POTENTIAL ACCURACY OF WIND-VELOCITY MEASUREMENTS WITH A COHERENT DOPPLER LIDAR

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The accuracy of wind-velocity measurements performed with a heterodyne lidar is fundamentally limited by signal fluctuations and by noise in the receiving channel. A statistical model of the post-photomixer signal and noise is constructed. It is shown that the component of the photocurrent determined by aerosol scattering of the laser radiation can be regarded as a Gaussian, narrow-band, random process and the noise can be regarded as white noise. The correlation function of this component is found. Shot noise and interference owing to "heterodyning" of the background radiation are taken into account in the analysis. An expression is derived for the maximum relative error in measuring the wind velocity. Model calculations of the SNR and the error as a function of the altitude for a ground-based lidar and a space-based lidar with an orbital of 300 km were performed.

Two lidar methods for performing spatially resolved measurements of the characteristics of the random vector field of the wind-velocity $v(r)$, have not been implemented: correlation and Doppler. The latter is based on the use of a CO_2-laser, heterodyning and post-detector spectral analysis for determining the Doppler shift w_d of the radiation frequency. In spite of the difficulties, associated with the high monochromaticity and frequency stability of the laser, in implementing the Doppler method as well as technically complicated photomixing under probing conditions, this method has the advantages over the correlation method. Its possibilities are significantly limited[1] by fluctuations of the transmission up to the scattering volumes and the low contrast of aerosol inhomogeneities as well as their variability, which make it much more difficult to interpret the measurements.

In this paper we analyze the limiting or potential accuracy of the average (over the scattering volume) random field of the velocity vector $\langle v(r)\rangle_{\text{vol}}$ with a heterodyne lidar based on a CO_2 laser. For the monostatic pulsed probing scheme studied below the Doppler shift is given by the expression

$$\omega_D = 4\pi \ |\langle \mathbf{v}(\mathbf{r})\rangle_{\text{vol}}| \cos\varphi/\lambda,$$

where λ is a wavelength φ is the angle between the wave vector k of a scattered wave and $\langle v(r)\rangle_{\text{vol}}$. The results obtained below determine the magnitude of the error irrespective of the structure of post-detector processing and permit evaluating the efficiency of different modifications of this structure.

MODEL OF THE SIGNAL AND NOISES

We shall take the signal as the component photocurrent

$$J(t)=2b\iint_{S'} A_1(\mathbf{r},t)A_2^*(\mathbf{r},t)d^2\mathbf{r}\exp(i(\omega_1-\omega_2)t. \tag{1}$$

owing to aerosol scattering of laser radiation with frequency w_0. Here $b = e_1e_2\eta q/h\nu$, e_1 is the unit polarization vector, $A_i(r, t)$ is the complex amplitude of the field at the point r at time t, w_i is the frequency ($i = 1$ is the scattered field and $i = 2$ is the heterodyne field), $w_1 = w_0 + w_D$, S' is the aperture of the receiving antenna with area S, η is the quantum efficiency of the photodetector, q is the electron. charge, and $h\nu$ is the average energy of the photons of the interacting fields. The complex amplitude of the heterodyne field is of the form

$$A_2(\mathbf{r},t) = A_2\exp(i\varphi_1(t)) \tag{2}$$

where A_2 = const and $\varphi_1(t)$ is the randomly time-varying phase. This model accepted corresponds to a single-mode laser far above the lasing threshold[2].

Assume that the scattered field $A_1(r, t)$ is Gaussian and has a narrow bandwidth, and that the width of the spectrum $\Delta w'$ of the process $j(t)$ satisfies the condition $\Delta w' \ll w_0 - w_2$. Satisfaction of the last condition is guaranteed by the stringent constraints imposed on the laser monochromaticity and by the appropriate choice of the intermediate frequency $w_0 - w_2$. With these assumptions the process $j(t)$ is Gaussian and narrow-band irrespective of the statistical characteristics $\varphi_1(t)$ (Ref. 3).

