## METHOD FOR DETERMINING THE OPTICAL THICKNESS OF THE ATMOSPHERE FROM MULTIANGLE LASER SOUNDING DATA

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A method is proposed for interpreting multiangle laser sounding data for the problem of determining the optical thickness and the attenuation coefficient of the atmosphere. The method is based on a regularizing algorithm for processing experimental information. The efficiency of the algorithm is studied in a numerical experiment. The results of the processing of the experimental data are presented.

In the last few years a number of papers on the practical application of the method of two-angle laser sounding and its modifications for determining the optical thickness and the aerosol scattering coefficients for the IR region of the spectrum have been published in foreign journals.<sup>1</sup>

In this sounding scheme, however, the angle between the directions of sounding must be  $60^{\circ}$  in order to determine the optical thickness of the atmosphere reliably. This, in its turn, obviously contradicts the assumption on which the method of two-angle sounding is based, namely, that the vertical layers of the atmosphere are horizontally homogeneous.<sup>1,2</sup>

In this paper it is suggested that the multiangle method of sounding combined with the regularizing algorithm of the logarithmic derivative method (LDM) be employed to determine the optical thickness of the atmosphere or the bottom boundary of dense cloud formations.<sup>3</sup>

We shall write the lidar equation, under the assumption that the atmosphere can be divided into horizontally homogeneous layers, in the form

$$S(R_{\mathbf{k}}, \mathbf{x}) = \beta_{\pi}(R_{\mathbf{k}}) \exp(-2\mathbf{x} \cdot \tau(R_{\mathbf{k}}), \qquad (1)$$

where  $S(R_k, x) = U(R_k, x) \cdot R_k^2 \cdot x^2 / P_0$ ,  $U(R_k, x)$  is the amplitude of the lidar signals reflected at different angles  $\gamma$  from the layer  $\Delta R_k = R_{k+1} - R_k$ , located at a height  $R_k$ ;  $x = \sec \gamma$ ;  $P_0$  is the instrumental constant of the lidar;  $\beta_{\text{ext}}(R_k)$ ,  $\beta_{\pi}(R_k)$  are the attenuation and backscattering coefficients in the layer  $\Delta R$ ; and,  $r(R_k) = \int_{k}^{R_k} R_k (R_k) dR_k$  is the optical thickness of the

 $\tau(R_{\rm k}) = \int_{0}^{R_{\rm k}} \beta_{\rm ext}(R) dR$  is the optical thickness of the

atmosphere in a direction toward the zenith. The following expression for determining  $\tau(R_k)$ 

can be derived from Eq. (1):

$$\tau(R_{\mathbf{k}}) = -0.5 d \ln S(R_{\mathbf{k}}, \mathbf{x}) / d\mathbf{x}.$$
<sup>(2)</sup>

Thus the problem of determining the optical thickness of the atmosphere reduces to determining the logarithmic derivative with respect to  $x = \sec\gamma$  of the signal  $S(R_k, x)$ , obtained from the layer  $\Delta R_k$  by sounding at different angles. It is well known<sup>4</sup> that the problem of differentiating empirical functions (lidar signals) containing random errors is an improperly posed problem and requires the use of regularizing algorithms for processing experimental data. To this end one of these authors previously developed a regularizing algorithm for the LDM.<sup>3</sup> According to Ref. 3 the information sought from  $\tau(R_k)$  is extracted from the following expression:

$$\tau_{\alpha}(R_{k}) = -0.5 d \ln S^{(a)}(R_{k}, x) / dx,$$
 (3)

where  $S^{(\alpha)}(R_k, x) = S(R_k, x_0) + \int_{x_0}^x \varphi'_{\alpha}(R_k, x) dx$ , and

 $\varphi'_{\alpha}(R_{k}, x) = d \ln S^{(\alpha)}(R_{k}, x) / dx$  is the regularized value of the logarithmic derivative of the signal with respect to  $x = \sec \gamma$  for a chosen value of the parameter  $\alpha$ . The process of transferring from  $S_{k}(R_{k}, x)/dx$  to  $S^{(\alpha)}(R_{k}, x)$  must in its turn be understood as a procedure for filtering and separating the regular (differentiable) component in the lidar signals received against the background of random noise.<sup>4</sup> According to Ref. 5 the numerical differentiation problem can be reduced to the solution of the Fredholm integral equation of the first kind for the derivative sought by Tikhonov's smoothing functional method.

