SELECTIVE PROPERTIES AND POLARIZATION CHARACTERISTICS OF AN ANISOTROPIC, LINEAR, THREE-MIRROR RESONATOR

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The effect of a linear polarizer on the selective properties of a resonator is studied. A relationship is established between the amplitude-frequency and polarization characteristics, and an explanation is found for the increase (decrease) in the sharpness of selection obtained with the use of a partial polarizer.

In the last few years a great deal of attention has been devoted to laser resonators with anisotropic elements[1–5] as the most promising means for selection of longitudinal modes and control of the polarization of the radiation. When phase elements are inserted into a resonator an additional degree of freedom in the control of the amplitude-frequency characteristics of the resonator is made available, which raises the possibility of improving the selective properties. Past investigations in this field are predominantly of a theoretical character, since multimirror, linear, anisotropic resonators did not give significantly better results than the currently widely employed isotropic resonators. The three-mirror, whose layout is shown in Fig. 1, with two phase elements and a polarizer is an exception.[3,4]

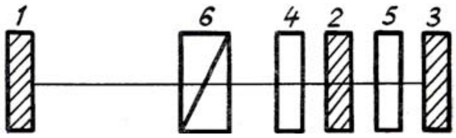

FIG. 1. Diagram of the resonator: 1, 2, and 3 are mirrors; 4 and 5 are phase elements; and 6 is a linear polarizer.

This resonator consists of the following: three mirrors 1, 2, and 3 with reflection coefficients R_1, R_2 and R_3; two phase elements 4 and 5; and, a linear polarizer 6 placed on the same optical axis. The phase elements consist of linear phase plates (LPP), placed on both sides of the mirror 2 at the center of the resonator. The linear polarizer lies in the large part of the resonator, where the active element of the laser is supposed to be placed.

Analysis of the effect of separate parameters of this system on its selective properties[3,4] showed that the best characteristics are obtained when the LPP4 is a quarter-wave plate and its optical axis makes an angle of $\pi/4$ with the axis of maximum transmission of the linear polarizer, the isotropic losses in the elements are minimum, and $R_1 \to 1$. The largest region of free dispersion is obtained for two values of the phase difference in the plate 5: $2\varphi_1 = \pi/2$ for a quarter-wave LPP and $2\varphi_2 = \arcsin[(1 - R) \times (1 + R)^{-1}]$, which we shall call the variant with a "thin" phase plate. The best selective properties are obtained when the principal axes of the LPP5 are also the axes of the LPP4[4,5]. Conversely, if the angle β between the principal axes of the plates equals $\pi/4$, then the worst selective properties of the system are obtained when the minimum transmission of the latter differs substantially from zero; this is especially noticeable with a partial polarizer[5]. In Ref. 5 it is also shown in a particular example that with a nonideal polarizer it is possible to achieve only high attenuation of the modes close to the selected mode but also high attenuation of modes whose frequencies differ considerably from that of the selected mode.

In this paper we shall study the effect of a linear polarizer on the selective properties of a three-mirror resonator and we shall show that the amplitude-frequency and polarization characteristics of this resonator are related.

In the plane-wave approximation the characteristics of the system under study can be determined with the help of Jones' matrix method[6] by solving the equation

$$A\hat{M}E = \Lambda_{1,2}E.$$

Here $\hat{M}$ is the generalized matrix of the resonator; $\vec{E}$ is the electric vector of the light wave propagating in it; $\Lambda_{1,2}$ are the eigenvalues of the matrix $\hat{M}$; A is a factor, equal to $\sqrt{R_1}\exp(2i\nu L)$, where $\nu = 2\pi/\lambda$ is the wave number, λ is the wavelength, and L is the length of the resonator. The polarization properties of the system can be judged according to the parameter κ, equal to the ratio of the components of the electric vector E, and the largest of the functions $|\lambda|^2$ corresponds to the amplitude-frequency characteristic (AFC) of the resonator.