In laser systems for probing the atmosphere the condition studied in Ref. 4, under which the space-time correlation function field can be factorized holds owing to the high monochromaticity and low divergence of the transmitter beam for such systems. Using the condition of the factorization and the expressions (1) and (2) the correlation function of the photocurrent can be written in the form

$$B(\tau=t_1-t_2,t_1)=$$
$$=4b^2\exp(-i(\omega_1-\omega_2)\tau)P_1(t_1)P_2\rho_{1t}(\tau,t_1)\rho_2(\tau)/\theta(ct_1/2). \quad (3)$$

Here

$$P_1(t=2R/c)=SEc\bar{\beta}_\pi(R)T^2(R)/2R^2,$$

P_2 are the average powers of the scattered field and the heterodyne field incident on the photodetector; $E=\int_0^\infty p(t)dt$ is the energy of the sounding pulse; $P(t)$ is the power of the pulse; R is the slant range up to the scattering volume; c is the velocity of light; $T(R)$ is the transmission function; $\bar{\beta}_\pi(R)$ is the average aerosol backscattering coefficient; and, $p_{1t}(\tau, t_1)$, $p_2(\tau)$ are the time correlation coefficients for the scattered field and heterodyne field. The parameter $\theta(ct/2)$ is determined by the expression

$$\theta(ct/2) = S^2/\iint_{S'}d\mathbf{r}_1\iint_{S'}d\mathbf{r}_2\rho_{1r}(\mathbf{r}_2-\mathbf{r}_1,t)$$

and characterizes the decrease in the signal power owing to turbulence in the propagation medium ($p_{1r}(\Delta r, t)$) is the coefficient of spatial correlation of the scattered field). It is shown in Ref. 5 that

$$\theta(R) = \left[\frac{\pi d^2}{2\lambda R}\right]\left[1-\frac{R}{F}\right]^2+1+\left[\frac{d}{\rho_0}\right]^2,$$

$$\rho_0=\left[1.45k^2\int_0^R C_n^2(x)\left[1-\frac{x}{R}\right]^{5/3}dx\right]^{-3.5} \quad (4)$$

is the coherence radius of the scattered field; d is the diameter of the receiving aperture, F is the focal length, $k = 2\pi/\lambda$ and $C_n^2(x)$ is the structure factor of the fluctuations of the refractive index.

In Eq. (3) p_{1t} can be written in the form

$$\rho_{1t}(\tau,t) = \rho_i(\tau)\rho_s(\tau,t) \quad (5)$$

where $p_i(\tau)$ and $p_s(\tau, t)$ are, respectively, the correlation coefficients of the laser radiation owing to amplitude-phase fluctuations and the scattered field in the case of sounding with a pulse with monochromatic fill and an effective duration τ_p. In Ref. 6 p_s was found for a Gaussian, homogeneous and isotropic wind-velocity field within the scattering volume. It can be shown that taking into account the fluctuations of β_π, whose relative magnitude $K_\beta \ll 1$ (Ref. 1),

$$\rho_s(\tau,t)=\gamma_p(\tau,t)\exp(-2k^2\sigma_v^2\tau^2)\times$$
$$\times\{1-K_\beta^2[1-\rho_\beta(c\tau/2,ct/2)]/4\} \quad (6)$$

where $p_\beta(\Delta R, R)$ is the coefficient of spatial correlation of β_π; σ_v^2 is the variance of $|v(r)|\cos\varphi b$ in the scattering volume; and,

$$\gamma_p(\tau,t) = \int_0^\infty\sqrt{P(t-2R'/c)P(t+\tau-2R'/c)}\,dR'/E. \quad (7)$$

Thus the "signal" component of the photocurrent is a Gaussian, narrow-band, random process, whose correlation coefficient, using (3) and (5), is given by the expression

$$\rho(\tau,t) = \rho_s(\tau,t)\rho_1(\tau)\rho_2(\tau). \quad (8)$$

The interference is due to shot noise and heterodyning of the background radiation with the angle-frequency spectral density $N(w)$. Because of its wide-band nature of the process the spectrum of each component can be regarded as "white" with the spectral density

$$N_1(t)=2q\{[P_1(t)+P_2+P_{ph}]\eta q/h\nu+j_0\}$$

for shot noise and

$$N_2=4b^2P_2N(\omega_1)\lambda^2$$

for background radiation. Here j_0 is the dark current of the photodetector, $P_{ph} = N(\omega_1)\ \Omega\ S\ \Delta\omega'$ is the power of the background radiation incident on the photodetector; Ω is the field of view of the detector; Δw is the bandwidth of the optical filter; and, $N(t) = N_1(t) + N_2$.

