Peculiarities in remote sensing of spatially distributed non-stationary objects using complex signals

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We consider peculiar features of using compound signals in remote sensing of spatially distributed non-stationary objects (SDNO) like, for instance, meteorological objects or vortex inhomogeneities in the atmosphere. We also analyze the influence of the shape of the ambiguity function (AF) of a vector sounding signal on the accuracy of the estimate of the matrix response function (MRF) of a SDNO, which is its compact description, allowing joint representation of range distribution, velocities, and polarization parameters of elementary reflectors, a set of which forms the SDNO. It is shown that for correct MRF estimation signals are needed, ambiguity function of which has a needle shape. The results of simulation of an algorithm for MRF estimation in cases of using three types of signals: phase-code-modulated signal, chirp signals with the bandwidth duration product $N = 10^4$, and a simple signal with a unity bandwidth duration product.

Introduction

The onrush development of the technology for formation and processing signals, including those in the optical range, enables one to use complex signals with large bandwidth duration product for solving the problems of remote sensing of the environment. Use of such signals makes it possible to decrease the peak power of radiation at keeping the potential of the system and the parameters of range and velocity resolution of sounding. To achieve this, one needs clear understanding of the physics of formation of the system response under matched filtering of complex signals reflected by spatially distributed non-stationary objects (SDNO) formed by an ensemble of "small" inhomogeneities, located at different distances and moving at different speeds. Correct estimation of the matrix response function of a SDNO, in which its polarization and coordinates are presented, is only possible if certain requirements to the shape of the uncertainty function (UF) of the sensing signal are met.

Matrix response function of a spatially distributed non-stationary object and algorithm for its estimation

The matrix response function (MRF) of an object sounded with radar is the development of the conception of its scattering phase matrix. Such a matrix $\mathbf{S}_i$ formalizes transformation of the vector $\mathbf{u}_0$ describing the complex amplitudes of orthogonal polarized components of a plane sounding wave to the vector $\mathbf{u}_{pi}$ the plane wave reflected backwards, due to diffraction of the field on an immobile point object considered as a spatial inhomogeneity: $\mathbf{u}_{pi} = \mathbf{S}_i \cdot \mathbf{u}_0$, where the 2×2 operator $\mathbf{S}_i$ is set by four complex coefficients $\dot{S}^i_{ij}$. In the general case, the vector $\mathbf{u}_0$ has the form

$$\mathbf{u}_0(t,\omega) = \left(\dot{f}_1(t,\omega);\ \dot{f}_2(t,\omega)\right)^{\mathrm{T}}, \quad (1)$$

where $\dot{f}_1(t,\omega)$ and $\dot{f}_2(t,\omega)$ are the complex functions (t is the current variable, and ω is the parameter) describing the frequency and time structure of the orthogonal polarized components of the emitted field, T is the symbol of transposition.

In the general case of motion of a point object with the radial velocity V_i, its coordinates (distance and radial velocity) and polarization properties can be presented together in the shape of a matrix response function:

$$\mathbf{g}_i(\tau,\Omega) = \delta(\tau_i,\Omega_i) \cdot \mathbf{S}_i, \quad (2)$$

where $\delta(\tau_i, \Omega_i)$ is the delta-function set at the point with coordinates (τ_i, Ω_i); $\tau_i = 2D_i/c$ is the delay time of the signal $\mathbf{u}_{pi}(t, \omega)$ reflected from it relative to the incident sounding signal $\mathbf{u}_0(t, \omega)$, and $\Omega_i = 2V_i/\lambda_0$ is the Doppler shift of the reflected signal frequency caused by the radial motion of the reflector (c is the speed of propagation of the wave, λ_0 is the wavelength of the incident wave). In such a description of the scattering properties of the object, correspondence between the incident and reflected vector signals is determined by the relationship in the form of a bilateral matrix convolution

$$\mathbf{u}_{pi}(t,\omega) = \mathbf{g}_i(\tau,\Omega) * \mathbf{u}_0(t,\omega) =$$

$$= \int\int \mathbf{g}_i(\tau,\Omega) \cdot \mathbf{u}_0(t-\tau,\omega-\Omega)\,\mathrm{d}\tau\mathrm{d}\Omega. \quad (3)$$

