RECONSTRUCTION OF THE PROFILE OF AN OPTICAL BEAM BY TOMOGRAPHIC METHODS

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This paper presents the results of an investigation into the problem of determination of the energy structure of a light beam based on tomographic methods of processing the data of light scattering experiments.

One approach to noncontact measurements of the energy structure of a light beam can consist in recording the radiation scattered from the beam at different angles with successive reconstruction of the power distribution over the cross-section of the beam by computer methods of the reconstructive tomography (CRT). The possibility of realizing such an approach has been quite thoroughly investigated in Refs. 1–3 on the basis of the radiation transfer equation. But the question of the manner of obtaining the ray sum used in the reconstruction algorithms is yet open. This question is of paramount importance in the problem under discussion because of the extremely low intensity of the scattered radiation and the possible influence of diffraction of the radiation by the receiving aperture.

This paper deals with an investigation into the possibilities of applying CRT methods to the diagnostics of the energetics of the scattering channel under conditions of photoelectric recording using the simplest method of collimation of the received radiation. The reception scheme and the sources of measurement errors are analyzed, based on an emission model which uses a wave approach for the description of the radiation.

Let us represent the scattering channel as a cylinder containing a uniform distribution of particles which are the sources of the secondary radiation. The intensity of the radiation from each such source is assumed to be proportional to the intensity of the incident radiation. The effects of particle screening in the direction perpendicular to the cylinder axis are considered to be negligible, which corresponds to the case of linear interaction of the radiation with the medium and single scattering of the light by the monodispersed ensemble of particles.

Let us consider the simplest scheme of reception in which the beam is collimated using a diaphragm of radius ρ. The center of this diaphragm coincides with the origin of the coordinate system, the x-axis is perpendicular to the diaphragm plane, and the z-axis is parallel to the axis of the investigated channel. The detector measures the intensity of the radiation scattered at the points $\mathbf{r}_1(y_1,\ z_1)$ of the which is at a distance x_1 from the $(y,\ x)$ plane and parallel to it. Let now the $(y_0,\ z_0)$ plane, which is at the distance x_0 from the $(y,\ z)$ plane and parallel to it, be the longitudinal section of the channel and $t(x_0,\ r_0)$ be the spatial distribution of the amplitude of the direct radiation field. Then, at an arbitrary point r_1 of the reception plane the field of radiation scattered from the $(y_0,\ z_0)$ plane may be represented as the superposition integral

$$U(x_0, x_1, r_1) = \frac{i}{\lambda x_1}\int t(x_0, r_0)\sqrt{\kappa(\theta)}\,\frac{P(r)}{x_0}\times$$
$$\times\exp\left\{ik\left[(x_1-x_0)+\frac{1}{2}\frac{(r_1-r)^2}{x_1}-\frac{1}{2}\frac{(r-r_0)^2}{x_1}\right]\right\}dr\,dr_0 \qquad (1)$$

where $\kappa(\theta)$ is the scattering function, $\theta = \theta(x_0,\ r_0,\ r)$ is the angle between the observation direction and the incident wave polarization vector, r_0 and r are two two-dimensional radius vectors in the $(y_0,\ z_0)$ and $(y,\ z)$ planes, respectively, $P(\mathbf{r}) = \mathrm{circ}(r/\rho)$ is the pupil function, and $k = 2\pi/\lambda$, where λ is the wavelength.

The contribution of all scattering centers of the channel to the radiation intensity at the point r_1 can be obtained as an integral of $U(x_0,\ x_1,\ r_1)$ over all cross-sections of the channel.

It is then necessary to determine the conditions under which the received radiation intensity radiation

$$I(x_1, \mathbf{x}_1, \mathbf{r}_1) = \int dx_0 \langle |U(x_0, x_1, \mathbf{r}_1)|^2\rangle$$

can be represented, at least asymptotically, as the Radon transform of the direct radiation intensity $I_0(x_0,\ \mathbf{r}_0) = \langle| t(\mathbf{x}_0,\ \mathbf{r}_0)|^2\rangle$ in the channel.

