PASSAGE OF A SIGNAL THROUGH A CLOUD LAYER TO A REMOTE DETECTOR

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Computational relationships are obtained for estimating the effect of cloud cover upon the intensity of a signal emitted by a surface source and detectedat high altitude by a sensor with a wide field of view. The treatment follows the narrow-angle approximation of radiation transfer theory applied to a beam of radiation with Gaussian intensity distribution in both the angular and lateral spatial coordinates. The model of a homogeneous scattering layer, its scattering phase function approximated by a Gaussian, is used.

The possibility of estimating the parameters of the cloud layer from the magnitude of the distortions introduced by it in the spectral distribution of the sounding pulse emitted by a flash-lamp, using the relationships from the present study, is discussed.

Earlier studies[1–3] have considered the passage through a uniform cloud layer of a beam of optical radiation with Gaussian intensity distribution in both the angular and spatial coordinates in the plane of the source $z = 0$:

$$I_0(\Phi,\ \vec{r},\ z = 0) = \frac{\beta^2\gamma^2}{\pi^2}\exp\left[-\ \beta^2\Phi^2 - \gamma^2r^2\right].$$

$$\Phi = (\sin\theta\cos\varphi,\ \sin\theta\sin\varphi),\ \vec{r} = (x,y). \qquad (1)$$

Analytical expressions were obtained for the beam characteristics upon exit from a layer, whose scattering and extinction coefficients are described by the following function:

$$\sigma(z) = \begin{cases} \sigma, & z \le 0 \le d, \\ 0, & z > d, \end{cases} \qquad (2)$$

and the scattering phase function of the medium is approximated by the Gaussian function:

$$P(\Phi) = 4\alpha^2\exp(-\ \alpha^2\Phi^2),\ \Phi = \sin\theta \simeq \theta \qquad (3)$$

with small variance

$$1/\alpha^2 \ll 1 \qquad (4)$$

In such a model the signal power recorded by a detector with wide field of view at a distance $z > d$ from the source is given by Eq. 3

$$P(R,\ z) = \frac{R\exp(-\tau)}{\Lambda}\int\limits_0^\infty du\ \exp(-u^2)J_1(Ru/\Lambda)\exp[\Omega(z)], \qquad (5)$$

$$\Omega(z) = \frac{\sqrt{\pi}\tau_s\alpha\Lambda}{ud}\left\{\mathrm{erf}\left[\frac{uz}{2\alpha\Lambda}\right] - \mathrm{erf}\left[\frac{u(z-d)}{2\alpha\Lambda}\right]\right\}, \qquad (6)$$

were $P(R,\ z)$ is the power recorded by the detector; $\tau = \sigma_{ex}d$ and $\tau_s = \sigma_{sc}d$ are the extinction and scattering optical thickness of layer, respectively;

$$\Lambda = \frac{z}{2\beta}\left[1 + \beta^2/\gamma^2z^2\right]^{1/2} \qquad (7)$$

is a parameter corresponding to the beam width at the distance z from the source in the absence of the perturbing action of clouds; $1/\beta$ is the angular size of the beam; $1/\gamma$ is the linear size of the beam in the plane $z = 0$; R is the radius of the detector iris; $J_1(\xi)$ and $\mathrm{erf}(\xi)$ are the Bessel function and the probability integral, respectively, of the argument ξ. The factor $\exp[\Omega(z)]$ describes the effects of multiple scattering.

Equations (5) and (6) are valid in the narrow-angle approximation for random media with highly anisotropic scattering.[4,5] Within the framework of the considered model these follow from the general equations

$$I(\xi,\ \zeta,\ z) = \exp(-\tau)\ I_0(\xi + z\zeta,\ \zeta,\ 0)\ \exp[\Omega(z)];$$

$$\Omega(z) = \int\limits_0^z dz'\sigma_{sc}(z')\ p[\xi - (z - z')\zeta];$$

$$\tau = \int\limits_0^z dz'\ \sigma_{ex}(z'),$$

which in the narrow-angle approximation, relate the spatial spectra of the perturbed (I) and unperturbed

(I_0) fields.[5] Bellow, we will analyze these equations for a case of practical importance in which the detector is located at a very remote distance from the scattering layer. For this case computational relationships suitable for practical use in evaluating the influence of clouds on the value of recorded signal will be derived from Eqs. (5) and (6). We will consider the detector removed "to infinity".

2. The condition of removal "to infinity" is equivalent to satisfying the inequality

$$\Lambda \gg d/\alpha, \quad (8)$$

which allows us, when calculating the parameter $\Omega(z)$ from Eq. (6), to consider

$$ud/2\alpha\Lambda \ll 1 \quad (9)$$

In going from Eq. (8) to Eq. (9) we have taken into account the fact that the main contribution to the integral in Eq. (5) comes from low values of u. The error of approximating the value of $\Omega(z)$ at high values of u is negligible when estimating $P(R, z)$.