In the frameworks of the conception of "sparkling points", one can present SDNO in the form of a set of elementary point reflectors distributed over space and having, in general case, different velocities of radial motion relative to the reference point. Also one can present the full response of a SDNO in the form of the sum $\mathbf{u}_\Sigma(t,\omega)$ of responses $\mathbf{u}_{pi}(t,\omega)$ from each of the elementary reflectors forming it:

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$$\mathbf{u}_\Sigma(t,\omega) = \sum_{i=1}^{N} \mathbf{u}_{pi}(t,\omega) =$$

$$= \underbrace{\sum_{i=1}^{N} \mathbf{g}_i(\tau,\Omega)}_{\mathbf{G}_\Sigma(\tau,\Omega)} * \mathbf{u}_0(t,\omega) = \mathbf{G}_\Sigma(\tau,\Omega) * \mathbf{u}_0(t,\omega), \quad (4)$$

the matrix response function $\mathbf{G}_\Sigma(\tau,\Omega)$ is equal to the sum of MRF of elementary reflectors, the set of which forms the SDNO. In the general case, the problem of joint estimation of polarization and coordinates of the objects sounded is to estimate the matrix response function $\mathbf{G}_\Sigma(\tau,\Omega)$, which involves these parameters, using the reflected signal $\mathbf{u}_\Sigma(t,\omega)$ recorded at the known sounding signal $\mathbf{u}_0(t,\omega)$.

As was shown[1] in the case when reflected signal $\mathbf{u}_\Sigma(t,\omega)$ is observed in the presence of a white-noise unpolarized component, the optimal estimate $\hat{\mathbf{G}}_\Sigma(\tau,\Omega)$ of the response function $\mathbf{G}_\Sigma(\tau,\Omega)$ is formed by use of a vector bilateral matrix convolution of the reflected and sounding vector signals set by the relationship

$$\mathbf{J}(t,\omega) = \mathbf{u}_\Sigma(t,\omega) \odot \mathbf{u}_0(\tau,\Omega) =$$

$$\iint \mathbf{u}_\Sigma(t-\tau,\omega-\Omega) \otimes \mathbf{u}_0^*(\tau,\Omega)\,\mathrm{d}\tau\mathrm{d}\Omega =$$

$$= \mathbf{G}_\Sigma(\tau,\Omega) * \underbrace{\mathbf{u}_0(t,\omega) \odot \mathbf{u}_0(\tau,\Omega)}_{\mathbf{X}_0(t,\omega)} =$$

$$= \mathbf{G}_\Sigma(\tau,\Omega) * \mathbf{X}_0(t,\omega) \Rightarrow \hat{\mathbf{G}}_\Sigma(\tau,\Omega), \quad (5)$$

where "$\odot$" is the symbol of vector convolution, "$\otimes$" is the symbol of Kroneker product. If certain requirements to the signals $\dot{f}_1(t,\omega)$ and $\dot{f}_2(t,\omega)$ have been met in Eq. (1), such that the following relationship is fulfilled

$$\mathbf{X}_0(t,\omega) = \mathbf{u}_0(t,\omega) \odot \mathbf{u}_0^+(\tau,\Omega) =$$

$$= \begin{pmatrix} \dot{B}_{11}(t,\omega); & \dot{B}_{12}(t,\omega); \\ \dot{B}_{21}(t,\omega); & \dot{B}_{22}(t,\omega); \end{pmatrix} \approx \dot{B}(t,\omega)\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad (6)$$

the error in estimating MRF formed according to Eq. (5) is proportional to the level of δ-correlation between the signals $\dot{f}_1(t,\omega)$ and $\dot{f}_2(t,\omega)$ for all possible shifts in frequency and time delay and is the best according to the criterion of the maximum signal-to-noise ratio at the output of the system processing the vector signal. The shape of the functions $\dot{B}_{11}(t,\omega)$ and $\dot{B}_{22}(t,\omega)$ is identical to that of the generalized autocorrelation functions of the signals $\dot{f}_1(t,\omega)$ and $\dot{f}_2(t,\omega)$, respectively. The shape of the functions $\dot{B}_{12}(t,\omega)$ and $\dot{B}_{21}^*(t,\omega)$ is identical to that of the generalized correlation functions of the signals $\dot{f}_1(t,\omega)$ and $\dot{f}_2(t,\omega)$. The signals, for which the relationship (6) has been fulfilled, are called orthogonal,[2] and the function $\mathbf{X}_0(t,\omega)$ is called the matrix uncertainty function (MUF) of the vector signal $\mathbf{u}_0(t,\omega)$.[1]

The general scheme of the algorithm (5) for optimal estimation of MRF of a spatially distributed object[1,3,4] is shown in Fig. 1.

