THE EFFECT OF THE INTERACTION OF OPTICAL PULSES IN A WEAKLY ABSORBING NONLINEAR MEDIUM

V.A. Donchenko, M.V. Kabanov, J.V. Lugin, A.A. Nalivaĭko, and A.V. Shapovalov

V.D. Kuznetsov Siberian Physico technical Scientific-Research Institute at the State University and Institute of Atmospheric Optics, Siberian Branch of the Academy of Sciences of the USSR, Tomsk

Received July 26, 1969

The effect of linear absorption on the dynamics of the initial pulse as it propagates in a weakly nonlinear medium under conditions of soliton formation is studied. It is shown by numerical methods that the process of decay of the initial pulse is stable. It was found that the soliton-like pulses into which the starting pulse decays, resulting in self-compression followed by spatial separation of the soliton pulses, merge. Some characteristic features of the phenomenon discovered, including the fact that it has a threshold with respect to the starting pulse width, are discussed.

Optical models are an important link in the study of the physical characteristics of the propagation of electromagnetic radiation in different media. Although many models have now been created for studying different optical properties and characteristics of the medium (absorption and refraction coefficients, scattering matrices, etc.), work in this direction is far from completion. Models that reflect the characteristic features of the dynamics of propagation of an electromagnetic pulse should continue to be investigated; this will lead to a deeper understanding of the mechanism of interaction of radiation with the medium. This is also true with regard to different nonlinear phenomena that accompany the propagation of an optical pulse in the atmosphere.

In our prior works[1,2] we studied the propagation of an optical pulse in a weakly nonlinear (cubic) medium in the region of resonance absorption. The model describing the evolution of the pulse was constructed based on the nonlinear Schrödinger equation (NS) with a small perturbation. Under the conditions of the atmosphere the small perturbation can be replaced by linear losses. The equation has the form

$$i\,\frac{\partial W}{\partial s} + \frac{1}{2}\,\frac{\partial^2 W}{\partial q^2} + \chi\,|W|^2 W = R(W), \tag{1}$$

where $s = t/T_0$; $q = (z - ct)/z_0$; t is the time; z is the spatial coordinate in the direction of propagation of the pulse; $z_0 = (T_0c/k)^{1/2}$; $W = E/E_0$; χ is the nonlinearity parameter; R is the perturbation, which also includes the transverse part of Laplacian; and, E is the field amplitude. The details of the derivation of Eq. (1) and an explanation of the notation employed in it can be found in Refs. 1 and 2 (see also the monograph Ref. 3 for a discussion of the derivation of Eq. (1)). In Ref. 2 the decay of the initial pulse into soliton-like pulses in the course of evolution in the absence of linear and nonlinear losses (the perturbation $R(W) = 0$) was investigated by the method of the inverse problem of scattering as well as by numerical methods.

In this paper the investigations described in Ref. 2 are continued under the assumption that linear losses exist. The perturbation has the form

$$R(W) = -\,i\alpha W, \quad \alpha > 0, \tag{2}$$

which corresponds to linear absorption. Thus the problem can be formulated in the form

$$iU_{,s} + \frac{1}{2}\,U_{,qq} + \left(|U|^2 + i\alpha\right) U = 0, \tag{3}$$

where $U = \rho \cdot W$, $\rho = \sqrt{\chi}$, $\chi > 0$. The initial pulse is assumed to be rectangular:

$$U(q) = \begin{cases} \rho, & a \le q \le a + b \\ 0, & a + b < q,\ q < a. \end{cases} \tag{4}$$

According to Ref. 2 the main parameter of the problem is the area of the initial pulse

$$S = \int\limits_{-\infty}^{\infty} |U(q)|\, dq = b\cdot\rho. \tag{5}$$

The parameter b corresponds to the width of the pulse (4) and ρ is the amplitude. The number N of solitons formed is estimated in the method of the inverse problem of scattering as the number of points in the discrete spectrum of the associated problem with the potential $U(q)$ of the form (4). A detailed calculation was performed in Ref. 1. For fixed S

$$N = \mathrm{int}\left[\frac{S}{\pi} + \frac{1}{2}\right]. \tag{6}$$

where int $(\cdot)$ means integer part of the argument.

A real pulse (the envelope) does not have steep fronts, but the direct analysis of a real pulse is complicated by mathematical difficulties. The model problem for a rectangular pulse can be analyzed[1,2], and the characteristics revealed by the numerical methods have a simple meaning: soliton formation, interaction of solitons, etc. The characteristics found numerically in Ref. 2 for a real pulse correspond to the dynamics of the model pulse; this makes it possible to understand and interpret the characteristics of the real pulse also. We point out once again that in this paper we continue our investigations of the model pulse (rectangle) in order to take into account the effect of additional factors (absorption by the medium and the width of the initial pulse).

