DERIVATION OF SIMPLE EXPRESSIONS FOR THE LIGHT INTENSITY FOR THE CASE OF DIFFRACTION BY A THIN SCREEN WITH A STRAIGHT EDGE. PART II.

Yu.I. Terent'ev

Institute of Atmospheric Optics,
Siberian Branch of the Academy of Sciences of the USSR, Tomsk
Received August 14, 1989

It is determined that the difference of the light amplitudes in the bands of the diffraction pattern due to the screen and the corresponding amplitudes of the incident wave is inverse proportional to the distance from the shadow boundary.

Simple expressions are derived for the intensity of the edge wave, and the intensity of the diffraction pattern, and are in good agreement with the experimental data.

In Part I of this paper[1] it was shown that the intensity of the components of the edge wave from a screen with a straight edge is described by the equation $J_e = A/h^2$.

The same relationship determines the dependence of the squared difference of the light amplitudes J_r in the bands of the diffraction pattern from the screen and the corresponding amplitudes of the incident wave on h. It is clearly confirmed by Tables I–IV in Ref. 1 with similar A values for diffraction maxima and first minima, where h_{exp} is the experimental value of the distance from the bands to the shadow boundary; J_b and J_i are the light intensities in the bands of the diffraction pattern and in the incident beam without screen respectively; $|J_r| = \left(\sqrt{J_b} - \sqrt{J_i}\right)^2$; and, $A = J_p h_{exp}^2$. The above values of h_{exp}, J_b, and J_i were obtained experimentally with a 30-μm wide slit (Fig. 4 from Ref. 1), illuminated by a parallel light beam (λ = 0.53 m), selected with the help of an interference filter from the radiation of a filament lamp or He-Ne laser, serving as the source of the cylindrical wave. The screen (a blade) was located up against the axis of the light beam, and the examined bands were located on the projection of the second half of the first maximum from the slit.

TABLE I.

l = 6 mm; L = 99.5 mm; λ = 0.53 μm					
Band	h_{exp}, mm	J_b	J_i	J_r	A
max_1	0.8	22.1	14.5	0.81	0.518
min_1	1.261	3	5.3	0.33	0.517
max_2	1.609	3.2	1.8	0.21	0.544

The decrease of A with the increase of the order of the minima in these experiments is easily explained by the decrease of the degree of overlap of the wave trains if the diffraction pattern is caused by the interference of the edge wave and the incident wave. This is facilitated by the small length of the wave trains in the light from the filament lamp, which appears to be comparable with the path difference between the edge rays and the direct rays. In the case of laser radiation (Table IV) the path difference between the interfering rays, within the limits of the examined pattern, is small in comparison with the length of the wave trains: the dependence of A on the order of the minima is weakened.

TABLE II

l = 12 mm; L = 95.5 mm; λ = 0.53 μm					
Band	h_{exp}, mm	J_b	J_i	J_r	A
max_1	0.582	39.7	28	1.02	0.345
max_2	1.145	17.3	13.2	0.28	0.371
max_3	1.540	7.7	5.7	0.14	0.340
max_4	1.830	2.9	1.9	0.11	0.353
min_1	0.9	13.7	19	0.43	0.350
min_2	1.350	6.1	8.2	0.16	0.282
min_3	1.670	2.3	3.1	0.06	0.166
min_4	1.970	0.7	0.9	0.01	0.049

While A is sensitive to the order of the minima, it is independent of the order of the tabulated maxima; which in all probability indicates the possibility of a complete interference of the diffracted and incident light at the photodetector, even in the presence of a path difference between the wave trains, if the corresponding time interval is substantially less than the decay time of the stimulated oscillations of the electrons in the cathode of the photodetector.

As the measurements show, J_r is equal to the light intensity in the shadow from the screen J_{sh} at the same distances from its boundary. It was impossible in these experiments to detect the presence of the phase shift of n between the light with intensity J_r and the light with intensity J_{sh}, which is the essential indication of an edge wave. Nevertheless, the equality of J_r and J_{sh} and the fact that their dependence on h obeys an edge-wave intensity distribution law allows one to conclude that the diffraction pattern from the screen and the light in the shadow region are really caused by the interference of one of its components with the direct beams and by the propagation of the other component into the shadow.

