rho_R = (\mathbf{i} \tan\alpha_R + \mathbf{j} \tan\beta_R + \mathbf{k}) \cos\epsilon_R$ is perpendicular to its aperture.

In that case, taking $d_x = d_y = d$, one has

$$\mathbf{d}_{x,y} \cdot \mathbf{n}_0(0) = d [\tan\{\alpha, \beta\}_n \cos\{\alpha, \beta\}_R - \sin\{\alpha, \beta\}_R] \cos\epsilon_n, \tag{11}$$

where angles α_n, β_n and ϵ_n are unknown. To take into account the influence of refraction on these angles, it is more convenient to use the equations of Ref. 9, which, using the previously adopted notation can be written as follows:

$$\frac{\partial}{\partial z} \{ x_n(z), y_n(z) \} \approx \tan\{\alpha, \beta\}_n + \left[\frac{\Delta T(z)}{2T_0} \tan\{\alpha, \beta\}_n \sec\epsilon_n + \frac{v_{x,y}(z)}{c_0} \sec^2\{\alpha, \beta\}_n + \frac{v_{y,x}(z)}{c_0} \tan\alpha_n \tan\beta_n \right] \sec\epsilon_n, \tag{12}$$

where $x_n(z)$ and $y_n(z)$ are the coordinates of a point on a sound ray as functions of z , $V_x = V\mathbf{i}$, $V_y = V\mathbf{j}$.

Consider now the passive methods of sensing. Since a sound ray originates from the point \mathbf{r}_0 , one can write

$$\int_0^{z_0} \frac{\partial x_n(z)}{\partial z} dz = x_0, \quad \int_0^{z_0} \frac{\partial y_n(z)}{\partial z} dz = y_0.$$

By substituting Eq. (12) into these relations, written in the form $\partial x_n(z)/\partial z = \tan\alpha_n + a_n(z)$ and $\partial y_n(z)/\partial z = \tan\beta_n + b_n(z)$, and bearing in mind that $|a_n(z)| \ll 1$ and $|b_n(z)| \ll 1$, one obtains

$$\tan\{\alpha, \beta\}_n \approx \tan\{\alpha, \beta\}_n - \{ \bar{a}, \bar{b} \}_n, \tag{13}$$

where $\mathbf{p}_0 = -\mathbf{r}_0/r_0$, and functional \bar{F} mean average values of the functions $F(z)$ over the interval from 0 to z_0 . Substituting Eq. (13) into Eq. (12) and noting that $|\epsilon_n - \epsilon_0| \ll \pi$, one has

$$\mathbf{d}_{x,y} \cdot \mathbf{n}_0(0) \approx \mathbf{d}_{x,y} \cdot \mathbf{p}_0 - \mathbf{d} \{ \bar{a}, \bar{b} \}_n \cos\{\alpha, \beta\}_R \cos\epsilon_0. \tag{14}$$

Thus, using Eqs. (14) and (12) in Eq. (10) one obtains

$$\Delta\Phi_{x,y} \approx \frac{\omega(0)}{c_0} \left[\mathbf{d}_{x,y} \cdot \mathbf{p}_0 \left[1 - \frac{v_0 \mathbf{p}_0}{c_0} \right] - d \cos\{\alpha, \beta\}_R \times \left\{ \frac{\Delta T}{2T_0} \frac{\tan\{\alpha, \beta\}_0}{\cos\epsilon_0} + \frac{\bar{v}_{x,y}}{c_0} \sec^2\{\alpha, \beta\}_0 + \frac{\bar{v}_{x,y}}{c_0} \tan\alpha_0 \tan\beta_0 \right\} \right] \tag{15}$$

Trigonometric functions of angles α_0, β_0 and ϵ_0 and the orientation P_0 are calculated in Eq. (15) using the coordinates of the point $\mathbf{r}_0(t)$ where the wave detected at time t was emitted. For example, $\tan\beta_0 = y_0/z_0, \tan\alpha_0 = x_0/z_0, \cos\epsilon_0 = z_0/r_0$, and so on. At the same time, if $\mathbf{v} \neq 0$ the sound source will have already moved to another point

$$\mathbf{r}_s(t) = \mathbf{r}_0(t) + \mathbf{v}\tau_0(t), \tag{16}$$

where $\tau_0(t)$ is the time for sound to propagate from the point $\mathbf{r}_0(t)$ to the receiving antenna. Optical measurements, if made in parallel with the acoustic ones, could enable one to determine only the coordinates $\mathbf{r}_s(t)$ in addition to \mathbf{v} , but not $\mathbf{r}_0(t)$.

For a homogeneous medium, $\tau_0(t) = r(t)/c_0$, and therefore $r_0^2(t) = [r(t) - (\mathbf{v}/c_0)r_0(t)]^2$. Determining the range $r_0(t)$ from the latter equation and excluding solutions with $r_0(t) < 0$, one can find the expression for $\tau_0(t)$. From Eq. (16), one has

$$r_0(t) = -r_s(t) \left[\mathbf{p}_s(t) + \frac{\mathbf{v}}{c_0} \left[\sqrt{1 - \frac{v_{s\perp}^2}{c_0^2}} + \frac{v_s}{c_0} \right] / \left[1 - \frac{v^2}{c_0^2} \right] \right], \tag{17}$$

where $\mathbf{p}_s(t) = -\mathbf{r}_s(t)/r_s(t)$ is the actual direction toward the sound source, $v_s = \mathbf{v}\mathbf{p}_s, v_{s\perp}^2 = v^2 - v_s^2$.

