ADAPTIVE SYSTEM FOR THE LIGHT BEAM PHASE MODAL CONTROL

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The theoretical basis and the results of an experimental investigation, performed on an operating model of the adaptive system, of multicriterional algorithm for modal control of the phase of a light beam based on the first five Zernike polynomials and analysis of the first and second moments of the intensity distribution in the observation plane are presented. The wavefront corrector, based on an elastic continuous mirror, and an image recording device with a controllable viewing angle, both employed in the adaptive system, are described.

Modal control based on the lowest phase aberrations is important in the problem of adaptive focusing of a light beam. Such aberrations predominate in nonlinear distortions accompanying the propagation of intense optical radiation in the atmosphere, and their adaptive compensation increases the efficiency of the transport of light energy over a long distance.[1,2] By changing the phase profile of the beam with the help of a modal corrector it is possible to control the propagation of the radiation power in space: this is of interest in laser technology.

The problem of adaptive focusing of laser radiation is usually formulated in terms of optimization of some scalar criterion, for example, the focusing functional, the sharpness, etc.[3] In the scalar criterion all control coordinates are interrelated, which makes the optimization process less stable.

In this paper we report on the development and testing of a model of an adaptive system for focusing a light beam using a vector criterion together with a modal control basis. In this case it is possible to separate the mode-control channels and thereby to improve the convergence of the process of focusing the radiation into a fixed region. The components of the vector criterion are generated on a computer based on the image of the beam in the observation plane stored in the computer.

In the process of modal control the phase of the beam $\varphi(x, y)$ in the plane of the corrector is generated in the form of a superposition of the basis modes $\omega_1(x, y)$, generated by the phase corrector,

$$\varphi(x,y) = 2k\sum_{i=1}^{N} U_i w_i(x,y), \tag{1}$$

where N is the dimension of the basis and coefficients U_i are the control coordinates. If the control vector

$$\vec{U} = \{U_i;\ i = \overline{1,N}\} \tag{2}$$

is introduced, we can write

$$\varphi(x,y) = 2k\cdot\vec{U}^T\cdot\vec{\omega}(x,y), \tag{3}$$

where $\vec{\omega}\ (x, y) = \{\omega_i(x, y);\ i = \overline{1, N}\ \}$ is the modal-control basis.

For a fixed beam profile the intensity distribution $I(x, y, z)$ of the light field in the observation plane z_0 is determined by the phase $\varphi(x, y)$, generated by the corrector. For a modal corrector the intensity $I(x, y, z_0)$ depends parametrically on the coordinates of the control vector U_i.

We shall assume that the modes $\omega_i(x, y)$ are orthogonal and that there exist functionals $\hat{F}_i$, $(i = \overline{1, N}\ )$ of the intensity $I(x, y, z_0)$ which in the case of a linear medium satisfy the following conditions:

$$\hat{F}_i[I(x,y,z_0)] = f_i(U_i),\ i = \overline{1,N}. \tag{4}$$

The functionals $\hat{F}$ can be termed conjugate to the basis modes, since the value of each of them is determined solely by the coordinate U_i of the corresponding mode $\omega_i(x, y)$. We note that for a basis of finite dimension N the class of functions $I(x, y, z_0)$ is limited and the collection of functionals $\hat{F}_i$, $(i = \overline{1, N}\ )$ completely determines the distribution of the intensity $I(x, y, z_0)$.

Assume that we are required to obtain in the observation plane the intensity distribution $I^0(x, y, z_0)$, which is characterized by the values of $\hat{F}_i[I^0(z_0)]$, $I = \overline{1, N}$. Then the goal of control is to minimize the components of the vector $\vec{J}$, which have the form

$$J_i = \hat{F}_i[I(x,y,z_0)] - \hat{F}_i[I^0(x,y,z_0)]. \tag{5}$$

It follows from Eqs. (4) and (5) that the control coordinates sought U_i^0, for which the values $\hat{F}_i[I^0(z_0)]$ are achieved, are determined as follows in the case when the dependence $f_i(U_i)$ is linear:

$$\vec{U}^0 = \vec{U} - \hat{A}\vec{J}, \tag{6}$$

where $\hat{A}$ is the control matrix, which is diagonal in the case of a linear medium and an orthogonal basis $\vec{\omega}$ and its elements do not depend on $\vec{J}$. In this case the criterion is optimized in two steps. In the first step the elements of the matrix $\hat{A}$ are determined and at the second step the criterion is optimized.

Thus the problem of generating the intensity profile $I^0(x, y, z_0)$ in the class of admissible functions is reduced to the problem of minimizing all components of the vector criterion $\vec{J}$.