TABLE III.

l = 24 mm; L = 99.5 mm; λ = 0.53 μm					
Band	h_{exp}, mm	J_b	J_1	J_r	A
max_1	0.433	32.3	23	0.78	0.146
max_2	0.865	20.7	17	0.19	0.140
max_3	1.126	13.8	11.5	0.11	0.136
max_4	1.351	9.8	8.1	0.08	0.139
min_1	0.686	15.1	19.6	0.29	0.135
min_2	1.006	11.5	14	0.12	0.125
min_3	1.249	8.5	10	0.06	0.095
min_4	1.443	5.8	6.8	0.04	0.083

TABLE IV.

l = 11.4 mm; L = 99.5 mm; λ = 0.6828 μm					
band	h_{exp}, mm	J_b	J_1	J_r	A
max_1	0.650	75	53.1	1.89	0.800
max_2	1.276	34.9	27.2	0.48	0.780
max_3	1.701	14.4	10.6	0.29	0.84
max_4	2.038	4.3	2.6	0.20	0.847
min_1	1.016	27.4	37.5	0.79	0.816
min_2	1.506	11.4	15.6	0.34	0.765
min_3	1.851	4.4	6.5	0.20	0.700

TABLE V.

α, deg	J_1	J_3
0.133	39.4	40.4
0.166	25.5	26.4
0.263	10.4	10.4
0.360	5.5	5.5
0.537	2.6	2.5

The identification of the light in the shadow from the screen with the edge wave is also confirmed by the fact that at equal values of l and for the same intensities of the incident light on the screen edge its intensity in the shadow J_3 (Fig. 4 from Ref. 1) is equal to the intensity of the edge wave J_1 (Fig. 1 from Ref. 1) at equal deviation angles from the initial direction α; this is confirmed by the data in Table V.

TABLE VI.

1, mm	L, mm	J_{max1}/J_c
6	99.5	1.377
12	99.5	1.374
24	99.5	1.374
35.5	99.5	1.4
∞	114.2	1.39
φ	279.5	1.365

Furthermore, this equality demonstrates the inconsistency of Fresnel's statements[2] to the effect that the energy of the edge wave is insufficient to form the diffraction pattern with the experimentally observed variations of the light intensity in the bands.

The relationship of the light in the shadow with the edge wave becomes especially clear under conditions of periodic variation of the light intensity in the plane of the screen in the direction perpendicular to its edge upon displacement of the screen in the same direction. In this case the variations of the light flux in the shadow region follow the intensity variations near the screen edge even when the distance between the maxima is ~ 15 μm.

It is well known that given a constant intensity over the width of the wave-front, the ratio of the intensity of the maximum of the diffraction pattern caused by the screen J_{max1} to J_1 does not depend on the parameters l and L (Fig. 4 from Ref. 1)) and is approximately equal to 1.374 according to theCornu spiral. The validity of the above-mentioned fact is confirmed, for example, by the experimental data in Table VI.

This dependence enables one to express A in terms of J_1 and the parameters of the diffraction scheme. Let us do this first for the case of a cylindrical incident wave. According to relation (3) from Ref. 1 the distance from the first maximum to the shadow boundary is written in the form $h_{max1} = \sqrt{0.69\lambda L\frac{L+l}{l}}$.

Taking this into account, one can write the intensity of the edge wave at h_{max1} in the form

$$J_{e1}=a_{e1}^2=A|h_{max1}^2=Al|0.69\lambda L(L+l). \quad (1)$$

As was noted above, $J_e = J_r = J_{sh}$, from which it follows that $a_{e1}=(\sqrt{J_{max1}}-\sqrt{J_1})=\sqrt{1.374J_1}-\sqrt{J_1})=$ $=0.1722\sqrt{J_1}$ and $a_{e1}^2 = 0.02965\ J_1$. Substituting the value a_{e1}^2 into Eq. (1), we find that A $= \frac{0.02046\lambda L(L+l)J_i}{l}$. It then follows that

$$J_e = \frac{A}{h^2} = \frac{0.02046\lambda L(L+l)J_1}{h^2 l}. \qquad (2)$$

Using this formula one can easily determine the intensity of the edge wave at any h provided that the J_1 values are known.

