

## ON THE LIDAR RECEIVED POWER IN SOUNDING A FOAM-COVERED SEA SURFACE THROUGH THE ATMOSPHERE

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*Received July 24, 1989*

*The power received by a lidar in sounding the sea surface, partially covered with foam, with a narrow laser beam though the atmosphere is studied. An analytical expression is derived for the average power recorded by the receiver through a transparent aerosol atmosphere and an optically dense aerosol atmosphere.*

*It is shown the roughness of the sea surface and the existence of the foam substantially affect the echo signal power.*

A promised method for sounding the ocean is laser sounding. Since laser methods are in direct they, do not give directly the characteristics of the sea surface or the optical characteristics of the sea water. The values of the latter are related in a complicated way with the parameters of the received signal. One factor determining the lidar signal is the presence of foam on the sea surface.

The power of the signal received by a lidar when sounding a foam-free sea surface has been studied in a number of works (see, for example, Refs. 1–3). In what follows we study the signal power received by a lidar in sounding a foam-covered sea surface through the atmosphere.

We assume that the source and receiver are separated and are located at distances  $L_s$  and  $L_r$  from the surface being sounded, and their optical axes make angles of  $\theta_s$  and  $\theta_r$  with the normal to the undisturbed (flat) sea surface. We assume that the wavelength of the radiation lies in the IR range (where the absorption by water is high) and is small compared with the radii of curvature and the heights of the irregularities of the sea surface.

Since in the process of scattering by a randomly uneven surface the reflected field at each point acquires a large random phase shift<sup>4</sup> the average (over an ensemble of surfaces) radiation power  $P$  received when sounding a partially foam-covered sea surface will be determined by the average radiation power received when sounding a sea surface without foam  $P_s$  and when sounding a sea surface continuously covered with foam  $P_f$ :

$$P = (1-S)P_s + P_f, \quad (1)$$

where  $S$  is the relative fraction of the sea surface covered with foam and whitecaps.

The sections of foam are usually regarded as isotropic reflectors.<sup>5–8</sup> We shall also assume that the

sections of foam are isotropic (lambertian) reflectors, but we shall take into account the fact that the sections of foam lie on the slopes of the waves.<sup>8,9</sup> In addition, for the average wind velocity the spots of foam are almost parallel to the wave slopes, so that it can be assumed that the distribution of the slopes of the foam spots is the same the distribution of the wave slopes.<sup>8</sup>

We shall use two models for a sea surface continuously covered with foam: the model of a randomly uneven surface with a locally lambertian scattering phase function of elementary sections and the model of a flat lambertian surface.

The power  $P_f$  is known for the model of a flat lambertian surface.<sup>10</sup> We shall find  $P_f$  for the model of a randomly uneven surface with elementary sections with locally lambertian scattering phase function.

We write, analogously to Ref. 11, the expression for the power records by the lidar on sounding a randomly uneven locally lambertian surface  $S$  (we assume that the sounding angles  $\theta_s$  and  $\theta_r$  are small enough, so that the shading of some elements of the surface by other elements can be neglected):

$$P_f = \frac{A}{\pi} \int_S dR E(\mathbf{R}) E_r(\mathbf{R}), \quad (2)$$

where  $E(\mathbf{R})$  and  $E_r(\mathbf{R})$  are the illumination of the surface  $S$  at the point  $\mathbf{R}$  in the atmosphere by real and fictitious sources (with the parameters of the receiver), respectively, and  $A$  is the albedo of an elementary section of the surface covered with foam.

Transferring in Eq. (2) from integration over a randomly uneven surface  $S$  to integration over a surface  $S_0$  (the project in of  $S$  on the plane  $z = 0$ ), averaging  $P_r$  over an ensemble of surfaces (by a method analogous to Ref. 4), and using the expressions for the illuminations by real and fictitious sources in the atmosphere<sup>10</sup>, we obtain the following expression for

the average power received by the lidar for a narrow beam illuminating the surface while sounding a randomly nonuniform locally lambertian surface in an aerosol atmosphere (for simplicity we assume that the source and receiver lie in the same XOZ plane):

$$P_f \approx \frac{A\alpha_s \alpha_r}{L_s^2 L_r^2} (C_s + C_r)^{-1/2} (C_s \cos^2 \vartheta_s + C_r \cos^2 \vartheta_r)^{-1/2} \omega Q, \tag{3}$$

where

$$Q = \frac{a \exp(1/2a)}{4(\gamma_x^2 \gamma_y^2)^{1/2}} \sum_{k=0}^{\infty} \frac{a^{-k}}{k!} \left[ \frac{\beta}{2} \right]^{2k} \left\{ \sin \vartheta_r \sin \vartheta_s a^{1/4} \times \right.$$