Two scalar signals are formed at the output of the device for formation of the orthogonal components of the sounding vector signal (FOCVSS): $\dot{f}_1(t,\omega)$ and $\dot{f}_2(t,\omega)$, obeying the relationship (6), which drive, through the decoupling device (circulator), the branches of the polarization divider PD of the transmitting antenna. The wave is emitted to space along the direction toward SDNO, which is described by the vector $\mathbf{e}_0(t) = \left(\dot{f}_1(t);\dot{f}_2(t)\right)^{\mathrm{T}}$.

The wave $\mathbf{e}_p(t)$ reflected from the SDNO is received by the same antenna (detector), and two scalar signals $\dot{f}_{p1}(t)$ and $\dot{f}_{p2}(t)$ are formed at the output of the orthogonal branches of the polarization divider, the set of which composes the observed vector signal $\mathbf{u}_p(t) = \left(\dot{f}_{p1}(t);\dot{f}_{p2}(t)\right)^{\mathrm{T}}$. Each of the orthogonal components $\dot{f}_{p1}(t)$ and $\dot{f}_{p2}(t)$ of this signal is processed by two-dimensional filters SF_{12} matched to the signals

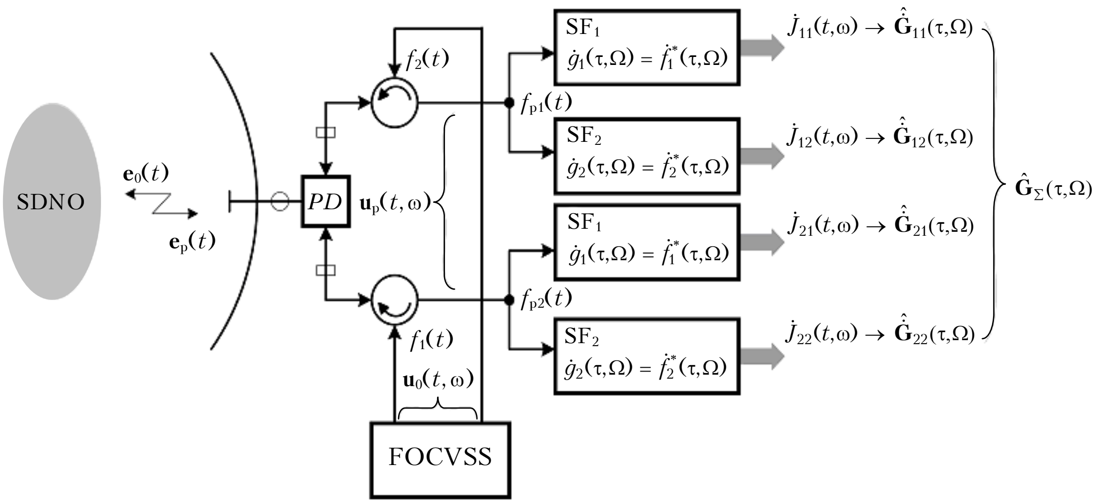

Fig. 1. Algorithm for estimation of the matrix response function of a spatially distributed non-stationary object.

$\dot{f}_1(t,\omega)$ and $\dot{f}_2(t,\omega)$, at the output of which the scalar responses $\dot{J}_{ij}(t,\omega)$ are formed, the set of which, in their turn, is the estimate $\hat{\mathbf{G}}_\Sigma(\tau,\Omega)$ of the matrix response function of the SDNO, provided that the relationship (6) is valid for the signals $\dot{f}_1(t,\omega)$ and $\dot{f}_2(t,\omega)$. In practice, the two-dimensional matched filter is realized in the form of a multi-channel (in frequency) system for joint processing of the received signals.[5]

Requirements to the signals determining the frequency and time structure of the orthogonally polarized components of the sounding flux

The following relationships follow from Eq. (6) for the signals $\dot{f}_1(t,\omega)$ and $\dot{f}_2(t,\omega)$, determining the frequency and time structure of the components of the sounding flux with the orthogonal polarization:

$$\dot{B}_{11}(t,\omega) \approx \dot{B}_{22}(t,\omega) = \dot{B}(t,\omega), \qquad (7)$$

$$\frac{\left|\dot{B}_{12}(t,\omega)\right|}{\left|\dot{B}(0,0)\right|} \approx \frac{\left|\dot{B}_{21}(t,\omega)\right|}{\left|\dot{B}(0,0)\right|} = \delta \ll 1 \to 0. \qquad (8)$$