The Zernike polynomials $Z_i(x, y)$, which optimize optical aberrations, form an orthogonal basis. For aberrations of the first and second order these polynomials determine the slopes (Z_1 and Z_2), the focusing (Z_3), and the astigmatisms (Z_4 and Z_5) of the phase of the beam. The functionals $\hat{F}_i[I]$, conjugate to the first five Zernike polynomials, are expressed in terms of the first and second moments of the intensity distribution $I(x, y, z_0)$ in the observation plane z_0. The first two of these moments have the following form:

$$\hat{F}_1[I] = x_c = M\{x\}; \quad \hat{F}_2[I] = y = M\{y\}, \tag{7}$$

where

$$M\{f(x,y)\} = \int_{-\infty}^{\infty}\int f(x,y) I(x,y,z_0)\,dxdy \Big/ \int_{-\infty}^{\infty}\int I(x,y,z_0)\,dxdy.$$

The functionals $\hat{F}_4$ and $\hat{F}_5$ characterize the roundness of the beam, $\hat{F}_3$ characterizes the degree of focusing of the beam, and $\hat{F}_4$ and $\hat{F}_2$ characterize the position of the energy center of gravity of the beam. Because the functionals $\hat{F}_1 - \hat{F}_5$ are conjugate to the polynomials $Z_1 - Z_5$ the coordinates $U_1 - U_5$ can be controlled independently. Similar to the operator, the adaptive system for control based on U_4 and U_5 generates a circular beam, the beam is focused by changing U_3, and, finally, the focused beam is positioned at a given point x^0, y^0 in the observation plane z_0 with the help of U_1 and U_2.

The adaptive system for control based on the vector criterion $\vec{J}$ includes a modal phase corrector with an optical channel for forming the beam, a device for recording the image in the observation plane, a computer with a package of programs for processing the image and forming the signals for controlling the phase corrector, as well as monitoring devices.[4,5]

In the model built (Fig. 1) the optical channel consists of a telescope, which provides the required expansion of the beam striking the corrector. The phase corrector consists of a flexible controllable mirror 50–70 mm in diameter whose center is clamped.[6] To control the shape of the reflecting surface of the mirror, six rods are rigidly mounted on the edge of the mirror. The rods are displaced with the help of return springs and pushers, connected to stepping motors with the help of a spiral couple (Fig. 2). The signals for controlling the stepping motors are generated by the interface of the modal corrector; this interface decodes the binary code from the computer mainline and transforms it into the current signal for the coils.

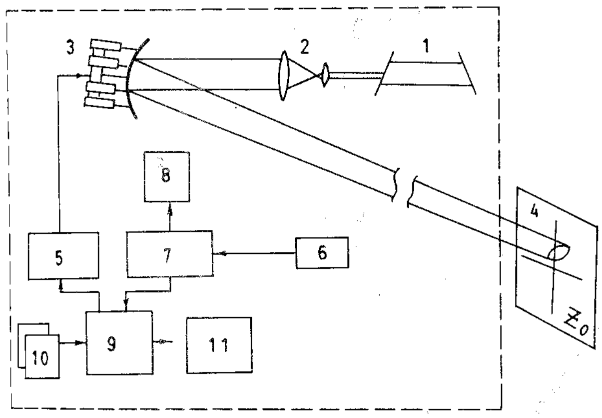

FIG. 1. Block diagram of the model of the adaptive optical system: laser (1); telescope (2); adaptive mirror (3); screen for fixing the position of the image of the beam on the screen (4); unit for controlling the stepping motors (5); television camera(6); image recording device (7); control monitor (8); computer (9); program package (10); plotter (11).

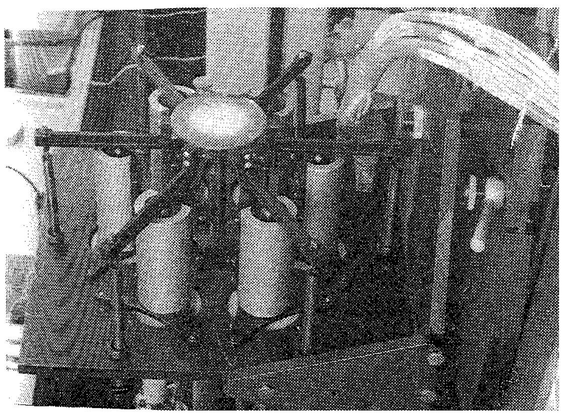

FIG. 2. Modal phase corrector.

