IMAGE RECONSTRUCTION FROM INCOMPLETE INFORMATION ON THE SPATIAL SPECTRUM IN A MULTIAPERTURE SYSTEM

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The problem of reconstructing the image of an object in a linear multiple-aperture optical system (MOS) is studied. The information about the spectrum of the object is incomplete owing to the form of the spatial frequency band synthesized by the given MOS and the recording conditions. Expressions are derived for the number of independent measurements of the spectrum of the object (its modulus and phase) that are sufficient for obtaining satisfactory reconstruction of the image with different signal/noise ratios in the starting data.

The problem of processing a series of short-exposure images of an astronomical object usually consists of compensating image distortions owing to the effect of the turbulent atmosphere and detection noise.[1] The processing reduces to estimating from a series of recorded distorted images the modulus and phase of the Fourier spectrum and reconstructing from them the undistorted image.[2] The potential accuracies of such estimates are comparable; in practice, however, one component of the spectrum is often significantly more accurate than another owing to different deviations from optimal conditions of formation and recording of the starting images, and in this case the image is reconstructed from one component only.[3,4]

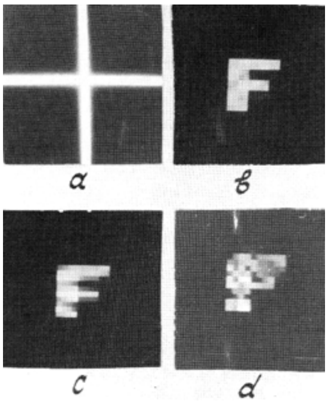

FIG. 1. a – the spatial frequency band in which information about the spectrum or its component is given; b – the image reconstructed from the spectrum given in the region of the "cross"; c – the image reconstructed only from the phase of the spectrum given in the region of the "cross"; d – the initial estimate of the image reconstructed from the modulus of the spectrum, extrapolated from the region of the "cross" to the entire frequency plane.

Multiple-aperture optical systems (MOS), to be used for spatial synthesis of apertures with an equivalent diameter exceeding 10 m,[5,6] have been under active development for the last few years. Because the telescopes comprising the MOS are finite in number and limited in size, information about the Fourier spectrum of the object can be obtained only in separate sections of the frequency band synthesized.[7,8] In this situation the processing problem becomes significantly more complicated: to reconstruct the image the obtained estimates of the spectrum must be extended to the entire band of spatial frequencies. In this paper, as a first step in the solution of this problem, we present the results obtained by modeling the reconstruction process in the case when information about the spectrum (or only about the components of the spectrum) is known in a "cross" (Fig. 1a) the width of whose arms D/λ (λ is the wavelength) is much smaller than the lengths L/λ of the arms. This situation can arise, for example, when a linear MOS, consisting of telescopes with diameter D with a maximum spacing L, is employed for recording the starting images in two mutually perpendicular positions for processing.

It can be shown that the given information about the spectrum determines the dimensions of a square matrix with an image containing the maximum number of independent real readings M_0 (i.e., the number of resolution elements of size λ/L). We studied the problem of evaluating the minimum number of uniform readings of the spectrum M_s which are uniform within the "cross", the modulus of the spectrum M_m, and the phase of the spectrum M_p that are sufficient for stable reconstruction of the image. In the modeling we employed the well-known iteration approach, which has performed well in the solution of amplitude and phase problems.[3,4] In this case the iteration scheme has the form

$$I_{k+1} = \hat{P}_2 \hat{F}^{-1} \hat{P}_1 \hat{F} I_k, \qquad (1)$$

where I_k is the estimate of the image at the k-th iteration; $\hat{F}$ and $\hat{F}^{-1}$ are the direct and inverse Fourier transform operators; the operator $\hat{P}_i$ replaces the values of the spectrum (or its components) in the "cross" by the given values; and, the operator $\hat{P}_2$ sets to zero the estimates of the image at points outside the square matrix as well as points where the condition of positiveness is not satisfied. The letter F, inscribed into a 10×10 square frame, placed at the center of a 32×32 field ($L/\lambda = 32$), was used as the given object in the modeling. The value of D/λ was varied from 1 to 11.

Figures 1b and c show the characteristic images obtained by reconstruction. Our investigations established the following.

1. Satisfactory quality of reconstruction (an error of less than 1%) and rate of convergence (not more than 100 iterations) are obtained with ratios $M_s/M_0 \approx M_p/M_0 \geq 1.5$ and $M_m/M_0 \geq 3$.

2. When white Gaussian noise is added to the given information the process of reconstruction from the spectrum and phase is stable for signal/noise ratios (SNR) equal to 10 and higher and reconstruction from the modulus is stable for SNR ≥ 30.

3. The process of reconstruction from the spectrum or the phase of the spectrum in the "cross" is virtually independent of the starting estimate, for which the following were chosen: a) a random field with uniform distribution of values from 0 to 1; b) an image corresponding to the Fourier spectrum given in the "cross" and equal to zero outside with superposed noise (the SNR was varied from 1 to 100). It was found that the convergence of the algorithm for reconstruction from the modulus with an initial estimate (a) is unsatisfactory if noise is present in the data.

Additional investigations showed that the convergence of the algorithm of reconstruction from the modulus can be improved substantially if the result of reconstruction of the image from the full, though noisy, Fourier modulus is used as the starting estimate.[3,9] To extrapolate the values of the modulus from the "cross" to the entire Fourier region it is best to employ an iteration algorithm, whose scheme can be written, analogously to (1), in the form

$$A_{k+1} = \hat{P}_4 \hat{F}^{-1} \hat{P}_3 \hat{F} A_k. \qquad (2)$$

Here A_k is an estimate of the autocorrelation of the image, the operator $\hat{P}_3$ replaces the square of the modulus in the "cross" by the given modulus, and the operator $\hat{P}_4$ sets to zero the autocorrelation outside the square frame (the autocorrelation frame is defined as twice the image frame). Investigation of this algorithm showed the following: 1) the error in the extrapolation of the noise-free modulus after 30÷50 iterations is equal to 1÷2% and 2) stable extrapolation is possible if $M_m/M_0 \geq 3$ and SNR ≥ 30. When these conditions are not satisfied the algorithm still works but strong distortions of the modulus appear in the high-frequency region, and this increases the error in the subsequent reconstruction of the image. The autocorrelation obtained with the inverse Fourier transform of the squared modulus of the spectrum with SNR ≥ 30, given in the "cross" and equal to zero outside it, was used as the starting estimate for the iterative extrapolation procedure. The starting estimate reconstructed from the modulus obtained with the help of extrapolation is presented in Fig. 1d.

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