The formed estimate of the MRF of the spatially distributed object is described by the formula

$$\mathbf{J}(t,\omega) = \begin{pmatrix} \dot{J}_{11} & \dot{J}_{12} \\ \dot{J}_{21} & \dot{J}_{22} \end{pmatrix} = \mathbf{G}_\Sigma(\tau,\Omega) * \mathbf{X}_0(t,\omega) \approx$$

$$\approx \begin{pmatrix} \dot{G}_{11}(\tau,\Omega)*\dot{B}(t,\omega); & \dot{G}_{12}(\tau,\Omega)*\dot{B}(t,\omega) \\ \dot{G}_{21}(\tau,\Omega)*\dot{B}(t,\omega); & \dot{G}_{22}(\tau,\Omega)*\dot{B}(t,\omega) \end{pmatrix} =$$

$$= \mathbf{G}_\Sigma(\tau,\Omega)*\dot{B}(t,\omega) \Rightarrow \hat{\mathbf{G}}_\Sigma(\tau,\Omega). \qquad (9)$$

It is obvious, that in the ideal case, when the mutual correlation $\left|\dot{B}_{12}(t,\omega)\right|$ of the signals $\dot{f}_1(t,\omega)$ and $\dot{f}_2(t,\omega)$ has been equal to zero for all possible relative shifts in frequency and time, and the forms of the generalized autocorrelation functions $\dot{B}_{11}(t,\omega)$ and $\dot{B}_{22}(t,\omega)$ of these signals have degenerated to delta-function, i.e., when the following relationship holds:

$$\mathbf{X}_0(t,\omega) = \mathbf{u}_0(t,\omega) \odot \mathbf{u}_0^+(\tau,\Omega) = \overbrace{\delta(0,0)\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}}^{\text{matrix delta-function}}, \qquad (10)$$

the formed estimate (9) is absolutely precise (the filtering property of the delta-function).

The effect of shape of the MUF of a sounding signal on the accuracy of estimation of the matrix response function of a SDNO

To qualitatively estimate the effect of the shape of the MUF of a sounding signal on the accuracy of measuring the MRF, computer simulation of the algorithm for measuring MRF was performed (see Eq. (9) and Fig. 1). Simulation was carried out using the MathCAD software package. A set of N independent elementary (point) spatially distributed reflectors with different radial velocities of motion V relative the transmitting-receiving device (antenna) and different polarization properties set by the backscattering phase matrix of the ith elementary reflector of the SDNO model was taken as the model of the SDNO. The matrix response function of the model of the SDNO was described by the formula

$$\mathbf{G}_\Sigma(\tau,\Omega) = \sum_{i=1}^{N} \mathbf{g}_i(\tau_i,\Omega_i) = \sum_{i=1}^{N} \Phi(\tau',\Omega')*$$

$$\underbrace{*\left\{\delta(\tau_i,\Omega_i)\cdot \overbrace{\tilde{\mathbf{R}}(\theta_0^i)\cdot\tilde{\mathbf{F}}(\varepsilon_0^i)}^{\tilde{\mathbf{L}}(\varepsilon,\theta)}\cdot \begin{pmatrix} \dot{\lambda}_1^i & 0 \\ 0 & \dot{\lambda}_2^i \end{pmatrix} \cdot \overbrace{\mathbf{F}(\varepsilon_0^i)\cdot\mathbf{R}(\theta_0^i)}^{\mathbf{L}(\varepsilon,\theta)}\right\}}_{\mathbf{g}_i(\tau,\Omega)}, \qquad (11)$$

where $\mathbf{F}(\varepsilon_0^i)$ and $\mathbf{R}(\theta_0^i)$ are the unitary operators, whose product determines the operator $\mathbf{L}(\varepsilon,\theta) = \mathbf{F}(\varepsilon_0^i)\cdot\mathbf{R}(\theta_0^i)$ of transformation of the coordinate system of description of the backscattering phase matrix of the ith elementary reflector in passing from its polarization eigenvector basis to the Cartesian basis[1]; $\delta(\tau_i,\Omega_i)$ is delta-function set at the point with coordinates τ_i on the plane "delay time – Doppler shift", Ω_i ("~" is the symbol of transposition); $\Phi(\tau',\Omega')$ is the two-dimensional smoothing function determining the correlation of the neighbor reflectors of the model (two-dimensional Gaussian function with the peak width on the time axis being $\Delta\tau' = 2$ ns and on the frequency axis $\Delta\Omega' = 50$ kHz).

