QUANTUM FLUCTUATIONS OF A LASER WITH INTRACAVITY SECOND-HARMONIC GENERATION

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The possibility of developing a macroscopic source of squeezed light based on a laser with intracavity second-harmonic generation is studied. It is shown that the radiation from such a source can result in the suppression of shot noise in the photodetection of the fundamental wave.

It is well known that some instruments employed in ultrasensitive laser spectroscopy, for example, spectrometers for intracavity laser spectroscopy, the maximum sensitivity, which is limited solely by the spontaneous noise of the laser source, has been achieved.[1,2] In this paper we study a scheme for a source of squeezed light with reduced quantum fluctuations of the intensity. The radiation is formed inside the common resonator, into which a transparent nonlinear crystal, which transforms the field at the frequency the laser source (LS) – the fundamental wave (FW) – into the second harmonic (SH), is inserted together with the active laser medium.

SEMICLASSICAL DESCRIPTION OF ICGSH

We shall describe the process of second-harmonic generation in the cavity of a laser (ICGSH) by the following system of equations:[3]

$$\dot{\alpha}_1 = -\frac{\gamma_1}{2}\alpha_1 + \frac{k\alpha_1}{2}(1+\beta|\alpha_1|^2)^{-1} - g\alpha_1^*\alpha_2;$$
$$\dot{\alpha}_2 = -\frac{\gamma_2}{2}\cdot\alpha_2 + \frac{g}{2}\alpha_p^2 \tag{1}$$

where $\alpha_{1,2}$ are the dimensionless amplitudes of the FW and the SH waves in the cavity: $\gamma_{1,2}$ is the cavity width: g is the nonlinear coupling constant between the FW and SH waves; and, k and β are the gain and saturation on the FW in the active laser medium.

The formal solution for α_2 has the form

$$\alpha_2(t) = \frac{g}{\gamma_2}\int_{-\infty}^{t}\exp\left[-\frac{\gamma_2}{2}(t-\tau)\right](\alpha_1^2)_\tau d\tau. \tag{2}$$

Integrating Eq. (2) by parts gives

$$\alpha_2(t) = \frac{g}{\gamma_2}\alpha_1^2 - \frac{g}{\gamma_2}\int_{-\infty}^{t}\exp\left[-\frac{\gamma_2}{2}(t-\tau)\right]2(\alpha_1\dot{\alpha}_1)_\tau d\tau. \tag{3}$$

Substituting into Eq. (3) the right side of the equation for $\dot{\alpha}_1$ from Eqs. (1), where instead of the unknown α_2 we employ the solution $\alpha_2^{(0)} = \frac{g}{\gamma_2}\alpha_1^2$ as the zeroth-order approximation, and integrating by parts once again gives

$$\alpha_2 = \frac{g}{\gamma_2}\alpha_1^2 + \frac{2g}{\gamma_2^2}\alpha_1^2\left[-\frac{\gamma_1}{2} + \frac{k}{2(1+\beta|\alpha_1|^2)} - \frac{g^2}{\gamma_2}|\alpha_1|^2\right] + \Pi, \tag{4}$$

where

$$\Pi = -\frac{2g}{\gamma_2^2}\int_{-\infty}^{t}\exp\left[-\frac{\gamma_2}{2}(t-\tau)\right]\cdot$$
$$\cdot\left[\left[-\frac{\gamma_2}{2} + \frac{k}{2(1+\beta|\alpha_1|^2)} - \frac{g^2}{\gamma_2}|\alpha_1|^2\right]_\tau + (\alpha_1\dot{\alpha}_1)_\tau +\right.$$
$$\left.+\alpha_1^2\frac{d}{d_\tau}\left[-\frac{\gamma_1}{2} + \frac{k}{2(1+\beta|\alpha_1|^2)} - \frac{g^2}{\gamma_2}|\alpha_1|^2\right]_\tau\right]d\tau. \tag{5}$$

We write the first two terms of Eq. (4) in the form

$$\alpha_2^{(1)} = \frac{g}{\gamma_2}\alpha_1^2\left[1 + \left[\frac{k}{(1+\beta|\alpha_1|^2)} - \frac{2g^2\alpha_1^2}{\gamma_2^2} - \frac{\gamma_1}{\gamma_2}\right]\right]. \tag{6}$$

We note that under the conditions

$$0 < \frac{k}{\gamma_2}(1+\beta|\alpha_1|^2)^{-1} - \frac{2g^2}{\gamma_2^2}|\alpha_1|^2 - \frac{\gamma_2}{\gamma_1} \ll 1 \tag{7}$$

the solution in the form $\alpha_2 = \frac{g}{\gamma_2}\alpha_1^2$ is a good approximation for the second harmonic. Under the conditions of stable simultaneous lasing and generation of the second harmonic the nontrivial solution for $n_0 = |\alpha_1^0|^2$ is found from the relation

$$\Psi = 2\varphi_1 - \varphi_2,\ \Psi = 0; \tag{8}$$

$$\gamma_1 + \frac{2g^2}{\gamma_2^2} n_0 = \frac{k}{1 + \beta I_0}. \tag{9}$$

We introduce the following notation: $\eta = \frac{2g^2}{\gamma^2} n_0$ is the SHC coefficient and $I = \beta n_0$. It is well known that in the absence of ICGSH there exists an optimal cavity width $\gamma_1^{\text{opt}} = \frac{k}{1+I_0}$. The SHC coefficient is related as follows with the parameters of the system:

$$\eta = \frac{1}{2}\left(\frac{\gamma_1^{\text{opt}}}{\gamma_2} - \frac{\gamma_1}{\gamma_2}\right). \tag{10}$$