Replacing h^2 by its value given by expression (3) in Ref. 1 allows us to write Eq. (2) in the form

$$J_e = \frac{0.02046 J_1}{0.69 + k}, \qquad (3)$$

which is convenient for determining J_e in the bands of the diffraction pattern.

The equation for the intensity of the edge wave enables one to derive a formula that describes the intensity of the diffraction pattern J_d, assuming that the latter is due to the interference of the edge rays 2 with the directly transmitted rays 1 (Fig. 4 from Ref. 1). Based on the rule of coherent interference we have

$$J_d = a_1^2 + a_e^2 + 2a_1 a_e \cos\Psi = J_1 + J_e + 2\sqrt{J_1}\sqrt{J_e}\cos\psi, \qquad (4)$$

where ψ is the phase difference between beams 1 and 2; $\psi = 2\pi\left(\Delta_{21} - 0.69\frac{\lambda}{2}\right)/\lambda$, here $0.69\ \lambda/2$ is the path difference between the interfering beams, due to the initial phase advance of the edge wave, propagating to the illuminated side, relative to the incident wave by $0.69\ \pi$ (Ref. 1); $\Delta_{21} = h^2 l/2L(L + l)$. Therefore, we finally have

$$\psi = \frac{[h^2 l - 0.69\lambda L(L+l)]\pi}{\lambda L(L+l)}. \qquad (5)$$

After substituting relations (2) and (5) into expression (4) we have

$$J_d = J_1\left[1 + \frac{0.02046\lambda L(L+l)}{h^2 l} + 2\sqrt{\frac{0.02046\lambda L(L+l)}{h^2 l}} \times \cos\frac{[h^2 l - 0.69\lambda L(L+l)]\pi}{\lambda L(L+l)}\right]. \qquad (6)$$

As can be easily seen, this equation demonstrates the simplicity of the dependence of J_d on λ, L, l, and J_1.

Simultaneously solving Eqs. (3) from Refs. 1 and 6, we obtain an equation which characterizes the light intensity in the maxima and minima of the diffraction pattern:

$$J_b = \left[1 \pm \sqrt{\frac{0.02046}{0.69 + k}}\right]^2 J_1. \qquad (7)$$

Note that $k = 0, 2, 4 \ldots$ correspond to the maxima, and $k = 1, 3, 5 \ldots$ correspond to the minima, of the pattern. In the case of $J_1(h)$, for example, if the bands are localized on the second half of the first maximum from the slit S, illuminated by the parallel beam, we have

$$J_d = J_1(h) + \frac{0.02046\lambda L)L+l)J_{1e}}{h^2 l} + 2\sqrt{J_1(h)} \times \sqrt{\frac{0.02046\lambda L(L+l)J_{1e}}{h^2 l}} \cos\frac{[h^2 l - 0.69\lambda L(L+l)]\pi}{\lambda L(L+l)}; \qquad (8)$$

$$J_b = \left[\sqrt{J_1(h)} \pm \sqrt{\frac{0.02046\lambda L)J_{1e}}{0.69 + k}}\right]^2, \qquad (9)$$

where J_{ie} is the intensity of the incident beam in at the edge of the shadow without a screen.

The correctness of Eq. (8) is confirmed by Table VII, which contains results of a comparison of the calculated light intensity J_{dr} in the range between the first maximum and the minimum of the diffraction pattern with its experimental values J_{dexp}, obtained for $l = 12$ mm, where $L = 99.5$ mm; $J_{ie} = 36$ rel.units, where $\Delta J_{expc} = J_{dexp} - J_{dc}$.

TABLE VII.