Let us first consider the case when the diaphragm is in immediate proximity to the investigated channel and the cross-section size d of the investigated channel is sufficiently small so that the following condition

$$\rho^2/\lambda|x_0| \gg 1 \qquad (2)$$

holds for all points lying within the detector field of view. Supposing $\kappa(\theta)$ to be a slowly varying function of r and making the projective transform of the pupil function, one obtains after integration over $\mathbf{r}$:

$$U(x_0, x_1, r_1) = \int t(x_0, r_0)\sqrt{\kappa(\theta)} \times$$

$$\times \frac{\operatorname{circ}\left[\frac{x_1}{\rho(x_1-x_0)}\left|r_0 - r_1\frac{x_0}{x_1}\right|\right]}{x_1 - x_0}$$

$$\times \exp\left\{ik\left[(x_1-x_0)+\frac{(r_1-r_0)^2}{x_1-x_0}\right]\right\}dr_0$$

It is then easy to obtain

$$I(x_1, r_1) = \int I_0(x_0, r_0)\kappa(\theta) \times$$

$$\times \frac{\operatorname{circ}\left[\frac{x_1}{\rho(x_1-x_0)^2}\left|r_0 - r_1\frac{x_0}{x_1}\right|\right]}{(x_1-x_0)^2}\, dx_0\, dr_0$$

if the scattering centers are statistically independent.

It is easy to make out the asymptotic behavior of this integral by making use of the well-known definition of the δ-function:

$$\delta(x, y) = \lim_{N\to\infty}\left[\frac{N^2}{\pi}\operatorname{circ}(Nr)\right].$$

Then, for $\rho/x_1 \to 0$, one obtains the approximate expression

$$I(x_1, r_1) \simeq \frac{S\kappa(\theta)}{x_1^2}\int I_0\left[x_0, r_1\frac{x_0}{x_1}\right]dx_0 \qquad (3)$$

where $S = \pi\rho^2$.

This approximate equality becomes more accurate with decrease of the viewing angle of the detector S/x_1^2. Decrease of the viewing angle is limited in practice by the threshold sensitivity of the detector. Thus, the additive errors associated with the use of formula (3) influence the spatial resolution, whose linear magnitude can be estimated as follows:

$$\Delta \approx 2(d + x_1)\rho/x_1.$$

It should be noted that violation of condition (2), which can occur, in particular, when studying excessively broad channels, will result in substantial distortions of the information. Thus, for example, if the radius ρ coincides for some points (x_0, $\mathbf{r}_0$) of the path with the size of the even Fresnel zones, the contribution from these points to the intensity of the scattered radiation will be less than that from the points for which ρ coincides with the size of the odd Fresnel zones. In this case one cannot obtain a relationship similar to Eq. 3, which obviously demonstrates the inapplicability of the given measurement technique under these conditions.

Let us now consider the case when $\rho^2/\lambda|x_0| \ll 1$ and $\rho^2/\lambda x_1 \ll 1$, which corresponds to the Frauenhofer approximation of this diffraction problem.

Integrating over r in formula (1) and taking into account the statistical independence of the scattering particles in the channel, we obtain

$$I = \frac{S}{x_1^2}\int I_0(x_0, r_0)\kappa(\theta) \times$$

$$\times \left[J_1\left(\frac{2\pi\rho}{\lambda}\left|\frac{r_1}{x_1} - \frac{r_0}{x_0}\right|\right)\right]^2 \Big/ x_0^2\pi\left(\frac{r_1}{x_1} - \frac{r_0}{x_0}\right)^2 dx_0 dr_0,$$

where $J_1(x)$ is the Bessel function of the first kind.

It is easy to show that the function $[J_1(Nr)]^2/\pi r^2$, where $r = \sqrt{x^2 + y^2}$, has all the features of the two-dimensional δ-function as $N \to \infty$. Therefore $I(x_1, \mathbf{r}_1)$ can be represented asymptotically by the linear integral (2) under the conditions $\rho/\lambda \to \infty$.