Mirrors with a steel backing 3–4 mm thick and glass or copper reflecting surface were tested. The range of displacement of the reflecting surface of the mirror is equal to 300 μm at the edge of the aperture and the positioning accuracy is equal to 1.5 μm per control step.

The basis modes are generated by feeding to the stepping motors the corresponding collection of basis signals, the proportional change in which varies the focal power of the corrected aberration. According to the calculations the system of concentrated forces and moments acting on the contour of the circular plate makes it possible to obtain basis modes $\omega_i(x, y)$ whose rms deviation from the corresponding Zernike polynomials $Z_i(x, y)$ is equal to about 6% for focusing Z_3 and 1.1% for astigmatisms Z_4 and Z_5. For the wavefront slopes Z_1 and Z_2, generated by a mirror whose center is clamped, the theoretical estimate of the error reaches 10%. Interference measurements show that the model of the mirror, prepared according to the average accuracy class, reproduces axisymmetric focusing with an error of 21%, astigmatism aberrations with an error of 15%, and phase slopes with an error of 5%.

The image recording device consists of a standard television camera and an interface which samples, digitizes, and stores readings on a grid of 64×64 bytes from the video signal and feeds them into a computer. The interface also includes an "electronic transfocator" scheme, which enables adaptive control of the spatial resolution and the value and position of the viewing angle in the scanning pattern of the television camera.

Recording schemes with controllable spatial resolution are necessary in adaptive systems, since the beam parameters vary over wide limits in the process of focusing. The use of high resolution simultaneously with a wide viewing angle in the recording system results in an excessive increase in the volume of information and therefore the information processing time also. In addition, the control algorithm based on analysis of the moments of the intensity distribution in the beam provides satisfactory accuracy of analysis when processing an image on a sparse grid with dimensions of 64×64 bytes.

The transfocator scheme developed makes it possible to separate from the television standard three reading regions with different dimensions, corresponding to three degrees of spatial resolution ("survey", "medium", and "fine" grids), and it gives a step for changing the coordinates of the center of the reading region equal to 1/8 of the linear size of a frame, which gives virtually smooth displacement of the grid over the scanning pattern of the camera. The volume of the information introduced remains unchanged.

In the process of focusing the beam the "electronic transfocator" is transferred automatically by computer command from the "survey" grid to the "medium" grid and then to the "fine" grid, simultaneously directing the center of the region of reading on the energy center of the beam.

A computer based on DVK-2M, which is additionally equipped with an OZU-256K electronic disk, a parallel interface, and a plotter, is used for image processing and formation of the control commands. The program package contains subprograms for calculating the components of the vector criterion from the stored image, control of the "electronic transfocator", adaptive control of the phase of the beam based on minimization of the vector criterion, and service programs for: graphical display of information, statistical analysis, and conducting the protocol of the experiment. The image recording time is equal to 20 ms, the control commands are processed and formed within 1-2 s, and the operating time of the III servomechanisms is equal to 0.5 s. The structure of the adaptive control algorithm is presented in Fig. 3.

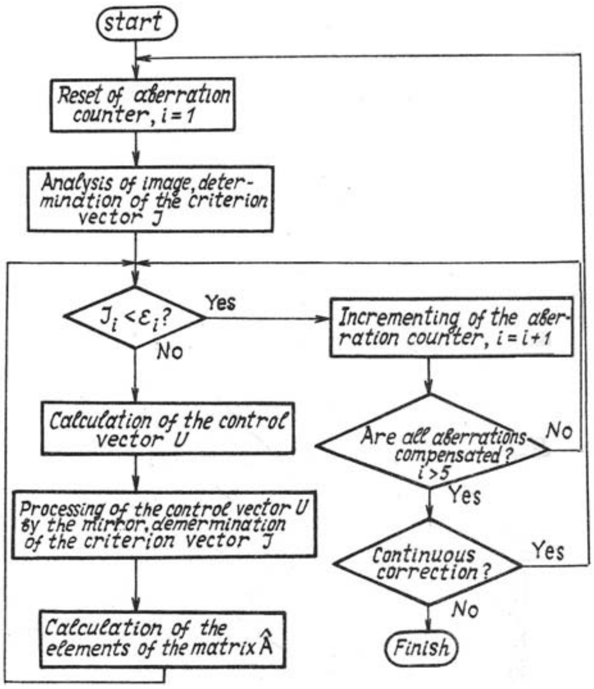

FIG. 3. Structure of the adaptive control algorithm.