Distributions of the values of the distance D_i, radial velocity V_i, eigenvalues $\dot{\lambda}_{1(2)}^i$ of the backscattering phase matrix, angles of ellipticity ε_0^i, and orientation θ_0^i of the polarization eigenvector basis for each of the elementary reflectors of the model were set by independent random number generators with the known distribution laws. The parameters of the distributions of the aforementioned values were selected so that the main volume of the two-dimensional function of the MRF norm is located in the limited area on the plane "delay time – Doppler shift". The shape of the MRF of the model and its position on the plane [τ, Ω] are presented in Fig. 2.

In modeling three kinds of vector signals were used as sounding signals: noise-like phase-code-modulated (PCM) signals, chirp signals, and simple signal with a unity bandwidth duration product ($N = \Delta f_s \tau_s$).

The matrix functions of uncertainty of such vector signals have qualitative differences in the form. Durations of PCM and chirp signals are identical and are equal to $\tau_p = 20 \cdot 10^{-6}$ s, the bandwidth duration products are equal to $N = 10^4$. With such parameters, the cross section widths of the main peak of their uncertainty function along the axes of the frequency shift and time delay are equal to 50 kHz and 2 ns, respectively, and the spectrum width is $\Delta f_s \simeq 500$ MHz. The duration of simple signal was also taken equal to 2 ns.

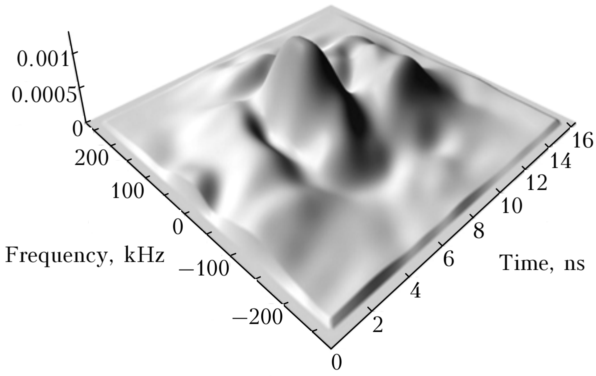

Fig. 2. Shape of the MRF norm of the model of spatially distributed object.

Thus, we have chosen, for modeling, the signals having equal frequency bands $\Delta f_s \simeq 500$ MHz while qualitatively different uncertainty functions (the shapes of UF of the selected signals can be found, for example, in Ref. 6). The vector PCM signal was formed using two orthogonal m-sequences of the length $N = 10^5 + 1$. The vector chirp signal was formed using two radio signals with counter directed linear change of frequency[1] relative to the common central point of the spectrum.

The results of simulation of the algorithm for estimation of MRF of the model of a SDNO (see Fig. 1) using the aforementioned sounding signals are shown in Fig. 3.

The results are presented in the form of two-dimensional functions on the plane "delay time – Doppler shift", gradations of gray scale represent the amplitude of the function.

Black color corresponds to the maximum value of the function marked by the corresponding figure. White color corresponds to the zero value of the shown function. The amplitude scale is transformed to gray scale according to linear law.

The forms of the absolute value of the element $\dot{G}_{11}(\tau,\Omega)$ and the norm $\sqrt{\| \mathbf{G}_\Sigma(\tau,\Omega) \|}$ of the model MRF, respectively, are shown in Figs. 3*a* and *e* for making a comparison of the initial MRF and its estimates obtained.

The shape of the obtained estimates of the absolute value of the element $\dot{G}_{11}(\tau,\Omega)$ and the absolute value of the norm $\sqrt{\| \mathbf{G}_\Sigma(\tau,\Omega) \|}$ of MRF in using the PCM sounding signal are shown in Figs. 3*b* and *f*.

Similar results obtained in using the chirp sounding signal are shown in Figs. 3*c* and *g*, and that using simple signal are shown in Figs. 3*d* and *h*.