We shall use the Taylor series expansion of the error integral erf $[u(z - d)/2\alpha\Lambda]$

$$\mathrm{erf}\,[u(z-d)/2\alpha\Lambda] = \mathrm{erf}(uz/2\alpha\Lambda) - \frac{ud}{\sqrt{\pi}\alpha\Lambda}\exp\left[-\left(\frac{uz}{2\alpha\Lambda}\right)^2\right] \quad (10)$$

Retaining only the first two terms of the expansion (recall inequality (9)), we obtain from Eq. (6)

$$\Omega(z) = \tau_s \exp(-u^2a^2);$$

$$a^2 = z/2\alpha\Lambda = (\beta/\alpha)\ (1 + \beta^2/\gamma^2z^2)^{1/2} \simeq \beta/\alpha \quad (11)$$

Then

$$P(R,\ z) = 2\beta\frac{R}{z}\exp(-\tau)\int_0^\infty du\ e^{-u^2}J_1(2\beta\ u\ R/z) \times$$

$$\times \exp\left[\tau_s e^{-u^2a^2}\right] \simeq \beta^2\frac{R^2}{z^2}\exp(-\tau)\int_0^\upsilon d\upsilon\ e^{-\upsilon}\exp\left[\tau_s e^{-\upsilon a^2}\right]. \quad (12)$$

The second equality in Eq. (12) follows from the first on the condition that the value $\beta\ u\ R/z$ remains small:

$$J_1(2\beta uR/z) \simeq \beta\ uR/z.$$

Equation (12) is basic to all the calculations for the remote detector. From it simple computational relationships are derived.

Integral (12) is exactly calculated by expanding the exponent exp $\left[\tau_s e^{-u^2a^2}\right]$ into a power series and then integrating thus obtained series term by term,

$$e^{-u^2}\exp\left[\tau_s e^{-u^2a^2}\right] = \sum_{k=0}^{\infty}\frac{\tau_s^k}{k!}\exp\left[-u^2(1 + ka^2)\right]$$

using the tabulated relations.[6]

$$\int_0^\infty d\xi J_1(\xi)\ \exp(-\xi^2/q^2) = \frac{\sqrt{\pi}}{2}\,q\,\exp(-q^2/8)\times$$

$$\times I_{1/2}(q^2/8) = 1 - \exp(-q^2/4).$$

The solution is then written in the form

$$P(R,z) = \exp(-\tau)\sum_{k=0}^{\infty}\frac{\tau_s^k}{k!}\left[1 - \exp\left[-\beta^2\frac{R^2}{z^2}\frac{1}{ka^2+1}\right]\right] \simeq$$

$$\simeq \beta^2\frac{R^2}{z^2}\exp(-\tau)\sum_{k=0}^{\infty}\frac{\tau_s^k}{k!}\frac{1}{ka^2+1}. \quad (13)$$

For $\tau_s \ll 1$, to first order in the small parameter τ_s the above expression yields:

$$P(R,\ z) \simeq \beta^2\frac{R^2}{z^2}\exp(-\tau)\left[1 + \frac{\tau_s}{1 + a^2}\right] \simeq$$

$$\simeq \beta^2\frac{R^2}{z^2}\exp(-\tau_a)\left[1 - \tau_s a^2/(1 + a^2)\right], \quad (14)$$

where

$$\tau_a = \tau - \tau_s \quad (15)$$

is the optical thickness of the layer due to inelastic scattering. In the visible spectrum absorption by the water droplets is small and the value τ_a can be neglected, assuming

$$\exp(-\tau_a) \simeq 1$$

The series in Eq. (13) is then convolved and for $\beta = \alpha$ the solution can be represented in closed form:

$$P(R,\ z) = \beta^2\frac{R^2}{z^2}\exp(-\tau_a)\frac{1 - \exp(-\tau_s)}{\tau_s}, \quad (16)$$

The solution (16) in this case can be obtained directly from Eq. (12), writing the integral contained therein in the form

$$\int_0^\infty d\upsilon\ e^{-\upsilon}\exp\left(\tau_s e^{-\upsilon}\right) = \int_0^1 d\xi\exp\left(\tau_s\xi\right) = \frac{\exp(\tau_s) - 1}{\tau_s}.$$

Equation (16) remains approximately valid even when β differs from α, but by not too large a value:

$$\left|\frac{\beta-\alpha}{\beta}\right| \ll \tau_s/e, \quad (17)$$

where e is the Napierian base. If $\tau_s \geq 5$, then this inequality is satisfied for practically all narrow beams with divergence $1/\beta$ comparable to or less than the width of the scattering phase function $1/\alpha$.