$$\times \frac{\Gamma(2k+2)}{\Gamma(k+1)} W_{-k-0.75, k+0.75}^{(1/a)} - \sin \vartheta_s \sin \vartheta_r a^{-1/4} \times$$

$$\times \frac{\Gamma(2k+3)}{\Gamma(k+2)} \left[ \frac{\beta}{2} \right] W_{-k-1.25, k+1.25}^{(1/a)} + 2 \cos \vartheta_s \cos \vartheta_r \times$$

$$\left. \times a^{-1/4} \frac{\Gamma(2k+1)}{\Gamma(k+1)} W_{-k-0.25, k+0.25}^{(1/a)} \right\};$$

$$\omega = (1 + x^{-2})^{-1/2}; \quad a = 4 \left[ \frac{1}{\gamma_x^2} + \frac{1}{\gamma_y^2} \right]^{-1}; \quad \beta = \frac{\Delta a}{2};$$

$$\Delta = \frac{1}{2\gamma_x^2} - \frac{1}{2\gamma_y^2}; \quad x = \left[ \sin^2 \vartheta_s C_s + \sin^2 \vartheta_r C_r - \frac{(C_s \sin \vartheta_s \cos \vartheta_s + C_r \sin \vartheta_r \cos \vartheta_r)^2}{C_s \cos^2 \vartheta_s + C_r \cos^2 \vartheta_r} \right]^{-1/2} \frac{1}{\sqrt{2}\sigma}.$$

For a transparent aerosol atmosphere<sup>10</sup>

$$\alpha_s = \frac{P_0 e^{-\tau_1}}{\pi \alpha_s^2}; \quad \alpha_r = \pi r_r^2 e^{-\tau_2};$$

$$C_s = (\alpha_s L_s)^{-2}; \quad C_r = (\alpha_r L_r)^{-2}; \quad \tau_{1,2} = \int_0^{L_{s,r}} \sigma(z) dz.$$

In an optically dense atmosphere we have the following estimates for  $\alpha_s$ ,  $\alpha_r$ ,  $C_s$ , and  $C_r$ :<sup>10</sup>

$$\alpha_s \approx \frac{P_0 \exp\left\{-\int_0^L (1-\lambda)\epsilon(z) dz\right\} C_s}{\pi L_s^{-2}};$$

$$\alpha_r \approx \frac{\pi r_r^2 \exp\left\{-\int_0^L (1-\lambda)\epsilon(z) dz\right\} C_r}{L_r^{-2} \alpha_r^{-2}};$$

$$C_{s,r} = [\alpha_{s,r}^2 L_{s,r}^2 + \mu_{s,r} L_{s,r}^2]^{-1};$$

$$\mu_{s,r} = L_{s,r}^{-2} \int_0^{L_{s,r}} \langle \tilde{\sigma}(z) \rangle \langle \gamma^2(z) \rangle (L_{s,r}^2 - z) dz; \quad \lambda = \frac{\tilde{\sigma}}{\epsilon},$$

where  $P_0$  is the power emitted by the laser source;  $\sigma^2$  and  $\gamma_{xy}^2$  are the variance of the heights and slopes of the sea surface, respectively;  $r_r$  is the effective radius of the receiving aperture;  $2a_s$  and  $2a_r$  are the angle of divergence of the laser radiation and the angle of view of the receiver;  $\epsilon(z)$  and  $\sigma(z)$  are the attenuation and scattering coefficients of the medium;  $\langle \gamma^2(z) \rangle$  is the variance of the deflection of the beam in an elementary scattering act;  $\sigma(z)$  is the effective scattering index;  $\sigma = (1 - x_0)$ ;  $x_0$  is the isotropic part of the scattering phase function;<sup>10</sup>  $W_{n,m}(z)$  is the Whittaker function; and,  $\Gamma(k)$  is the gamma function.

The formula (3) was derived in the approximation  $\beta \ll 1$ , which holds well for a wide range of conditions of wind-driven sea waves.

In the limiting case of an isotropic randomly uneven surface ( $\gamma_x^2 = \gamma_y^2 = \gamma_0^2$ ) the formula (3) is identical to the expression defined in Ref. 11.