If the analysis is confined to the case of the best values of the parameters of the LPP4, $R_3 = 1$, and no losses in the elements and R_1 is set equal to unity (decreasing the value of this parameter, just like the value of the maximum transmission of the polarizer, merely gives a proportional change in $|\Lambda|^2$), then the squared modulus of the eigenvalue and the parameter κ for two extreme angles β will be as follows:

1) for $\beta = 0$ (the principal axes of the LPP4 and 5 are identical)

$$|\lambda_{1,2}|^2=0,25(1+F)^{-1}\left|1+P\pm(1-P)\sqrt{1-SF}\right|^2; \tag{1}$$

$$\kappa_{1,2} = -\frac{i}{\sqrt{SF}}\left(1\pm\sqrt{1-SF}\right). \tag{2}$$

where $S = 4P(1 - P)^{-2}$, P is the ratio of the minimum transmission of the polarizer 6 to its maximum transmission; $F = F_1\cos^2 2\nu t$, if the LPP5 is a quarter-wave plate, and $F = F_2^2 \sin^4 \nu l$ for the case with a "thin" LPP5, where $F_1 = 4R(1 - R)^{-2}$, $F_2 = 4\sqrt{R}\,(1 + R)(1 - R)^{-2}$, and l is the average optical length of the passive gap (between the mirrors 2 and 3);

2) for $\beta = \pi/4$

$$|\lambda_{1,2}|^2=0,25(1+F)^{-1}\left[1-P\pm(1+P)\sqrt{1+HF}\right]^2; \tag{3}$$

$$\kappa_{1,2} = \frac{1}{\sqrt{HF}}\left(1\pm\sqrt{1+HF}\right), \tag{4}$$

where $H = 4P(1 + P)^{-2}$.

We shall analyze the expression (1). If SF $\leq$ 1, which happens near the maximum of the AFC, the modulus symbol can be replaced by parentheses. In the case when SF > 1, however, we have $|\Lambda_{1,2}|^2 = P$, i.e., in a definite interval of νl the squared of the eigenvalue is a constant. The calculations showed that the highest degree of selection, characterizing attenuation of the modes far from the selected mode and defined as the ratio of the maximum to the minimum value of the AFC, is achieved when the equality $|\lambda_1| = |\lambda_2|$ holds only for two values of νl per period (AFC). In the case of the quarter-wave plate 5 this occurs when $P = P_1 = (1 - \sqrt{R}\,)^2(1 + \sqrt{R}\,)^{-2}$. When the LPP5 is "thin", the highest degree of selection is obtained when

$$P=P_2=\left[\sqrt{F_2^2+1} - F_2\right]^2.$$

It is not difficult to show, that with the use of a linear polarizer high degrees of selection is achieved with $2\varphi_1 = \pi/2$ by a factor of $(1+\sqrt{R}\,)^4\ (1 + R)^{-2})$ and $2\varphi_2 = \arcsin\,[(1 - R)(1 + R)^{-1}]$ (by a factor of $\left(\sqrt{F_2^2+1}-F_2\right)^{-2}\left(1+F_2^2\right)^{-1}$.

If R $\to$ 1, the relative increase in the attenuation of modes far from the selected mode approaches the value 4 in both cases, but an appreciable absolute value, which is important from the viewpoint of practical applications, is observed only with a quarter-wave LPP5 and low densities of the middle mirror of the resonator($R \leq 0,35$). Thus the largest difference in the minimum values of $|\Lambda|^2$ (for a partial polarizer and an ideal polarizer) obtains, according to the calculations, for $R = 0,061$ and equals ~ 0.4185. As the parameter P is increased (with $\beta = 0$ and R fixed) from zero to an optimal value (P_1 or P_2) the degree of selection increases. Increasing P further decreases the attenuation of the modes lying far from the selected mode. If $\beta = \pi/4$, however, any increase in P is accompanied by a decrease in the degree of selection, which follows from an analysis of Eq. (3).

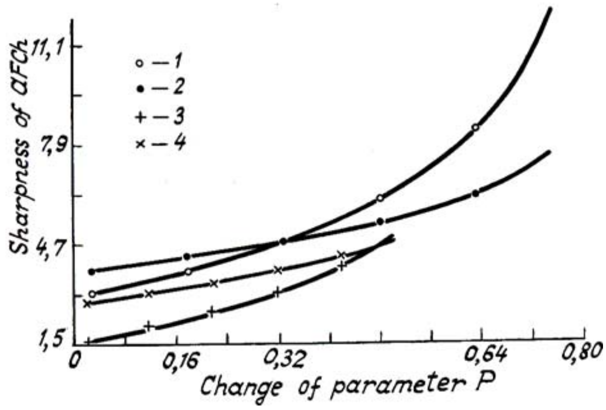

FIG. 2. The sharpness of the AFC of the optical system under study as a function of the properties of the polarizer: curves 1 and 3 are for a quarter-wave LPP and curves 2 and 4 are for a "thin" LPP.
Key: 1) Sharpness of AFC
2) Change in the parameter P.