The heterodyne detector always includes an intermediate-frequency filter. The constant component of the photocurrent and the spectra of the fluctuations of the intensity of the scattered field and the field of the heterodyne do not fall within its transmission band. For this reason the enumerated components are ignored in the statistical model of the noise. The component of the photocurrent, due to molecular scattering, is neglected because for λ = 10.6 μm the scattering coefficient is much smaller than the aerosol scattering coefficient.

POTENTIAL ACCURACY

The minimum variance of the estimate of the central frequency of the energy spectrum of a Gaussian, narrow-band random process, whose realization has a duration T, against a white-noise background was determined in Ref. 7. The results, neglecting the nonstationary nature of $j(t)$ on the interval T, permit writing the following expression for the limiting relative error δ_v of the estimate of $\left|\langle v(r)\rangle_{vol}\right|\cos\varphi$:

$$\delta_v^2=2\left[Q\int_{-\infty}^{\infty}\tau^2\tilde{\rho}(\tau)\rho(\tau)d\tau\right]^{-1}. \quad (9)$$

where $Q=2P_cT/N$ is the SNR; $P_c = 2b^2P_1P_2/\theta$, $T = 2\Delta R/c$; ΔR is the slang-range resolution;

$$\tilde{\rho}(\tau)=F^{-1}\left\{\frac{1}{1+P_cF[\rho(\tau)]/N}\right\}, \tag{10}$$

where F and F^{-1} denote the direct and inverse Furrier transform. The expression (9) was derived for $\xi = T/\tau_0 \gg 1$, where

$$\tau_0=\int_0^\infty \rho(\tau)d\tau$$

is the correlation time. The assumption that jit) is stationary is predicated on the assumption that $\Delta R/R$ and the relative change in $\bar{\beta}_\pi$ over the altitude interval $(R,\ R+\Delta R)$ are small.

In order to perform model calculations the form of $\rho(\tau)$ must be specified. Since (9) is valid for $T \gg \tau_0$ the computational results will not depend significantly on the form of $\rho(\tau)$ for fixed τ_0. Based on this and in order to simplify the subsequent analysis consideration we shall employ an exponential approximation for the correlation coefficient of the photocurrent

$$\rho(\tau)=\exp(-|\tau|/\tau_0). \tag{11}$$

Carrying out the integration in (9), using (10) and (11), we find

$$\delta_v^2=\sqrt{1+Q'}\,[1+\sqrt{1+Q'}\,]^3/\xi(Q'\psi)^2, \tag{12}$$

where $\psi = w_dt_0$, $Q' = Q/\xi$ is the ratio of the average signal power to the white-noise power in the effective frequency band of the useful signal. To determine τ_0 we shall assume that the sounding pulse has the exponential form

$$P(t)=E\exp(-t/\tau_p)/\tau_p \quad \text{and} \quad \rho_1(\tau) = \exp(-|\tau|\gamma_1),$$

$$\rho_1(\tau) = \exp(-|\tau|\gamma_1)$$

where τ_p is the effective width of the pulse, and γ_1 and γ_2 are the widths of the spectra of the radiation source and the heterodyne.

Then

$$\tau_0=\sqrt{\pi}p\xi_1\exp(\xi_1^2)[1-\Phi(\xi_1)],$$

where $\xi_1 = (2\sqrt{2}k\sigma_vp)^{-1}$, $p = \gamma_1 + \gamma_2 + 1/2\tau_p$, and $\Phi(x)$ is the probability integral. The following asymptotic relations hold:

$$\tau_0\approx\sqrt{\pi}p\xi_1(1-2\xi_1/\sqrt{\pi}+\xi_1^2), \quad \xi_1<<1;$$

$$\tau_0\approx p(1-1/2\xi_1^2), \quad \xi_1>>1.$$

The expression for τ_0 does not take into account the fluctuations of β_π, since their contribution to τ_0 for $K_\beta \ll 1$ is of the order of 3–5% and can be neglected.