We shall study the case when radiation at the frequency w is trapped in the cavity ($\gamma_1 = 0$) and is completely converted into the second harmonic. In order that the system described above generate the second harmonic in the optimal regime the condition $\gamma_2 = \gamma_1^{\text{opt}}$ must be satisfied. From here it follows that $\eta^{\max} = 0.5$. Starting from Eq. (1) we shall write an equation for $n = |\alpha_1|^2$ and based on it the linearized equation for ν ($n = n_0 + \nu$; $\nu \ll n_0$):

$$\nu = -\Gamma_1 \nu; \tag{11}$$

$$\Gamma_1 = I_0 \frac{\gamma_1 + 2\eta\gamma_2}{1 + I_0}. \tag{12}$$

For $\eta = 0$ the expression (12) transforms into the well-known expression for the rate of damping of fluctuations of the number of photons generated by the laser.[4]

QUANTUM DESCRIPTION OF ICGSH

We shall use the Fokker-Plank equation (FPE) for the positive-definite Glauber phase density $\rho_{\alpha\alpha} = \langle \alpha | \rho^{F} | \alpha \rangle$[5,6] to analyze the statistical properties of the light source:

$$\frac{d\rho_{\alpha\alpha}}{dt} = \left[\frac{d}{d\alpha_1}\left(\frac{\gamma_1}{2}\alpha_1 - \frac{k\alpha_1}{2(1+\beta|\alpha_1|^2} + g\alpha_1^*\alpha_2\right) + \text{c. s}\right]\rho_{\alpha\alpha} +$$
$$+\left[\frac{d^2}{d\alpha_2}\left(\frac{\gamma_2}{2}\alpha_2 - \frac{g}{2}\alpha_1^2\right) + \text{c.s.}\right]\rho_{\alpha\alpha} + \gamma_1(\langle n_1^{\text{T}}\rangle + 1)\frac{d^2\rho_{\alpha\alpha}}{d\alpha_1 d\alpha_1^*} +$$
$$+\left[\frac{g}{2}\alpha_2^*\frac{d^2}{d\alpha_1^{*2}} + \text{c. s.} + \hat{D}_{\text{las}}\right]\rho_{\alpha\alpha}. \tag{13}$$

Analysis of the semiclassical system of equations (1) allows us to use in Eq. (13) the operation of adiabatic elimination of variables.[4] We shall transform in the equation obtained to polar coordinates $\alpha = \sqrt{n}e^{i\varphi}$. Under the conditions of stationary lasing stable values of n_0 and Ψ_0 are established. For this reason it can be assumed that the fluctuations $\upsilon = n - n_0$, $\delta\Psi = \Psi - \Psi_0$, determined by $\rho_{\alpha\alpha}$, are small. Taking this into account and making the assumption that the amplitude fluctuations, are independent of the phase fluctuation, i.e., $\rho_{\alpha\alpha} = R\Phi$, we obtain based on Eq. (13) a linearized equation for R:

$$R = \Gamma_1\left[\frac{d\upsilon R}{d\upsilon} + \langle\upsilon^2\rangle_{A}\frac{d^2R}{d\upsilon^2}\right], \tag{14}$$

where

$$\langle\upsilon^2\rangle_{A} = n_0\left[I_0^{-1} - \frac{1 + I_0^{-1}}{2(1 + \gamma_1/\gamma_2\eta)} + 2\right]. \tag{15}$$

Here the expression for Γ_1 has the form (12). Using the relation $\langle\upsilon^2\rangle_{A} = \langle\upsilon^2\rangle_{N} + 2n_0 + 1$ we obtain an expression for $\langle\upsilon^2\rangle_{N} = n_0\delta_1$, where δ_1 is a statistical parameter defined as $\delta_1 = \frac{\langle n_1^2\rangle - \langle n_1\rangle^2 - \langle n_1\rangle}{\langle n_1\rangle}$. For our case δ_1 has the form

$$\delta_1 = I_0^{-1} - 0.5(I_0^{-1} + 1)/(1 + 2\gamma_1/\gamma_2\eta). \tag{16}$$

We note that in the absence of ICGSH, i.e., for $\eta = 0$, Eq. (16) transforms into the well-known expression $\delta_1 = I_0^{-1}$ for a laser with no technical noise.[4] In the opposite case, i.e. when $\frac{\gamma_1}{\gamma_2\eta} = 1$ (highly efficient ICGSH) the squeezing of the FW inside the cavity can reach the limiting value $\delta_1 = -0.5$. To calculate δ_2 we shall start from the operator analog of the system (1). The solution for $\hat{a}_2$ is obtained using the scheme (4)–(9). This makes it possible to determine δ_2:

$$\delta_2 = 4\eta\delta_1. \tag{17}$$

It follows from Ref. 7 that shot noise can be suppressed in the low-frequency region of the power spectrum of the photocurrent, i.e., for $\Gamma_2 \ll \Omega^2$. If $\Omega \to 0$, then

$$\langle i^2\rangle = q\gamma n_2\left(1 - \frac{2q\delta\gamma}{\Gamma}\right). \tag{18}$$

We substitute Eq. (12) into Eq. (18) and write the expression in the parentheses in Eq. (18) as

$$k = 1 - 2q\delta_1(1 + I_0)/I_0(1 + 2\eta\gamma_2/\gamma_1). \tag{19}$$

For real parameters of the source and receiver ($q \approx 0.9$; $\eta = 25\%$; $\gamma_2/\gamma_1 = 5$; $I_0 = 10$) $1 - k = 0.12$, i.e., the shot noise in photodetection of the FW is reduced by 12%.

The theoretical study performed above shows that it is possible to build a macroscopic source of squeezed light based on a laser with ICGSH. The radiation of such a source can result in suppression of the shot noise in photodectection of the FW.

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