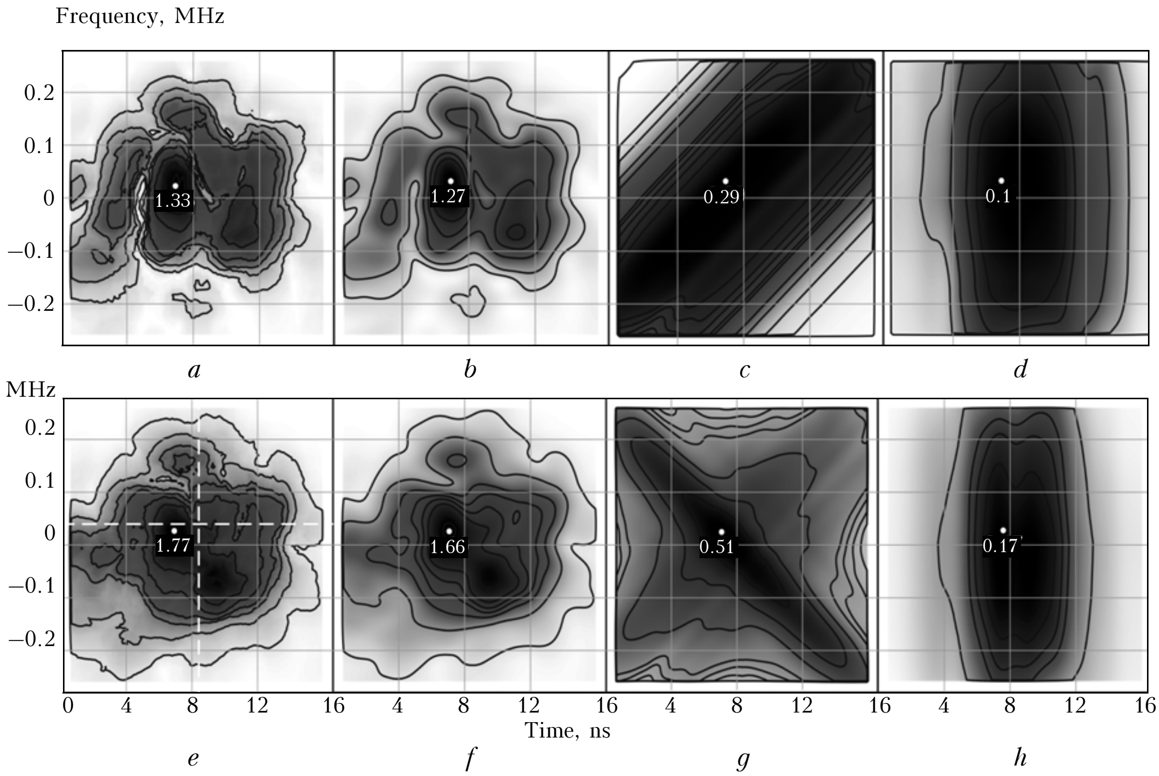

Fig. 3. Initial MRF of spatially distributed object and its estimates for different kinds of signal: absolute value of the element $\dot{G}_{11}(\tau,\Omega)$ of MRF of the SDNO model (*a*); norm of MRF of the SDNO model (*e*); estimate of the element $\dot{G}_{11}(\tau,\Omega)$ for FCM signal (*b*); estimate of the norm of MRF for FCM-signal (*f*); estimate of the element $\dot{G}_{11}(\tau,\Omega)$ for LFM-signal (*c*); estimate of the norm of MRF for LFM-signal (*g*); estimate of the element $\dot{G}_{11}(\tau,\Omega)$ for "simple" signal (*d*); estimate of the norm of MRF for "simple" signal (*h*).

As follows from the results shown in Fig. 3, the most accurate estimate of the MRF shape of the model is formed while using the PCM sounding signal.

In all other cases, it is practically impossible to obtain the data on the joint distribution of the velocities and the distances to the elementary reflectors, the ensemble of which forms the spatially distributed temporally non-stationary object.

Quantitative characteristics of the accuracy of estimates of the MRF of a SDNO were obtained by calculating the deviations of the formed estimates, normalized by the amplitude, from the initial one, also normalized.