We shall study the change in the sharpness of the AFC (also called the sharpness of selection[5,7]). If it is defined as the coefficient of the quadratic term in the expansion of the AFC in powers of νl near the maximum[7], which is valid for the case with a quarter-wave LPP5, then it can be shown that for $\beta = \pi/4$ the sharpness of selection in a resonator with a partial polarizer is $(1 + P)\ (1 - P)^{-1}$ times lower than for the case of an ideal resonator and for $\beta = 0$ it is larger by the same factor. The actual increase in the sharpness (at some level ξ), defined as the minimum ratio of the region of free dispersion to the frequency band in this region, where the AFC lies above the selected level, is somewhat smaller. This can be seen from Fig. 2, which shows four numerically computed curves illustrating the sharpness of selection versus the parameter P for the two best values of the phase shift in the LPP5, for $\beta = 0$ at 0.5 (curves 3 and 4) and 0.75 (curves 1 and 2) levels. The increase in sharpness only for $2\phi_2 = \pi/2$ at the 0.5 level (curve 3) corresponds to the analytically predicted increase. For the same LPP at the 0.75 level the sharpness of AFC increases significantly less (by a factor of 4 and not 7 as expected when P changes from 0 to 0.75), as shown by curve 1 in Fig. 2. When a "thin" phase plate is employed the increase in the sharpness of selection is significantly smaller (by a factor of 1.7–2.3, see curve 2 for $\xi = 0.75$ and curve 4 for $\xi = 0.5$). It should be noted that for low values of P a higher sharpness of the AFC is achieved for the same value of R in the case of a "thin" LPP5, while the opposite situation obtains for large values of P. It follows from here that the variant with a quarter-wave plate is preferable for lasers with an active medium whose gain is low.

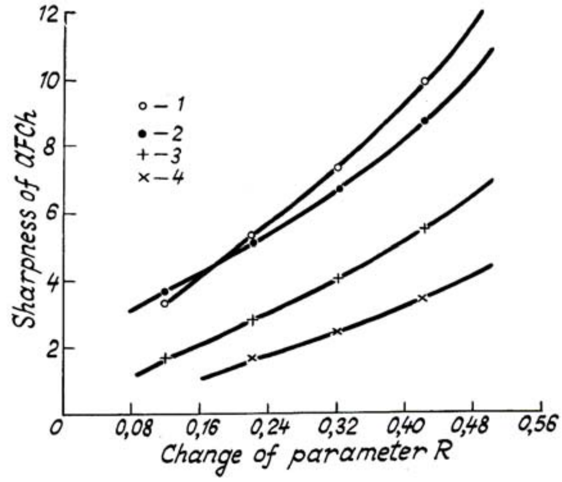

FIG. 3. Comparison of the selective properties of an anisotropic resonator and resonators with Fabri-Pero and Fox-Smith interferometers.
Key: 1) Sharpness of AFC
2) Change in the parameter R.

The numerical calculations showed that the sharpness of selection increases practically up to the point at which the parameter P reaches the level ξ, so that in order to make a laser based on the resonator studied the value of P should be chosen somewhat smaller (by 0.01–0.05) than $(1 - G + B)$, where G is the per—pass gain of the active medium and B are the total nonselective losses, including half the losses due to the transmission of the outer mirrors (1 and 3).

Comparing the selective properties of widely employed resonators with the selectors in the form of a Fabry-Perot interferometer (FPI), a Fox-Smith interferometer (FSI), and the system under study showed that the latter gives both a higher degree and a higher sharpness of selection. Fig. 3 shows the sharpness of the AFC versus the reflection coefficient of the corresponding mirror (mirrors) for four resonators: 1) for the resonator under study with a quarter-wave plate in the passive gap; 2) for the case with a "thin" LPP5; 3) for a resonator with an FPI; and, 4) for a resonator with an FSI. It follows from the calculations that the AFC of a three-mirror linear resonator with quarter-wave plates and a partial resonator is 1.7–2 times sharper than the AFC of a resonator with an FPI and 2.8–4 times sharper than that of a resonator with an FSI.