Band	h_{exp}, mm	J_1	J_{dexp}	Ψ	$\cos\Psi$	J_e	J_{dc}	ΔJ_{expc}
max_1	0.582	27.5	39.2	14'	1	1.07	39.4	-0.2
	0.601	27	38.6	8°29'	0.989	0.99	38.3	0.3
	0.676	25.2	32.3	43°40'	0.723	0.79	32.4	-0.1
	0.751	23	24.4	82°59'	0.122	0.64	24.6	-0.2
	0.722	22.5	22.5	94°44'	-0.083	0.61	22.5	0
	0.826	21.2	17.6	126°26'	-0.594	0.53	17.8	-0.2
	0.901	19	13.7	174°1'	-0.995	0.45	13.7	0
min_1	0.910	18.8	13.6	180°	-1	0.44	13.5	0.1

Note that in order to obtain equality between J_{dc} and J_{dexp} it is necessary to carefully determine the location of the shadow boundary.

The validity of Eqs. (7) and (9) is confirmed by the experimental data shown in Tables VIII and IX ($l = 12$ μm, $L = 99.5$ μm, $I_{ie} = 36$ rel. units).

TABLE VIII.

Band	h_{exp}, mm	J_i	J_{dexp}	J_{dc}	ΔJ_{expc}
max_1	0.582	36	49.13	49.47	-0.34
max_2	1.145	36	42.8	42.55	0.25
max_3	1.540	36	40.7	40.9	-0.2
max_4	1.830	36	40	40.1	-0.1
min_1	0.910	36	28.54	28.5	0
min_2	1.350	36	31.43	31.84	-0.41

TABLE IX.

band	h_{exp}, mm	J_i	J_{dexp}	J_{dc}	ΔJ_{expc}
max_1	0.582	28	39.7	40	-0.3
max_2	1.145	13.2	17.45	17.28	0.17
max_3	1.540	5.7	7.65	7.75	-0.1
max_4	1.830	1.9	2.9	2.93	-0.03
min_1	0.910	19	13.7	13.68	0.02
min_2	1.350	8.2	6.1	5.84	0.26
min_3	1.670	3.1	2.3	1.96	0.34

In the case of a parallel incident beam $l = \infty$, and Eqs. (2), (6), and (8) simplify to

$$J_e = \frac{0.02046\lambda L J_i}{h^2}; \tag{10}$$

$$J_d = \left[1 + \frac{0.02046\lambda L}{h^2} + 2\sqrt{\frac{0.02046\lambda L}{h^2}} \cos\frac{(h^2 - 0.69\lambda L)\pi}{\lambda L}\right] J_i, \tag{11}$$

and

$$J_d = J_i(h) + \frac{0.02046\lambda L J_{ie}}{h^2} + 2\sqrt{J_i(h)\,\frac{0.02046\lambda L J_{ie}}{h^2}} \times$$

$$\times \cos\frac{(h^2 - 0.69\lambda L)\pi}{\lambda L}. \tag{12}$$

Relations (3), (7), and (9) remain unchanged. The validity of Eq. (9) in the case of a parallel incident beam, obtained using a collimating objective after the slit, is shown in Tables X and XI.

TABLE X.

L = 114.2 mm; J_{ie} = 33.5					
Band	h_{exp}, mm	J_i	J_{dexp}	J_{dr}	ΔJ_{expc}
max_1	0.204	27.2	38.25	38.59	-0.34
max_2	0.4	17.9	22.05	22.43	-0.38
max_3	0.526	10	12.35	12.56	-0.21
max_4	0.632	5.65	7	7.27	-0.27

TABLE XI.

L = 279.5 mm; J_{ie} = 60.9					
Band	h_{exp}, mm	J_i	J_{dexp}	J_{dc}	ΔJ_{expc}
max_1	0.320	41.4	60	60.5	-0.5
max_2	0.656	13.95	19	19.5	-0.5
max_3	0.864	3.72	5.77	5.57	0.2
min_1	0.520	23	15.9	15.5	0.4
min_2	0.766	7	4.6	4.26	0.34

The above results demonstrate quite definitely that the diffraction pattern from the screen is really formed by the interference of the edge and incident waves. At the same time, the location of bands and the values of J_d obtained in accordance with Fresnel's ideas for J_i = const are confirmed experimentally.