The value of the following functional was taken as the integral measure of the error:

$$\Lambda = d\{\mathbf{G}_{\Sigma}(\tau,\Omega);\ \hat{\mathbf{G}}_{\Sigma}(\tau,\Omega)\} =$$

$$= \frac{1}{V_0}\iint\limits_{-\infty}^{+\infty} \left\| \frac{\mathbf{G}_{\Sigma}(\tau,\Omega)}{\|\mathbf{G}_{\Sigma}(\tau,\Omega)\|} - \frac{\hat{\mathbf{G}}_{\Sigma}(\tau,\Omega)}{\|\hat{\mathbf{G}}_{\Sigma}(\tau,\Omega)\|} \right\| \mathrm{d}\tau \mathrm{d}\Omega, \frac{\pi}{3} \quad (12)$$

which determines the volume of the difference of the normalized MRF of the model $\mathbf{G}_{\Sigma}(\tau,\Omega)$ and its normalized estimate $\hat{\mathbf{G}}_{\Sigma}(\tau,\Omega)$. The value V_0 is equal to the volume of the normalized MRF of the model of the object.

The results obtained are the following: for the PCM signal: $\Lambda = 0.043$ (4.3%); for a chirp signal 0.76 (76%); and for simple signal 0.52 (52%).

To illustrate the amplitude deviations of the formed estimates from the initial MRF of the model of the object, the cross section of the obtained estimates and the initial MRF of the model of the object are shown in Fig. 4. The coordinates of the cross section planes are shown in Fig. 3*e* by dotted lines.

Conclusions

Comparative analysis of the calculated data makes it possible to draw the following conclusions. Correct joint estimation of coordinates and velocities as well as of the full reflectivity of spatially distributed non-stationary objects requires that certain requirements to the shape of the uncertainty function of the sound signal be met in the measurement system. In an active radar system (both optical and radio-wave), the uncertainty function of the sounding signal appears as the instrumental function of the device, the peculiar hypothetical "window", through which the response function of the spatially distributed non-stationary object is viewed. In the frameworks of such an approach, the fact becomes almost clear, that complex signals, the uncertainty function of which has symmetrical, "needle" form, enable one to minimize the instrumental error in estimating the response function of the object sounded.

The results obtained can be useful for analysis of polarization parameters and the full specific reflectivity of real SDNO, for example, meteorological objects or vortex formations in the atmosphere at lidar sensing using complex signals, which would enable one to essentially increase the power potential of the system.

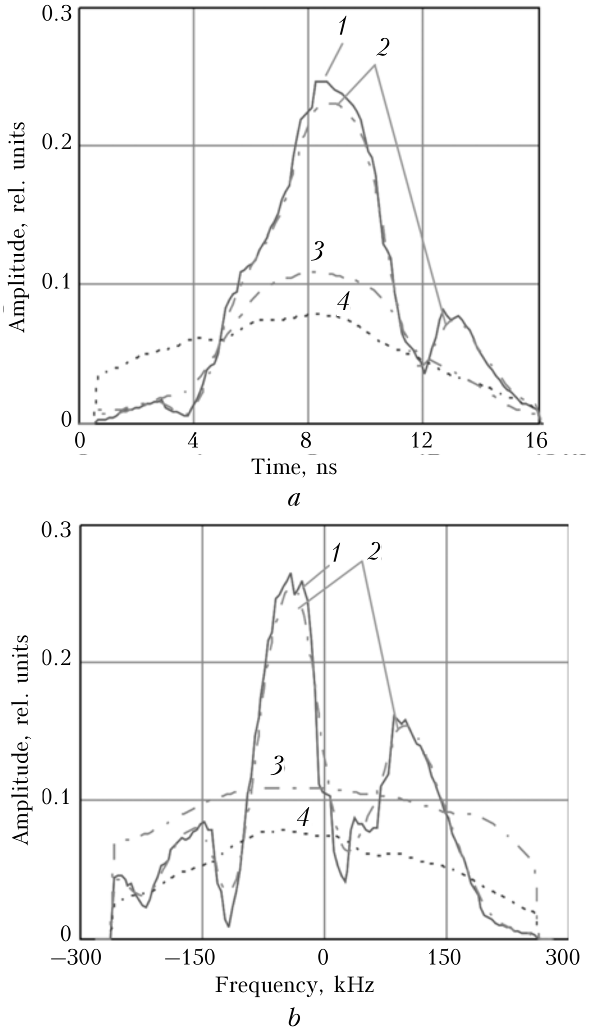

Fig. 4. Cross sections of the envelope of normalized MRF of the model (curve *1*) and of the normalized estimates of MRF formed while using PCM-signal (*2*), chirp-signal (*3*), and simple signal (*4*).

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