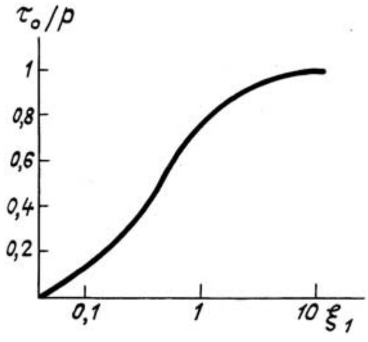

FIG. 1. τ_0/p versus ξ_1.

Figure 1 shows τ_0/p as a function of ξ_1. The results obtained can be employed for arbitrary ρ_1, ρ_2, and $P(t)$, if τ_0 is evaluated beforehand

MODEL CALCULATIONS

The profiles of the aerosol and molecular attenuation coefficients from Ref. 8 for the summer middle latitudes and the profile of the horizontal component of the average wind velocity from Ref. 9 were employed for the calculations.

$$v(H)=\begin{cases}2.5H+5 & 0<H\le2\\ 3.63H+2.74, & 2<H\le7.5\\ 30, & 7.5<H\le12.5\\ -2.66H+63.2. & 12.5<H,\end{cases}$$

(H in km, v in m/sec). The vertical component of v equals zero, and the vectors k and v lie in the same plane. The calculations of the relative error were performed for a ground-based lidar (variant 1) and a space-based lidar with an orbital altitude of $H^* = 300$ km (variant 2).

VARIANT 1. The vertical profile $C_n^2(H)$ is determined by the expression (10)

$$C_n^2(H)=C_{n0}^2(H/H_0)^{-2/3}\exp(-H/\bar{H}), \tag{13}$$

where

$\bar{H} = 3200$ m, $H = 2.5$ m,
$10^{-15}\ \text{m}^{-2/3} < C_{n0}^2 < 10^{-12}\ \text{m}^{-2/3}$.

Substituting (13) into (4) and carrying out the integration gives

$$\rho_0=\left\{2.29k^2C_{n0}^2\frac{H^{1/3}H_0^{2/3}}{\sin\varphi}\ {}_1F_1\left(\frac{1}{3};3;-\frac{H}{\bar{H}}\right)\right\}^{-3/5},$$

where ${}_1F_1$ is the degenerate hypergeometric function, φ is the angle of inclination of the sounding path relative to the ground, and $H = R/\sin\varphi$. The parameters of the lidar are: $P_2 = 10^{-2}\ Bt$, $d = 0.3$ m, $\eta = 0.6$. Figures 2 and 3 show the results of the calculations of the SNR and the relative error as a function of altitude for a number of sounding pulses $L = 1$, $\varphi = 60°$. For $L > 1$ $\delta_v(L) = \delta_v(L = 1)/\sqrt{L}$ for the case of independent measurements. The values of the parameters were chosen so as to show the degree of dependence on each parameter separately. In particular, curves 2 and 9 and curves 3 and 4 illustrate the significant effect of C_n^2 on the accuracy of the measurements, while curve 6 shows the dependence on the altitude with constant velocity. In the calculations the resulting correlation time was given.

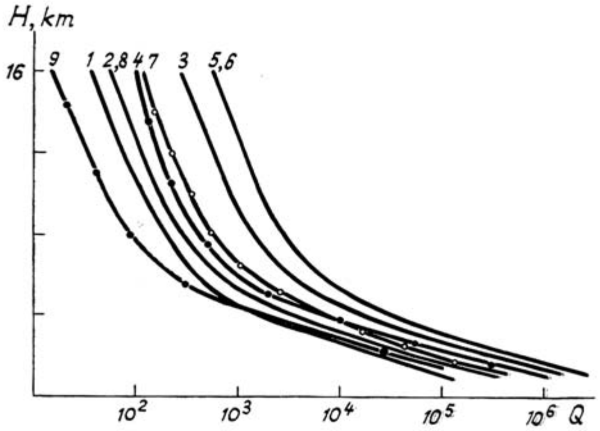

FIG. 2. Profiles of SNR Q for ground-based lidar. E = 0.065 J (1), 0.1(2, 8, 9), 0.5(3, 4, 7), and 1(5, 6); for curves 4 and 9 $C_{n0}^2 = 10^{-12}\ m^{-2/3}$ and for the other curves $10^{-14}\ m^{-2/3}$; for curve 8 ξ = 200 and for the other curves ξ = 100; for curve 7 ΔR = 300 m and for the other curves 750 m.