For beams with wider directional diagrams, i.e., when $\beta < \alpha$, another approximation may be used. Expanding the exponent $\exp(-\upsilon a^2)$ in the power series:

$$\exp(-\upsilon a^2) = 1 - \upsilon a^2 + \upsilon^2 a^4/2 - ..$$

from Eq. (12) we obtain to first order in the small parameter $a^2 \simeq \beta^2/\alpha^2$

$$P(R, z) = \beta^2 \frac{R^2}{z^2} \exp(-\tau_a) \frac{1}{1 + \tau_s \beta^2/\alpha^2} . \tag{18}$$

All the above approximations of the exact solution (13) are encompassed by the equation:

$$P(R, z) = \beta^2 \frac{R^2}{z^2} \exp(-\tau_a) \frac{1}{1 + \tau_s p_1} , \tag{19}$$

in which

$$p_1 = a^2 / (1 + a^2). \tag{20}$$

The parameter p_1 is defined by Eq. (20) so that Eq. (19) will be applicable to beams with arbitrary directional diagrams. For small values of τ_s it coincides with Eq. (16) or (18), depending on the value of a^2.

A more exact expression for $P(R, z)$ can be had by retaining the second-order term $\upsilon^2 a^4/2$ in the expression of the exponent $\exp(-\upsilon a^2)$. In this case

$$\int_0^\infty d\upsilon\, e^{-\upsilon} \exp\left[\tau_s e^{-\upsilon a^2}\right] \simeq e^{\tau_s} \int_0^\infty d\upsilon\, e^{-\upsilon\left(1 + \tau_s a^2\right)} +$$

$$+ \upsilon^2 \tau_s a^4/2 = \frac{1}{a^2\sqrt{\tau_s/2}} e^{\tau_s} F(V),$$

and Eq. (12) may be written in the form

$$P(R, z) = \beta^2 \frac{R^2}{z^2} \exp(-\tau_a) \frac{1}{a^2\sqrt{\tau_s/2}} F(V), \tag{21}$$

where

$$F(V) = \exp(-V^2) \int_0^V dt\ t^2$$

is Dawson's integral of the argument

$$V = \frac{1}{a^2\sqrt{\tau_s/2}} (1 + \tau_s a^2). \tag{22}$$

Using the asymptotic approximation[6]

$$F(V) \simeq \frac{1}{2V} (1 + 1/2V^2 + \ldots),$$

we obtain from Eq. (22)

$$P(R, z) = \beta^2 \frac{R^2}{z^2} \exp(-\tau_a) \frac{1}{1 + \tau_s a^2} \times$$

$$\times \left[1 + \frac{\tau_s a^4}{(1 + \tau_s a^2)^2}\right]. \tag{23}$$

For large τ_s the correction to Eq. (18) is not significant. For small values of $\tau_s a^4$, Eq. (19) follows from Eq. (23) to the accuracy of the first-order terms in $\tau_s a^4$ and the second-order terms in a^2.

3. Equation (19) is a calculational approximation suitable for practical applications when the condition of detector remoteness (8) is satisfied and the narrow-angle approximation of radiative transfer theory is justified within the context of the model developed in Ref. 3. Specifically, it can be used to estimate the parameters of the cloud layer τ_s and α^2 from the results of high-altitude measurements of signals emitted by a surface flash-lamp with know directional diagram and beam spectral energy distribution. The layer parameters are then determined from the distortions induced along the beam path in the spectral composition of radiat ion.

Let us consider the case in which the | distribution of radiation intensity at the bottor of the layer, described by Eq. (1), is generated by an isotopic surface source of small dimensions. In this case the angular distribution of energy in the light spot at the bottom of the layer is described by the expression[7]

$$I_0(\theta) \sim \cos^3 \theta$$

Approximating this expression by the Gaussian| function

$$I_0(\theta) = \exp(-2\,\theta^2), \tag{24}$$

we obtain $\beta^2 \simeq 2$. At distances z considerable larger that the altitude h of the bottom of the cloud layer, the value $\beta^2/\gamma^2 z^2$ can be neglected it comparison with unity, since $1/\gamma$ is of the order of h. The computational formulas for the considered case can be represented in the form

$$P(R, z, \lambda) = AE(\lambda) \frac{1}{1 + \tau_s p_1(\lambda)},$$

$$p_1(\lambda) = \frac{\beta^2}{\beta^2 + \alpha^2(\lambda)} \simeq \frac{2}{2 + \alpha^2(\lambda)}; \tag{25}$$

where $E(\lambda)$ and $P(\lambda)$ are the spectral densities of the radiation in the plane $z = 0$, at the detector entrance aperture; A is a constant that is independent of wavelength. Assuming certain model ideas about the dependence of α^2 on λ, e.g. setting[8]

$$\alpha^2 \sim 1/\lambda^2,$$

We can determine ts and a2 from a set of measured values $P(\lambda_1)$, $i = 1, 2, \ldots$ n, using Eq. (25), $n \geq 3$.

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