To better understand the reason for this, consider Fig. 1, where the distribution of the light intensity in the shadow from the screen is shown, where curve 1 characterizes the distribution J_{sh} based on the Cornu spiral[3]; 2 is the experimental curve, and curve 3 corresponds to the dependence $J_{sh} = A/h^2$. In the corresponding experiments $l = 35.5$ μm, $L = 99.5$ μm, and $h_{max1} = 0.372$ μm. As can be seen from the figure, at distances $h_\delta \geq 0.4$ μm all three curves coincide. Therefore, for $h \geq h_\delta$ the distribution J_{sh}, based on the Cornu spiral, is equivalent to the intensity distribution in the edge wave.

At distances $h \leq h_C$ Eq. (2) loses meaning. Since under such conditions $h_C < h_{max1}$, relations (6)–(9) are valid for all of the bands of the diffraction pattern.

The deviation angle of the diffracted beams (from the initial direction) corresponding to the critical distance h_C is $\varepsilon_C = h_C/L = 0.162°$.

If ε_C is independent of L, then $h_C \sim L$. According to Eq. (3) from Ref. 1 for $l \ll L$ h_b is also proportional to L. Therefore, in the case of diverging incident beams, the diffraction bands should not pass into the anomalous region as L increases.

If the incident beam is parallel, h_b increases more slowly than h_C as L increases because $h_b \sim \sqrt{L}$. As a result, at large L the bands can be found in the region where the band contrast range decreases. For example, the calculation made in accordance with the experimental dependence $J_{sh} = f(h)$ for $L = 279.5$ mm shows that at $L = 300$ m $J_{max1}/J_1 = 1.033$ and not 1.374.

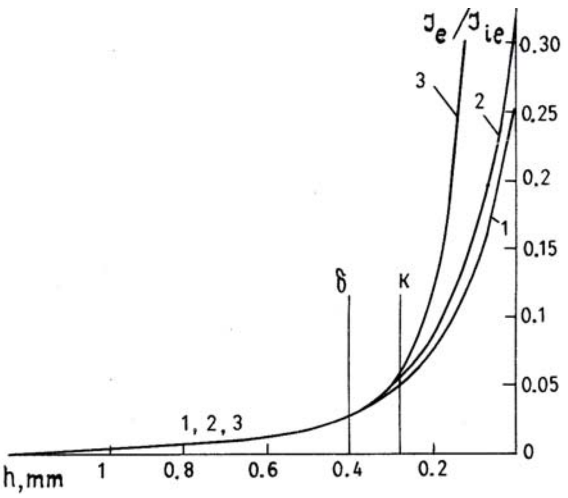
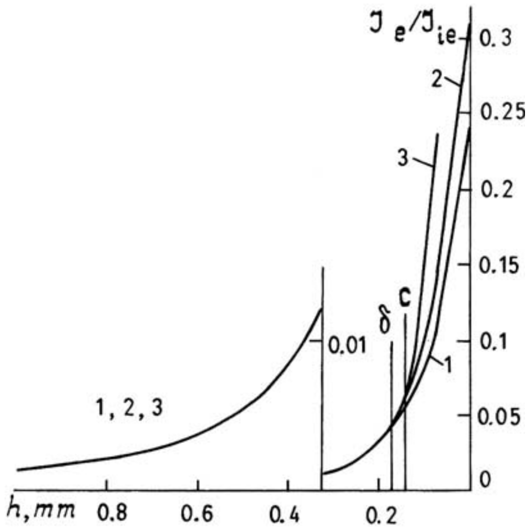
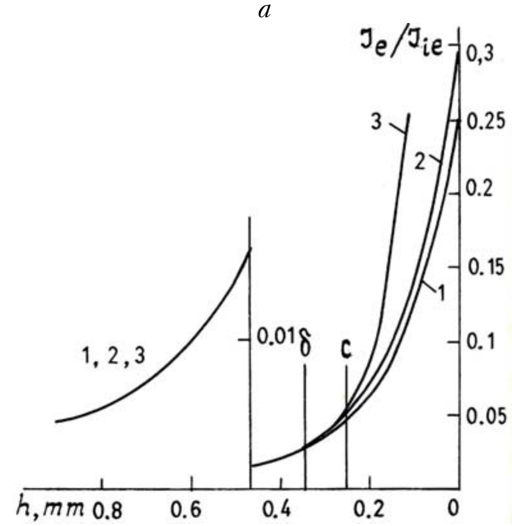


FIG. 1. Distribution of the light intensity in the shadow from the screen in the case of a cylindrical incident wave.