Using expression (17), one can determine the coordinates $r_0(t)$ with a relative error of $|\mu - 1| \ll 1$, provided that the components $r_s(t)$ and v are known. One can easily see that at low source velocities ($v \ll c$), this accuracy for $r_0(t)$ in Eq. (15) is quite acceptable. However, formula (17) is applicable to (15) in an even more general case, when the receiving antenna is positioned so that $v_{c\perp}/c \ll 1$. In this case $|\mathbf{p}_s(t) - \mathbf{p}_0(t)| \ll 1$ for any value of $v < c$. The simplest case is that of a fixed sound source, for which it is unnecessary to use Eq. (17) ($\mathbf{r}_0(t) \equiv \mathbf{r}_s(t)$), while Eq. (15) reduces to the form

$$\Delta\Phi_{x,y} \approx -\frac{\omega(0)}{c_0} d \left[\frac{\Delta\bar{T}}{2T_0} \frac{\sin\{\alpha, \beta\}_s}{\cos\epsilon_s} + \frac{\bar{v}_{x,y}}{c_0} \sec^2\{\alpha, \beta\}_s + \frac{\bar{v}_{y,x}}{c_0} \sin\alpha_s \tan\beta_s \right] \tag{18}$$

where the receiving antenna is assumed to be looking directly at the sound source ($\mathbf{p}_R = \mathbf{p}_s$).

Obviously, if one uses two spaced antennas for sounding the atmosphere, it is possible to separate out the contributions of $T(z)$ or $\mathbf{v}(z)$ into $\Delta\Phi_x$ and $\Delta\Phi_y$. At the same time, the profiles $\Delta T(z), v_x(z)$ and $v_y(z)$ can only be reconstructed from the functional $\Delta T, v_x$ and v_y if the height of the source varies.

In active methods, the sound is radiated in any desired direction \mathbf{p}_T using the transmitting antenna of a sodar. The vector $\mathbf{r}_0(t)$ then characterizes the position

of the atmospheric scattering volume, whose coordinates are known only approximately due to the influence of the refraction. Therefore one must eliminate from Eq. (15) the dependence of $\Delta\Phi$ on x_0, y_0 and z_0 in order to use it in active techniques.

Let the sounding be performed using a bistatic scheme with the receiving and transmitting antennas are spaced by $D \gg d$. Then $\beta_R = \beta_T = 180^\circ$ and $\alpha_n = \alpha_R$. Accordingly, it only makes sense to measure $\Delta\Phi$ in the y channel. Making use of the condition that the sounding beam trajectory (in this case curve $y_T(z)$) passes through the point r_0 prior to scattering, one obtains $\tan\beta_0 \approx \tan\beta_T + \bar{b}_T$. In this situation $d_y p_s \approx d b_T \cos\beta_T \cos\epsilon_R$ in Eq. (15), and hence one can write

$$\Delta\Phi_y = \omega(0) \frac{d}{c_0} \left\{ \frac{1}{2} \sin(2\beta_T) \left[(\sec^2\epsilon_T - \sec^2\epsilon_R) \frac{\Delta\bar{T}}{2T_0} + \left(\frac{\tan\alpha_T}{\cos\epsilon_T} - \frac{\tan\alpha_R}{\cos\epsilon_R} \right) \frac{\bar{v}_x}{c_0} \right] + (\sec\epsilon_T - \sec\epsilon_R) \frac{\bar{v}_y}{c_0} \right\} \cos\epsilon_R / \cos\beta_T. \tag{19}$$

For the monostatic scheme, when $p_R = -p_T$, and Eq. (19) reduces to which takes an extremely simple form for the case of vertical sounding,

$$\Delta\Phi_y = 2d\omega(0) (\bar{v}_y \sec\beta_T + \bar{v}_x \sin\beta_T \tan\alpha_T) / c_0^2. \tag{20}$$

By interchanging subscripts x and y and angles β_T and α_T , we obtain the analogous formula for $\Delta\Phi_x$.

$$\Delta\Phi_x = 2d\omega(0) \bar{v}_y / c_0^2. \tag{21}$$

Equation. (20) is well-known^{4,5}. In practice, this formula has been used for interpreting sodar sensing data. The data in this paper have demonstrated good agreement between sodar data on the horizontal components of the wind velocity vector and analogous data obtained with a 300 m high meteorological tower, under conditions of steady atmospheric stratification, large deviations between those measurements are observed due to the decrease in signal-to-noise ratio and growth of $\Delta\Phi$ fluctuations caused by atmospheric turbulence, the algorithm for data processing used in Ref 4 could not provide high-quality final results.

Note, in conclusion, that since bistatic sounding provides higher signal-to-noise ratios both in passive and active methods, it can give better results than those in Ref. 4. The feasibility of obtaining information on temperature profiles in addition to wind velocity constitutes another advantage of these techniques.

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