We note further that in the case of laser sounding it is of great interest to determine the optical thickness of the lower boundary of dense cloud formations or the optical thickness of the entire cloud. This problem can be solved either by successive (layerwise) determination using the computational scheme given by Eq. (3) of the LDM or with the help of a modification of the LDM proposed in Ref. (6) and making it possible to use, in constructing the smoothing functional, a priori data on the solutions sought in the form of a given model. In this case it is not the profile itself that' is determined, but rather its deviation from the a priori model under study. For our problem of determining  $\Delta \tau(R_{k+1}, R_k)$  this approach results in the following working relation:

$$\Delta \tau(R_{k}, R_{k+1}) = -0.5 d \ln(S^{(\alpha)}(R_{k+1}, x) / S_{mod}(R_{k}, x)) / dx,$$
(4)

where the value of  $S^{(\alpha)}(R_k)$  reconstructed by the scheme (3) or from independent measurements plays the role of the a priori profile  $S_{mod}(R_k)$ . In addition, the computational scheme (4) also permits determining the optical thickness of the entire cloud in the case when the lidar can record the signal arriving from the top boundary. In so doing it may be assumed that the contribution of multiple scattering to this signal will not depend significantly on the sounding angle and therefore will not affect the value of the derivative of the signal with respect to the angle.

To study the efficiency of the algorithm described above for reconstructing  $\tau(R_k)$  we performed a closed numerical experiment on layer-by-layer determination of the optical thickness for a model of a cloud with homogeneous layers, as shown in Fig. 1. The experimental procedure is described in detail in Ref. 5. We note that the degree of horizontal inhomogeneity of the cloud was modeled with the help of a random number generator by varying the backscattering coefficient  $\beta_{\pi}(R_k, x)$  in a range of 20–30%.



FIG. 1. The optical model of a cloud consisting of homogeneous layers.

The quantity 
$$\overline{\tau}(R_k) = \sum_{l=1}^n \tau_{\alpha}(R_k, x_l)$$
 was

determined by the scheme (3) of the LDM with n = 6and a uniform grid of angles. The maximum deviation of a ray from the vertical was equal to 15 and 30°. The corresponding results of the reconstruction of  $\overline{\tau}(R_k)$ for  $\Delta \gamma = 15^\circ$  and 30° are presented in Fig. 2. The figure also shows, for comparison, the values of  $\tau(R_k)$ determined by the two-angle method. The numerical results show that the proposed approach makes it possible to determine stably the optical thickness of the K.I. Gobrusenko and B.P. Ivanenko



The algorithm developed was tested on real laser sounding data for cloud formations. The sounding was performed in the region of the village of Katsiveli in the Crimea during an expedition in October of 1988 in which the authors directly participated.

Figure shows the results of the determination of the optical thickness and the attenuation coefficient of a cloud from multiangle sounding data. The quantity  $\bar{\tau}_{\alpha}(R_{\rm k})$  was determined using four angles and the computational scheme (4) of the LDM and the attenuation coefficient  $\beta_{\rm ext}(R_{\rm k})$  was determined by subsequent differentiation of  $\tau_{\alpha}(R_{\rm k})$  by the method of integral equations.<sup>5</sup>



FIG. 3. The results of processing of experimental data obtained by multiangle laser sounding of a cloud:  $\blacktriangleright$  – the values of  $\tau_{\alpha}(R_k)$  determined by the scheme (4) of the LDM with  $\Delta \gamma = 15^{\circ}$  and n = 4; — – the values of  $\beta_{\text{ext}}(R_k)$  obtained by numerical differentiation of  $\tau_{\alpha}(R_k)$ .

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The calculations were performed on a DVK-3 microcomputer using the IKAR program complex which we developed for interpreting laser sounding data obtained with single-frequency, multifrequency, and multi angle sounding of the atmosphere.

In conclusion we note that the proposed algorithm is quite efficient and noise-resistant, and it can be used in systems for real-time processing of lidar signals.

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