In practice the modal basis w of a real phase corrector is not orthogonal. In addition, there are significant difficulties in generating during the calculation the second-order functionals $\hat{F}_1 - \hat{F}_5$ conjugate to the basis modes. In this case it is possible to introduce quasiconjugate operators $\hat{F}_i$, for which the formula (4) assumes the form

$$\hat{F}_i[I(x,y,z_0)] = f_i(U_i;\ \mu U_i),\quad i \neq j,\ \mu \ll 1. \qquad (8)$$

For second-order aberrations the functionals $\hat{F}_3 - \hat{F}_5$ can be expressed in terms of the moments of intensity distribution $I(x, y, z_0)$ in the observation plane z_0 as follows:

$$\hat{F}_3[I] = a_x^2 + a_y^2 = M\{(x - x_c)^2 + (y - y_c)^2\};$$
$$\hat{F}_4[I] = a_x^2 - a_y^2 = M\{(x - x_c)^2 - (y - y_c)^2\}; \qquad (9)$$
$$\hat{F}_5[I] = a_{xy} = M\{(x - x_c) + (y - y_c)\}.$$

Then the components of the vector criterion assume the form

$$J_1 = x_c - x^0;\ J_2 = y_c - y^0;\ J_3 = a_x^2 + a_y^2 - a_{ef}^{02};$$
$$J_4 = a_x^2 - a_y^2;\ J_5 = a_{xy},$$

where

$$a_{ef}^{02} = a_x^{02} + a_y^{02};\ a_x^0 = a_y^0;\ a_{xy}^0 = 0. \tag{10}$$

In the case when the basis ω is not completely orthogonal and the functionals $\hat{F}_i$ are quasiconjugate and also under conditions of nonlinearity on the propagation path the control matrix $\vec{A}$ is not diagonal and its elements depend on the criterion vector $\vec{J}$. However when the nonlinearity is weak and the parameter $\mu \ll 1$, which reflects the degree of orthogonality of the basis and the degree to which the functionals are conjugate, the matrix $\vec{A}$ is well-conditioned. The focusing process assumes an iterative character, for which the control vector at n-th iteration step $\vec{U}^n$ is determined from the formula

$$\vec{U}^n = \vec{U}^{n-1} - \hat{A}^{n-1}\vec{J}^{n-1}. \tag{11}$$

The diagonal elements of the matrix $\vec{A}$ can be determined at the preceding control step as follows

$$a_{ii}^n = (U_i^{n-1} - U_i^n)/(J_i^{n-1} - J_i^n). \tag{12}$$

When matrix elements $a_{ij} \neq 0$ for $i \neq j$ exist changes can be observed in the components of the criterion J_1 in the process of control on the coordinates U_j, $i \neq j$. The method of successive compensation of the aberrations is used to overcome this undesirable effect as well as to improve the convergence stability of the control process. In the problem of focusing a light beam the astigmatisms were compensated first, after which the focusing and then the slopes were compensated.

The transfer to control on the next component of the control vector U_{j+1} was performed after the preceding criterion J_j was minimized with a fixed accuracy ε_j, where j is the number of the aberration in the order of their compensation. Since in the experiment the already optimized criteria J_j can fall outside the limits of the given ε_j neighborhood in the process of control on the coordinate U_j $(i \neq j)$, in the algorithm it is possible to return to control on the modes ω_i. The gal of compensation was regarded as reached when all criteria fall within the given ε neighborhood.

The trial experiment consisted of focusing radiation in a linear medium into a given point on a screen. The accuracy of focusing ε_3 based on the spherical aberration ω_3 was set equal to $\varepsilon_3 = 1.5\ J_3^{\min}$, where $J_3^{\min}$ is the smallest value of the square of the effective width of the beam achieved in the stand described.

The behavior of the criterion vector $\vec{J}$ (Fig. 4a) and the control vector $\vec{U}$ (Fig. 4c), whose components are the optical powers of the basis functions formed in the control process, and the behavior of the characteristics x_c, y_c, a_x, a_y, and a_{xy} were studied.