For a parallel incident beam, ε_C has smaller values, and the behavior of curves 1, 2, and 3 is analogous to that for a diverging beam. This is confirmed by Fig. 2a ($L = 114$ μm, $h_{max1} = 0.204$ mm, $h_c = 0.144$ mm, $\varepsilon_c = 0.072°$, $h_\delta = 0.163$ mm) and by Fig. 2b ($L = 279.5$ mm, $h_{max1} = 0.32$ mm, $\varepsilon_c = 0.053°$, $h_\delta = 0.345$ mm). According to the Cornu spiral, the amplitude difference of the bands in the diffraction pattern and the incident light without a screen, as in the case of interference between the edge wave and the incident wave, is equal to the amplitude of the light in the shadow at the same values of h, that is, $\sqrt{J_{SH}} = \sqrt{J_b} - \sqrt{J_1}$. This is the second cause of the equality between the intensities of the diffraction bands according to Fresnel and their values based on Young's idea. However, Fresnel's notions are valid only for $J_1 = \text{const}$ and for a larger width of the incident beam in the plane of the screen. When J_1 is not constant, for example, it decreases with approach to the edge of the beam (across its width) and is constant on the shadow boundary, then the form and dimensions of the Cornu spiral are changed. Consequently, J_{SH} and ($\left(\sqrt{J_b} - \sqrt{J_1}\right)^2 = J_{SH}$ also differ. If the light in the shadow and in the bands is due to the edge wave, then the indicated quantities remain their former values, as confirmed by experiment.



a



b

FIG. 2. Distribution of the light intensity in the shadow from the screen for an incident plane wave for L = 114.2 mm (a) and L = 279.9 mm (b).

TABLE XII.

$J_i = f(h);\ J_{1e} = 26.1$						$J_C = J_b = 26.1$				
Band	J_b	J_i	J_b/J_i	J_c	α_c	$\alpha_{ie} \pm \alpha_r$	J'_b	J'_b/J_{ie}	J_C/J_{ie}	J_b/J_C
max_1	32.3	23.1	1.4	0.773	0.88	5.988	35.9	1.374	1.374	1
min_1	15	19.6	0.76	0.308	0.555	4.554	20.7	0.794	0.773	1.03
max_2	20.8	17	1.23	0.194	0.44	5.549	30.8	1.18	1.201	0.98
min_2	11.3	14	0.81	0.143	0.379	4.73	22.4	0.857	0.835	1.03
max_3	13.9	11.5	1.21	0.114	0.338	5.447	29.7	1.137	1.157	0.98
min_3	8.1	10	0.82	0.093	0.305	4.804	23.1	0.884	0.897	0.99
max_4	9.8	8.1	1.21	0.08	0.282	5.391	29.1	1.113	1.126	0.99

We shall demonstrate this result using data from Table XII, where J_b is the intensity of the bands in the experiment with $J_i(h)$ at l = 24 mm, L = 99.5 mm, λ = 0.53 mm, $a_r = \sqrt{J_r}$, $J_r = \left(\sqrt{J_b} - \sqrt{J_i}\right)^2$, $a_{1e} = \sqrt{J_{ie}}$, J_C is the band intensity according to the Cornu spiral. Using the values of a_r, one can calculate the intensity J'_b of the bands at constant $J_i = J_{ie}$ from the formula $J'_b = (a_{ie} \pm a_r)^2$. As can be seen from the ratio J'_b/J_C, the obtained values are practically equal to J_C. So, $J_r = J_{SH}$ does not depend on the distribution of J_i across the wave front.

The inadequacy of Fresnel's ideas follows also from the lower values of J_d at the shadow the boundary compared with the experimental values (see Figs. 1–3).

REFERENCES

1. Yu.I. Terent'ev, Optika Atmosfery **2**, No. 11, 1141 (1989).
2. O. Fresnel, *Selected Works in Optics* [translated from the French by G.S. Landsberg], Moscow (1955).
3. S.E. Frish and A.V. Timoreva, *A Course of General Physics* (GITTL, Moscow, 1951).