Since the conditions $\rho^2/\lambda|x_0| \ll 1$ and $\rho/\lambda \to \infty$ cannot be fulfilled simultaneously, the errors of representing $I(x_1, r_1)$ in the form of a ray sum are mainly caused by the diffraction. The linear scale of the spatial -resolution in this case can be estimated as $\Delta \approx 0.61\ \lambda L/\rho$, where L is the maximum distance from the diaphragm to the points of the channel.

In the experiment the radiation sources were LG-75 gas lasers operating in the regime of fundamental mode generation. Two parallel beams separated by a distance comparable to their dimension d at half maximum power (FWHM) were formed by two beam expanders Bind a system of mirrors. The intensity distribution of the scattered radiation (in air) at an angle of 90° was measured by a scanning receiving system. Such a system permits one to obtain projections with a digitization interval of 1 mm, which for observation angles (aspect angles) from 0° to 360° did not exceed 20% of d. The photomultiplier FEU-79 was used as the detector. To improve the signal-to-noise ratio (SNR), the laser radiation was modulated at 1 kHz frequency and the photocathode of the PMT was cooled down to t = –20°C. A collimator 100 mm long and 0.8 mm in diameter was placed in front of the entrance window of the PMT.

The chosen geometry of the experiment ensures the satisfaction of inequality (2) and conditions of reliable recording of the scattered radiation at SNR levels much greater than unity.

Typical projections of the scattered radiation are shown in Fig. 1. Curves 1, 2, and 3 characterize the radiation from the first beam, from the second beam, and from the two beams placed one after the other, respectively, for an observation angle θ of 0°. Curve 4 is the arithmetic sum of curves 1 and 2. Curve 5 represents the radiation from the two beams at θ = 90°. The almost perfect coincidence of the combined curve (3) and the beam scattering arithmetic sum (4) demonstrates the adequacy of the adopted model of interaction between the radiation and the medium. At the same time, the strong dependence of the projections on θ (see curve 1, 2 and 5) makes it necessary to take into account the scattering phase function κ(0).

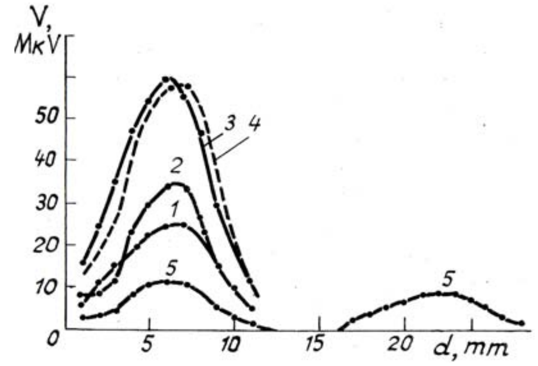

FIG. 1. Experimentally measured projections of the scattered radiation.

These results demonstrate the possibility of using computer-assisted tomographic methods in this application. In the realization of these methods, the light beam cross-section scanning data were inputs to the corresponding computer programs. Taking into account the κ(θ) values and the experimental digitization interval of the projections, the direct Radon transform was calculated, giving to within a proportionality constant the set of projections of the scattered radiation. To determine the degree of adequacy of the calculated projections to the experimental data, the correlation coefficient K_1 was calculated, which for θ = 0° and θ = 90° was greater than 0.99 (the beams were placed side by side and one after another, respectively). This fact permitted us to consider K_1 to be highly significant at a significance level of 1×10^{-3} and to be able to speak of complete agreement between the calculated projections and the experimental data. The reconstruction of the optical beam profile from the calculated projections was made by the Fourier inversion method.[4] The matrix P of discrete readings $P(m \times \Delta S, \theta_n)$ comprised the input data for this method. Here $\theta_n = \Delta\theta \times n$, Δθ is the step of the projection orientation angle, $n = 1, N$, N is the number of such steps, and ΔS is the discretization step in taking the M readings of the distance s along the projections, where $m = \overline{-M/2, M/2-1}$ if M is even.