We shall now examine the results of the numerical solution of Eqs. (3) and (4) on a computer. Figures 1–5 show graphs illustrating the process of decay of a rectangular pulse under conditions of linear absorption. The interval of integration over the variable q is the same for all graphs: (0, 20). The time step s is equal to $0.314 \cdot 10^{-2}$. The graphs of $|u|$ are presented for the following values of the variable s: $s = 0$ (1), 1.5710 (2), 2.3565 (3), 3.142 (4), 3.9275 (5), and 4.7130 (6). Figure 1 shows the dynamics of a pulse of area $S = 2$, for which according to Eq. (6) $N = 1$. The parameter $\alpha = 0.1$. It is observed that a stable configuration is formed (curve *2*, *3*, and *4* in Fig. 1); this configuration decays owing to absorption (the curve *1* is the starting pulse).

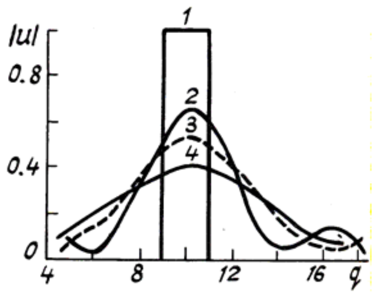

FIG. 1.

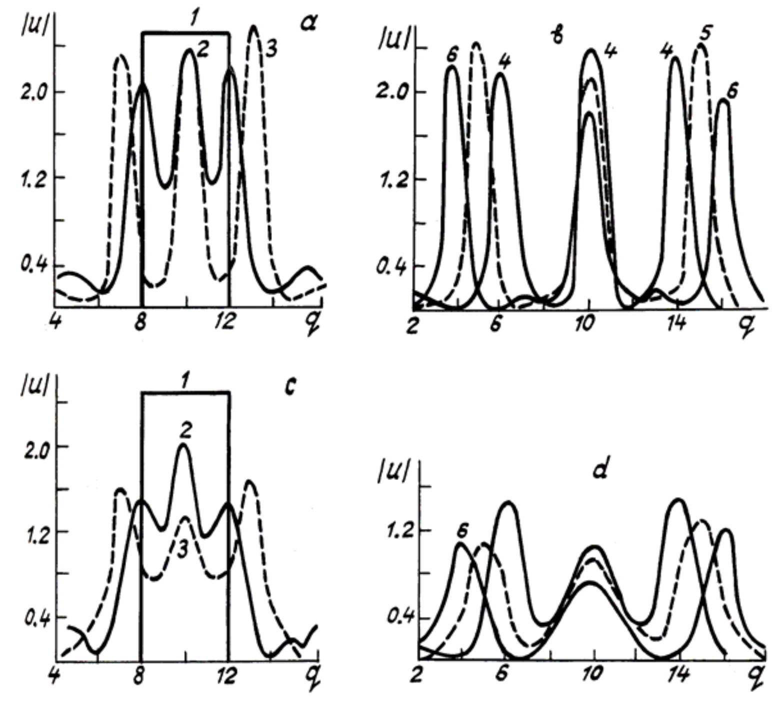

FIG. 2.

Figures 2*a* and *b* show the dynamics of a pulse for $S = 9$, which, according to Eq. (6), corresponds to $N = 3$. For Figs. 2*a* and *b* the parameter $\alpha = 0.001$. As absorption increases (in Figs. 2*c* and $\alpha = 0.1$) the qualitative picture of the decay process remains the some; only the degree of the decay increases. We note that the decay process is observed even for large values of the absorption ($\alpha = 0.3$), for which a perturbation $R(U)$ of the form Eq. (2) cannot be regarded as small in Eq. (3). This indicates that the decay process is stable with respect to perturbations and confirms the conclusion drawn in Ref. 1 based on qualitative estimates.

A more detailed analysis of the problem (3) makes it possible to study other aspects of the dynamics of the decay of the starting pulses and to draw additional conclusions. We performed a series of computer calculations of the dynamics of the pulse (4) determined by Eq. (3) for some fixed values of S in the case of constant linear absorption. In particular, we trace the dynamics of the pulse (4) with $S = 8$, which corresponds to the formation of two (possible three) soliton-like pulses in accordance with Eq. (6). The parameter α was set equal to 0.01. The calculations were performed for values of the parameter b (the width of the initial pulse) ranging from 2 to 12 with a step equal to 1. The value used for the absorption parameter $\alpha = 0.01$ (which corresponds to the absorption coefficient $\kappa \sim 10^{-4} - 10^{-5}$ cm^{-1} for the frequency $\omega \sim 10^{14}$ s^{-1} of a ruby laser; see estimates in Ref. 1) can differ by several orders of magnitude from the real values of the absorption coefficient for quite strong lines under atmospheric conditions. This indicates that in order of magnitude the values of α studied (for which the effect is most clearly observed in numerical experiments) are consistent with the real conditions of absorption in the atmosphere. The conclusion that the decay of the starting pulse is stable should remain valid for real conditions of absorption in the atmosphere. In the estimates presented the pulse width $\sim 10^{-9}$ s.