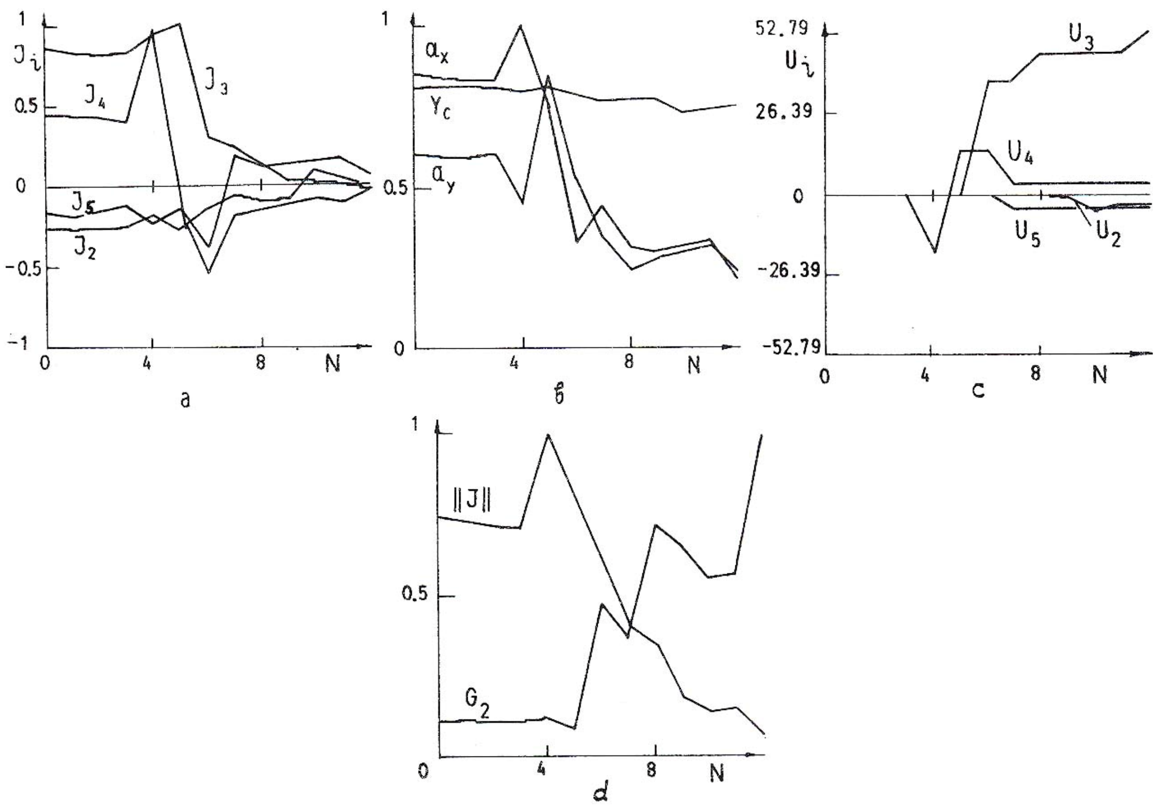

FIG. 4. The behavior of the components of the criterion vector $\vec{J}$ (a), the beam characteristics a_x, a_y, and y_c (b), the components of the control vector $\vec{U}$ (c), and the quantity J and the criterion G_2 (d) in the process of adaptive focusing of a beam into a given point in the observation plane.

In the realization presented in Fig. 4 the chain of control $\vec{J}$ of the motors was broken at the first three steps. When the chain is closed into a feedback loop in the first two steps the astigmatism ω_4 is compensated and the component J_4 falls within a ε_4 neighborhood of zero. The beam is focused at the fifth step. Then the system once again corrects the astigmatisms, reducing J_4 and J_5. At the final step' the beam is pointed at a fixed point. The behavior of the beam characteristics, presented in Fig. 4b, makes it possible to follow the process of minimization of the half-widths and the pointing of the focused beam at a fixed center.

The experiment showed that the initial compensation from the maximum distortion to the first time the criterion falls within an ε neighborhood according to all aberrations was achieved within 5–15 steps. Repeated compensation, caused by the criterion J_1 falling outside the ε_1 neighborhood owing to regular perturbation in any aberration ω_j, was observed within 2–4 steps. The algorithm employed, the recording system, and the phase corrector permit reducing, when focusing a beam, the transverse linear dimensions of the beam by a factor of 7–15 and aiming the center of the beam into a given region with an accuracy up to the size of the diameter of the focused beam.

In addition, the behavior of the quantity

$$\|\vec{J}\| = \sum_{i=1}^{5} J_i^2, \tag{13}$$

characterizing the quantity of focusing with respect to all optical aberrations simultaneously, and the criterion

$$G_2 = \int\int_{\Omega} I^2(x, y, z_0)\,dx dy, \tag{14}$$

which can be used in the control of adaptive optical systems, were also recorded.[3] One can see that the behavior of G_2 is substantially nonmonotonic, which makes it difficult to use this criterion (Fig. 4d).

The investigations performed show that modal control based on a vector criterion makes it possible to construct an efficient stable algorithm for adaptive focusing of a beam into a fixed point.

The setup developed was used for adaptive formation of a fixed intensity distribution determined by the lowest-order aberrations of the phase.

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