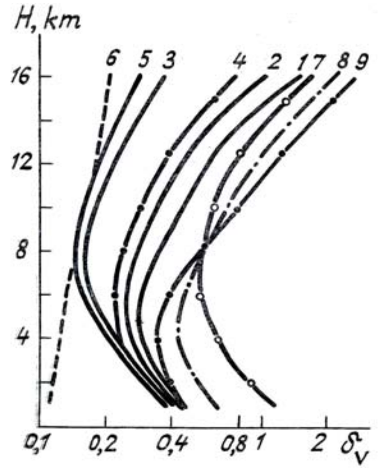

FIG. 3. The relative error δ_V versus the altitude H for a ground-based lidar. For curve 6 v(H) = 30 m/sec; for the other curves see caption to Fig. 2.

VARIANT 2. The effect of turbulence, which is most significant on the starting section of the path, can be neglected in the calculations. If φ* is nadir angle, then φ = π/2 = φ*. The parameters of the lidar are: $P_2 = 10^2$ Bt, $d = 1$ m, η = 0.6. The results of the calculations with $L = 1$ are presented in Figs. 4 and 5.

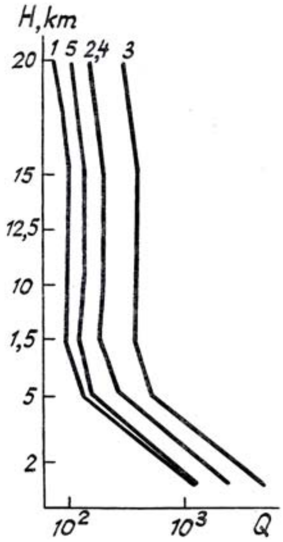

FIG. 4. Profiles of the SNR Q for a space-based lidar. E = 0.5 J (1, 7), 1(2, 4, 5, 6), and 2(3); for curves 5 and 6 φ = 45 and for the other curves φ* = 30°; for curve 4 ξ = 200 and for the other curves ξ = 100; ΔR = 1500 m.*

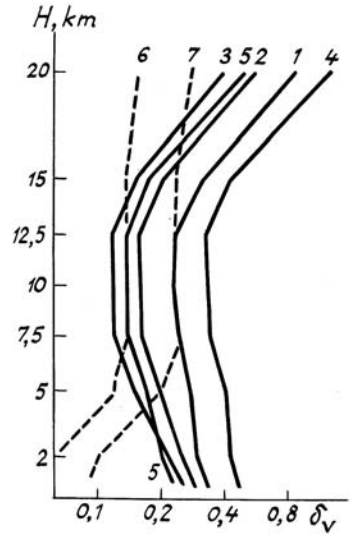

FIG. 5. The relative error δ_V versus the altitude H for a space-lidar. For curves 6 and 7 v(H) = 30 m/sec; for the other curves see the caption to Fig. 4.

The width of the spectrum of the "signal" component of the photocurrent $\psi \approx \tau_0^{-1} = 2 \times 10^7$ Hz, i.e., quite stringent constraints were imposed on the spectral characteristics of the heterodyne and the source of sounding pulses. The spectral characteristics were singled out because, on the one hand, their quality imposes stringent constraints on the accuracy of the wind-velocity measurements while, on the other hand, decreasing γ_2 and γ_1 is a difficult technical problem. The results obtained permit determining the efficiency of different structures of post-detector processing in coherent Doppler lidars, and there are also strong grounds for imposing constraints on the parameters of the lidar based on the maximum admissible errors.

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