As an illustration Figs. 3–5 show graphs of $|U(sq)|$ which reflect some characteristic features of the dynamics of the pulse (4).

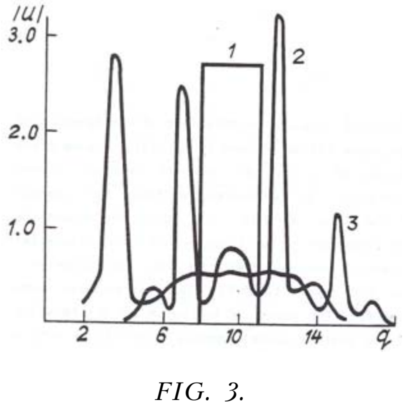

FIG. 3.

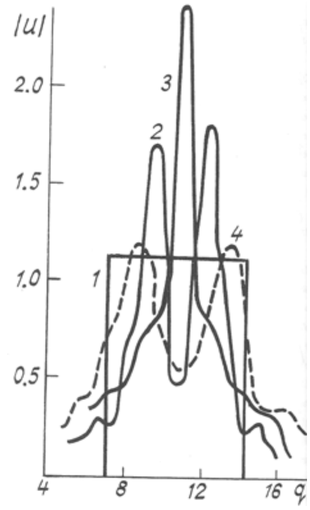

FIG. 4.

The graphs in Fig. 3 are presented for $b = 3$ and $s = 0$ (1), 1.571 (2), and 3.142 (3). Two sharp peaks are observed. These peaks separate in space as s increases. Figure 4 corresponds to $b = 7$ and $s = 0$ (1), 1.157 (2), 3.927 (3), and 6.284 (4). At first two soliton-like pulses are formed (curve *2* in Fig. 4). Then these pulses merge, which leads to self-compression (curve *3*); the maximum value $|U| = 2.42$ is achieved when $q = 10.5$. Then the pulse once again separates into two peaks (curve *4*), which separate in space as the time s increases. Figure 5 illustrates the dynamics of the pulse for even larger values of the initial pulse width.

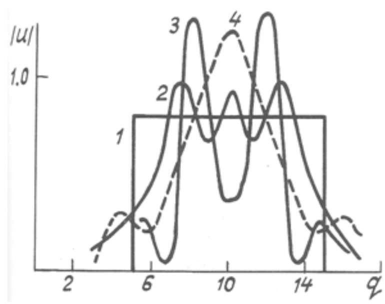

FIG. 5. The dynamics of a pulse for $S = 8$, $\alpha = 0.01$, $b = 10$, and $S = 1$ (1), 500 (2), 1500 (3), and 2000 (4) steps.

We can draw the following conclusions fro» the numerical calculations and Figs. 3–5. When the threshold conditions of soliton formation are satisfied (for example, $S = 8$ is sufficient for the formation of several solitons) self-compression is not observed for narrow initial pulses. It can be concluded that a threshold value of this parameter exists. Comparing Figs. 4 and 5 shows that the moment at which the field $|U|$ reached a maximum value also depends on the parameter b; this moment occurs increasingly later as b increases. We note once again that the existence of linear absorption does not destroy the process of formation of soliton-like pulses. On the contrary, the picture of soliton formation becomes clearer. This is explained by the fact that for $\alpha \neq 0$ a nonzero relative velocity of the solitons along the q axis appears; this causes the soli tons to be more sharply separated in space.

Thus the evolution of the pulse has been traced in detail by numerical methods. Three stages of the evolution of the signal were identified: decay of the starting pulse, interaction of the soliton-like pulses formed, and spatial separation of the latter pulses. Under certain conditions an effect similar to self-compression of a pulse arises at the second stage; this effect can apparently be explained by the nonlinear interference of soliton-like pulses. A qualitative investigation of the dynamics of this phenomenon showed that it depends on the width of the initial pulse and that it has a threshold as a function of this width. We note that although the nonlinear Schrödinger equation with a perturbation correction (2) cannot be integrated by the method of the inverse problem of scattering, the numerical solution of the problems shows that the decay process does exist and that it is stable; this fact is undoubtedly of interest for applied investigations.

The question of the conditions under which solitons are formed and can be observed was discussed in Ref. 2. The energy and other parameters of the pulses can be estimated based on the results presented in Refs. 1-3 and additional investigations of the conditions the model under study describes the real atmosphere, but this falls outside the scope of this work and will be discussed in a separate paper.

REFERENCES

1. E.V. Lugin, and A.V. Shapovalov, Izv. Vysch. Uchebn. Zaved. SSSR, Fiz. No. 9, 102 (1987); ibid., No. 2, 36 (1989).
2. V.A. Donchenko, M.V. Kabanov, E.V. Lugin, A.A. Nalivaiko, and A.V. Shapovalov, Atm. Opt., **1**, No. 1, 67 (1988).
3. G.M. Zaslavskii and R.Z. Sagdeev, *Introduction to Nonlinear Physics* (Nauka, Moscow, 1988).