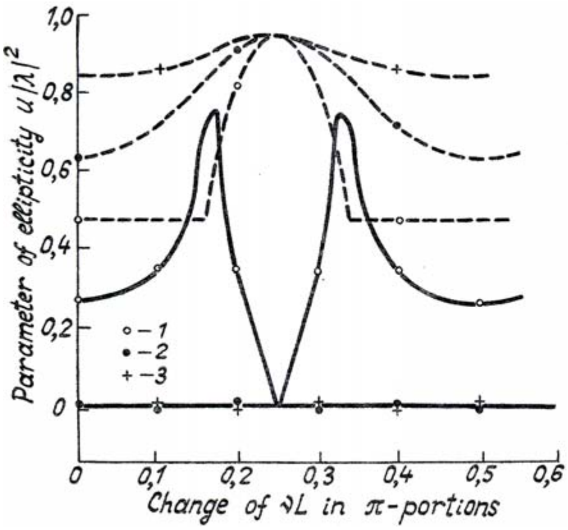

FIG. 4. The ellypticity and the values of the AFC for different values of β and P: 1) $\beta = 0$; $P > 0$; 2) $P = 0$; 3) $\beta = \pi/4$, $P > 1$.
Key: 1) Ellipticity and $|\Lambda|^2$
2) Change in νl as a fraction of π.

We shall examine some of the polarization I characteristics of the optical system under study. At the calculations showed the polarization at the resonance frequencies is linear for any real values of the parameters β, φ and P. Off resonance and for $P \neq 0$ the polarization changes, and these changes can be different depending on the value of β. For $\beta \neq \pi/4$ the light waves remain linearly polarized, which follows directly from analysis of the expression (4) for κ and is supported by numerical calculations (see the solid curve 3 in Fig. 4). In this case the orientation of the electric vector, i.e., the azimuth, changes.

If $\beta = 0$, off resonance predominantly the ellipticity changes. It reaches its maximum value, as one can see from the behavior of the solid curve 1 in Fig. 4, for values of νl close to the values where $|\Lambda_1|$ no longer equals $|\Lambda_2|$ (i.e., at points were SF $\neq 1$). One can also see from the figure that in this case the sharpness and degree of selection are higher (see the broken curve 1) than for $P = 0$ (curve 2), and they are significantly higher than for the variant with $\beta = \pi/4$ (broken curve 3) for $P > 0$. Thus the relation between the characteristic polarization of the resonator and its selective properties can be traced. It should be noted that when a partial polarizer is inserted into the resonator under study the polarization of the radiation can be controlled to some degree (by varying the parameters β and νl). Control does not extend over a wide range, since an appreciable change in the polarization is observed only considerably off resonance, but in this case the value of $|\Lambda|^2$ also drops sharply, which makes the resonator virtually unusable.

If the light oscillations in the resonator are represented as a superposition of two orthogonally polarized waves, whose electric vectors fall along the distinguished directions of the linear polarizer, then the increase (decrease) of the selective properties can be explained as follows. A selective reflector, including the mirrors 2 and 3 and anisotropic elements, changes the polarization of radiation at nonresonant frequencies, converting one wave into another (orthogonally polarized) wave and vice versa. For $\beta = 0$ the polarization is elliptical, and the phase shift between the orthogonal components of the electric vector approaches $\pi/2$. The wave with the lower amplitude incident on the selective reflector is partially converted into an orthogonally polarized wave, which is also shifted in phase by n relative to the wave with the higher amplitude. The latter wave is suppressed in the process, and both the degree and sharpness of selection increase. If $\beta = \pi/4$, the polarization is linear and the phase shift between the orthogonal components approaches π. The wave with the lower amplitude is partially converted on reflection into an orthogonally polarized wave shifted in phase by 2π relative to the wave with the higher amplitude. In the process the latter wave is amplified, and the degree and sharpness of selection correspondingly decrease. With an ideal polarizer ($P = 0$) neither effect is observed, since the amplitude of one of the waves is zero.

The investigations permit drawing the following basic conclusions.

1) For a certain choice of the parameters of the and using a partial polarizer both the sharpness and degree of selection are higher than in the case with an ideal linear polarizer.

2) The selective properties of the system under study are 1.7–4 times higher than for resonators with selectors in the form of Fabry-Perot or Fox-Smith interferometers.

3) The selective properties of an anisotropic three-mirror resonator are related with its polarization characteristics.

4) In the resonator studied in this work the sharpness and degree of selection as well as, to some degree, the polarization of the radiation can be controlled, and the sharpness and degree of selection can be controlled efficiently by varying the parameter β.

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