ON LIDAR RETURN PHOTOCOUNT STATISTICS

V.G. Astafurov

*Institute of Atmospheric Optics,
Siberian Branch of USSR Academy of Sciences, 634055, Tomsk*
Received December 12, 1989

The use of asymptotic distributions to describe the unconditional statistical characteristics of photoelectrons produced by lidar returns is discussed. Results of model calculations of the introduced degeneracy parameter are presented. This parameter takes into account the fluctuations of the backscattering coefficient, the transmission function, and the energy of the sounding pulse.

In many lidar applications conditions occur which enable one to use the asymptotic probability distribution $P(n)$ of the number of photoelectrons n recorded per unit time interval Δt. These include the negative-binomial (NB), the McLean–Pike (MP), and the Poisson (P) distribution. The basis of the application of these distributions and the determination of their parameters are normally made without taking into account the fluctuations of the backscattering coefficient $\beta_\pi(R)$ at the distance R, the transmission function $T(E)$, or the energy of the sounding pulse E (Ref. 1). That is to say, the analysis extends only to the conditional statistical characteristics of the photoelectron flux. In the present paper these fluctuations are taken into account.

In order to prescribe the P distribution, it is necessary to determine the mean number of photoelectrons $\bar{n}$, and for the KB and MP distributions, $\bar{n}$ and the number of space-time phase cells

$$s = \frac{\Delta t}{\tau_c}\,\frac{S}{s_c},$$

which characterize the coherence properties of the scattering field in the plane of the receiver aperture with area S, Here $s_c = \lambda^2/\Omega$ and $\tau_c \sim \alpha^{-1}$ are, respectively, the coherence area and time; Ω is the effective solid angle in which the recorded radiation is concentrated; and λ is the wavelength. The effective width of the spectrum a is determined from the width of the spectrum of the laser pulse and the effective spectral broadening due to scattering.[1] The mean number of photoelectrons is found from the lidar equation

$$\bar{n} = \frac{EcS\beta_\pi T^2(R)}{2R^2}\,\frac{\eta\Delta t}{h\nu}$$

where c is the velocity of light, $\beta_\pi(R)$ is the value of the backscattering coefficient at the range R, $T(R)$ is the atmospheric transmission as a function of the range, η is the quantum efficiency of the photodetector, and $h\nu$ is the mean energy of the photons. For the NB and MP distributions the variance of the number of photoelectrons is given by

$$D(n) = \bar{n}(1+\xi), \tag{1}$$

where $\xi = \bar{n}/s$ is the degeneracy parameter. Such an approach to the determination of the parameters of the asymptotic distributions in the case of lidars does not take into account the fluctuations of the values of $\beta_\pi(R)$, $T(R)$, and E. For this reason the above asymptotic distributions $P(N|E, \beta_\pi, T)$ must be considered as nominal while the unconditional distributions are obtained as follows

$$P(n) = \langle P(|E,\beta_\pi,T)\rangle_{E,\beta,T}, \tag{2}$$

where the angular brackets denote averaging over the indicated parameters. The same is valid for the moments of the distributions. Averaging expression (1) over β_π, e, and T and taking into account the fact that the mean value

$$\bar{n} = bE\beta_\pi(R)T^2(R)$$

entering into expression (1) is a conditional one, we obtain

$$D(n) = \langle D(n|E,\beta_\pi n,T)+(bE\beta_\pi(R)T^2(R))^2\rangle - \langle n\rangle^2 =$$
$$= \langle n\rangle + \langle n\rangle^2\left[\frac{1}{s} + \frac{1}{ss^*} + \frac{1}{s^*}\right],$$

where $\langle n\rangle$ is the unconditional mean value, b is the proportionality factor, and

$$K_T^2 = D(T^2)(\overline{T^2})^2,\quad K_E^2 = D(E)/\overline{E}^2,\ \text{and}\ K_\beta^2 = D(\beta_\pi)/\beta^{-2}$$

are the relative variances of T^2, E, and β_π, respectively. If one defines the equivalent number of "cells" s_{eq} by

$$s_{eq} = ss^*/(1+s^*+s),$$

then it is also possible to introduce the equivalent degeneracy parameter $\xi_{eq} = \langle n \rangle / s_{eq}$. However, this analogy is purely conventional and does not mean that the NB statistics are acceptable for describing the probability $P(n)$.