In the first stage of the reconstruction, based on the fast Fourier transform (FFT) procedure applied to the matrix P, the matrix of values of the spatial frequency spectrum was formed on the polar grid, the elements of which are given by the expression

$$p_1(\mu\cdot\Delta R, \theta_n) = \Delta s \sum_{m=-M/2}^{M/2-1} p(m\Delta s, \theta_n)\ \exp(i2\pi\mu m/M),$$

where $\mu = \overline{1, M}$ and $\Delta R = 1/M\Delta S$ is the spectrum digitization interval.

To ensure the possibility of repeated application of the FFT procedure to obtain the inverse Fourier transform in the second stage of the reconstruction, the interpolation of values from the polar grid to a rectangle grid in the x, y spatial frequency plane was carried out. The elements of the matrix $f_1(u\cdot\Delta X, \mathbf{v}\cdot\Delta Y)$, $u = \overline{-U/2, U/2-1}$, $v = \overline{-V/2, V/2-1}$ were determined as a weighted sum of the quantities $\rho_1(\mu\cdot\Delta R, \theta_n)$ at four adjacent points where ΔX, ΔY were the spectrum digitization intervals, U, V were the dimensions of the spectrum matrix on the rectangle grid, for U and V even. At the final stage of the reconstruction, the elements f of the $L\times K$ matrix of the intensity distribution were given by

$$f(k_1\Delta x, l_1\Delta y) = \Delta X\Delta Y \sum_{u=-U/2}^{U/2-1}\ \sum_{v=-V/2}^{V/2-1} f_1(u\Delta X, v\Delta Y)\times$$
$$\times \exp\{i2\pi[(k_1\Delta x)(u\Delta X)+(l_1\Delta y)(v\Delta Y)]\},$$

where $l_1 = \overline{1, L}$, $k_1 = \overline{1, K}$, and $\Delta x = 1/U\Delta X$ and $\Delta y = 1/V\Delta Y$ were the discretization steps of the representation of f.

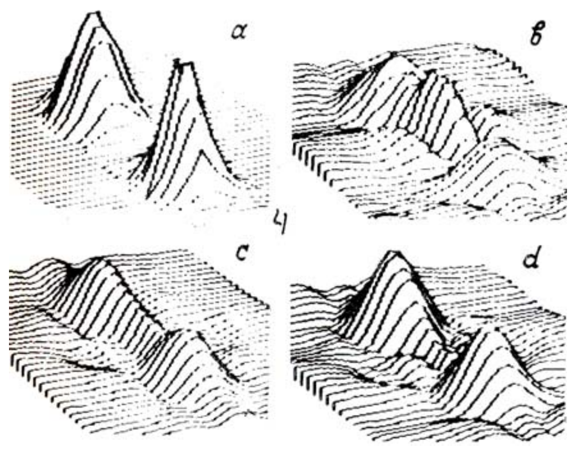

FIG. 2. Initial and reconstructed intensity distributions.

The reconstruction algorithm was realized on the microcomputer "Elektronika-60." Reconstruction time did not exceed 1.5 min for $M = N = 32$. An estimate of the quality of the reconstruction was made based on the value of the correlation coefficient K_2 between the initial and reconstructed distributions. An example of the functioning of the algorithm is presented in Fig. 2, where the initial (a) and the reconstructed intensity distributions (b) for $N = 2$, $M = 32$, $\Delta\theta = 90°$ ($K_2 < 0.5$) are depicted. In addition, Fig. 2c presents reconstruction results ($K_2 > 0.8$) for $N = 4$, $M = 32$, $\Delta\theta = 45°$, and correction of $p(m\cdot\Delta S, \theta_n)$ obtained by multiplying it by the function $f_k = [\kappa(\theta_n)]^{-1}$, and Fig. 2d presents reconstruction results ($K_2 > 0.6$) for $N = 4$, $M = 32$, $\Delta\theta = 45°$ without this correction.

It can be seen that if one allows for $\kappa(\theta)$ and assumes a simple beam shape formed by a superposition of two Gaussians with d_1 comparable to d, the chosen geometry of the experiment ensures satisfactory reconstruction already for $N = 4$.

On the whole, this experimental study demonstrates only the possibility of reconstructing the beam energy structure for sufficiently simple profiles under the conditions of the above-mentioned approximations.

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