We shall estimate quantitatively the effect of the foam on the power received by the lidar. For the quantity  $N$  (equal to the ratio of the power received by the lidar on sounding a sea surface partially covered with foam to the power received by the lidar on sounding a sea surface with the same driving wind velocity but no foam) we obtain from the formulas (1) and (3), taking into account the results of Ref. 3, which were obtained for the power recorded by the lidar on sounding a sea surface without foam,

$$N = (1-S) + S \frac{A16}{V^2(1+K)^2} \exp\left\{\frac{K}{\gamma_x^2}\right\} \left[ \sin \vartheta_s \sin \vartheta_r 2J^{-2} - \sin \vartheta_s \sin \vartheta_r 2\beta J^{-2} + \cos \vartheta_s \cos \vartheta_r J^{-1} - \cos \vartheta_s \cos \vartheta_r 2J^{-2} \right], \tag{4}$$

where

$$J = \frac{1}{\gamma_x^2} + \frac{1}{\gamma_y^2}; \quad \beta = \left( \frac{2}{\gamma_x^2} - \frac{1}{\gamma_y^2} \right) \frac{1}{J};$$

$$K = \left( \frac{\sin \vartheta_s + \sin \vartheta_r}{\cos \vartheta_s + \cos \vartheta_r} \right)^2;$$

and  $V^2$  is the Fresnel coefficient of a sea surface without foam.

For the model of foam in the form of a flat lambertian surface, using the results of Refs. 3 and 10 we have

$$N = (1-S) + S \frac{AB(\overline{\gamma_x^2} \overline{\gamma_y^2})^{1/2}}{V^2(1+K)^2} \exp\left[\frac{K}{2\overline{\gamma_x^2}}\right] \cos\theta_s \cos\theta_r \times \left[1 + 2\sigma^2 C_r \frac{\sin^2(\theta_s - \theta_r)}{\cos^2\theta_s}\right]^{1/2} \quad (5)$$

The formulas (4) and (5) were derived for a narrow beam illuminating the surface;  $\alpha_s \ll \alpha_r$ ,  $\alpha_s^2 \ll \gamma_x^2$ .

Figures 1 and 2 show the results of calculations of  $N$  for  $V^2 = 0.02$ ,  $A = 0.5$ , and  $C_r = 0.02$ . The calculations were performed using the formulas (4) (solid lines) and (5) (dashed lines).

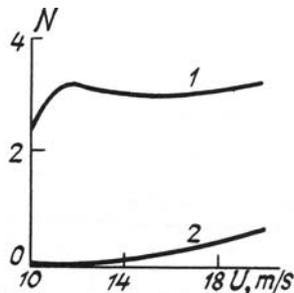


FIG. 1.  $N$  versus the velocity of the driving wind for monostatic sounding: 1)  $\theta_s = \theta_r = 30^\circ$ ; 2)  $\theta_s = \theta_r = 0$ .

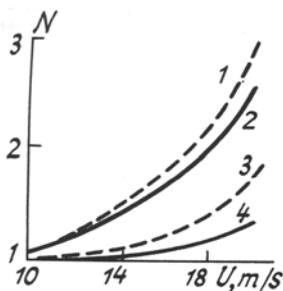


FIG. 2.  $N$  versus the velocity of the driving wind for bistatic sounding: 1) calculation using the formula (4); 2) calculation using the formula 5) with  $\theta_s = 0$  and  $\theta_r = 30^\circ$ ; 3) calculation using the formula (4); 4) calculation using the formula (5) with  $\theta_s = 45^\circ$  and  $\theta_r = -45^\circ$ .

The quantities  $\overline{\gamma_x^2}$ ,  $\overline{\gamma_y^2}$  were found using the formulas of Cox and Munk;<sup>12</sup>  $S$  and  $\sigma$  were calculated using the following expressions:<sup>9,13</sup>

$$S = 0.09 U^3 - 0.3296 U^2 + 4.549 U - 21.33$$

$$\sigma = 0.016 U^2,$$

where  $U$  is the velocity of driving wind, in m/s.

One can see from the figures that the presence of the foam strongly affects the echo signal. This effect increases as the angle between the direction of mirror reflection (from the sea surface undisturbed by the wind) and the direction toward the receiver increases. In the case of monostatic sounding  $N$  is virtually independent of the foam (the solid and dashed lines practically merge in Fig. 1). In the case of bistatic sounding and narrow beams of the source and receiver the echo signal recorded by the receiver depends strongly on the form of the optical model employed for the foam (see Fig. 2). In this case the optical model of the foam in the form of a flat lambertian surface gives values which are too high compared with the model in the form of a randomly uneven locally lambertian surface.

It is interesting that for the model of foam in the form of a randomly uneven locally lambertian surface  $N$  does not depend on the parameters of the atmosphere (i.e., the power received by the lidar on sounding a sea surface partially covered by with foam depends on the parameters of the atmosphere just like the power received by the lidar on sounding a sea surface without foam). For a model of foam in the form of a flat lambertian surface an increase in atmospheric turbidity (increase of  $\mu_r$ ) results in a reduction of the parameter  $\sigma^2 C_r$  and therefore a reduction of  $N$ , i.e., the effect of the foam on the received power is reduced.

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