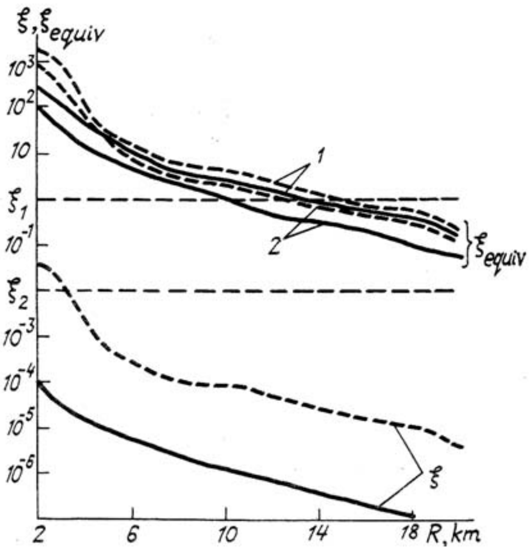

FIG. 1. The values ξ_{eq} and ξ as a functions of height for the aerosol (dashed lines) and the molecular (solid lines) components of a Iidar return: 1) K_{eq} = 0.1 and 2) K_{eq} = 0.04.

The unconditional distributions can be approximated by the binominal Charlier formula:

$$P(n) = \frac{\langle n\rangle e^{-\langle n\rangle}}{n!}\left\{1 + \left[(\langle n\rangle - n)^2 - n\right]\left[D(n) - \langle n\rangle\right] / 2\langle n\rangle^2\right\}, \quad (3)$$

which generalizes the HP distribution and is based on the equality of the first two moments of the exact and approximate distributions as well as the proximity of the latter to the Poisson distribution[2]. For $\xi_{eq} \ll 1$ expression (3) reduces to the P distribution. If the critical values ξ_1 and ξ_2 ($\xi_1 \ll 1$, $\xi_1 < \xi_2$) are prescribed, then it is possible to determine the regions of applicability of the P distribution for $\xi_{eq} < \xi_1$ and of distribution (3) for $\xi_1 < \xi_{eq} < \xi_2$. For $\xi_{eq} < \xi_2$, to determine $P(n)$, according to Eq. (2) it is necessary to average $P(n|E, \beta_\pi, T)$. The values of ξ_{eq} and ξ, calculated as functions of altitude for the aerosol and molecular scattering components of lidar returns are shown in Fig. 1.

The calculations were carried out using the atmospheric, model from Ref. 3 for λ = 0.69 μm, E = 0.01 J, S = 0.5 m^2, Δt = 2.10^{-6} sec, η = 0.05, Ω = 10^{-8} sr, $K_{\beta\alpha}$ = 0.1, $K_{\beta m}$ = 0.04, α = 10^8 Hz (the indices "α" and "*m*" denote the parameters for the aerosol and molecular components, respectively). K_T^2 was estimated by the approximation expression

$$K_T^2 = 4K^2\beta_\pi\sigma_a^2 R_0 R_c\left[1 - \exp(-2R/R_0)\right],$$

obtained assuming Gaussian fluctuations of the extinction coefficient with spatial correlation radius $R_c \ll R$. Here σ_α^2 = 0.2 km^{-1} is the coefficient of extinction due light scattering by aerosols in the ground layer of the atmosphere, R_0 = 1.2 km is the height of the homogeneous atmosphere for the aerosol, and R_c = 200 m. The results obtained reveal significant differences between the values of ξ and ξ_{eq}. Thus, depending on the value of the equivalent degeneracy parameter, one can use P or MP statistics to describe the unconditional distributions. In addition, the determination of $P(n)$ at large ξ_{eq} requires explicit forms of the probability densities $f(E)$, $f(\beta_\pi)$, and $f7(T)$ over a wide range of altitudes.

REFERENCES

1. G.N. Glasov, *Statistical Problems of Lldar Sensing of the Atmosphere* (Nauka, Novosibirsk, 1987).
2. G.N. Glasov and V.M. Dubyagin, Kvant. Elektron., **6**, No. 11, 2422 (1979).
3. R.A. McClatchey, R.W. Fenn, F.E. Volz et al., *Optical Properties of the Atmosphere*, Report AFCRL-71-0279. Environmental Research Papers, No. 354, 85 (1971).