Statistical simulation of radiative transfer in optically anisotropic ice clouds

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We present a mathematical model of radiative transfer through the atmospheric clouds with optical anisotropy with respect to zenith angle of incident radiation. An algorithm of statistical modeling has been developed to simulate solar radiative transfer through anisotropic scattering medium taking into account multiple scattering. We present the results of numerical experiment for media comprising ice crystals having the shape of hexagonal cylinders of different orientation in space. The local optical characteristics of anisotropic medium (scattering phase functions and scattering cross sections) were calculated based on the geometric optics approach. We have compared the radiative properties of clouds for crystals isotropically orientated in a horizontal plane and in space. Possible influence of crystal orientation on albedo of the cloud layer and the shape of halo is demonstrated.

Introduction

It is well known that optical properties of ice clouds in the atmosphere depend on the direction of radiation propagation, that is, the ice clouds are optically anisotropic scattering medium.[1,2] The optical anisotropy occurs due to irregular shapes of ice crystals and specific features of their orientation in space. A number of optical phenomena associated with ice clouds such as different halos and parhelion are explained taking into account the optical anisotropy (see, e.g., Ref. 3 studying the transfer of IR radiation through optically anisotropic media).

At present, the physical and optical properties of ice clouds and their influence on atmospheric radiation balance have been studied insufficiently (the current state of the studies in this field has been discussed, for instance, in Refs. 2 and 4).

In this paper we describe the mathematical model and algorithm of statistical simulation of radiative transfer through optically anisotropic clouds taking into account multiple scattering. We also present some results of numerical experiment, which demonstrate how strongly the orientation of ice cloud particles can influence the properties of radiation fields.

1. Mathematical description of the radiative transfer through an optically anisotropic medium

The process of optical radiative transfer (without the account of polarization) can be described using the following integral equation with the generalized kernel[5–11]:

$$I(r,\ \omega,\ t)=\int\limits_{-\infty}^{t}\int\limits_{\Omega}\int\limits_{R^3} q(r',\omega')\,\sigma(r',\omega')\,g(\omega',\omega,r')\frac{e^{-\tau(r',\,r)}}{|r-r'|^2}\times$$

$$\times\delta\left(\omega-\frac{r-r'}{|r-r'|}\right)I(r',\omega',t')\delta\big(S(t-t',r)\big)\mathrm{d}r'\mathrm{d}\omega'\mathrm{d}t'+I_0(r,\omega,t);$$

$$\tau(r',r)=\int\limits_{r'}^{r}\sigma(\rho,\omega)\,\mathrm{d}\rho;\ r',\ r\in R^3;\ \omega',\ \omega\in\Omega=\{\omega\in R^3:|\omega|=1\}.$$

Here, $I(r,\ \omega,\ t)$ is the intensity of radiation along ω direction at the point r and time t, $q(r',\omega')$ is the photon survival probability (single scattering albedo) in collision at the point r' for ω' direction, along which the photon traveled before the collision, $\sigma(r',\omega')$ is the extinction coefficient at the point r' along the direction ω', $g(\omega',\omega,r')$ is the scattering phase function at the point r' (ω' is the direction before scattering and ω is the direction after scattering); $\int\limits_{\Omega} g(\omega',\omega,r')\mathrm{d}\omega=1$; $\tau(r',r)$ is the optical length of the interval $[r',\ r]$; δ is the Dirac delta function, $S(t,r)$ is the set of points in the three-dimensional space from which the photon can arrive at the point r along a straight line during time t, $I_0(r,\omega,t)$ is intensity of emission of source from the point r at the time t.

In the below discussion, we will consider the stationary transfer equation with the source emission constant in time

$$I(r,\omega)=\int\limits_{\Omega}\int\limits_{R^3} q(r',\omega')\,\sigma(r',\omega')g(\omega',\omega,r')\frac{e^{-\tau(r',r)}}{|r-r'|^2}\times$$

$$\times\delta\left(\omega-\frac{r-r'}{|r-r'|}\right)I(r',\omega')\mathrm{d}r'\mathrm{d}\omega'+I_0(r,\omega). \qquad (1)$$

Let us proceed to the ice cloud model and assume that the scattering medium is optically anisotropic and the *optical properties of the medium in the cloud depend only on the zenith (and not on the azimuth) angle of radiation propagating*. Then the equation (1) can be written as follows

$$I(r,\omega)=\int_0^{2\pi}\int_{-1}^{1}\int_{R^3} q_c(r',c')\,\sigma_c(r',c')g_{c\psi}(c',\psi',c,\psi,r')\frac{\mathrm{e}^{-\tau(r',r)}}{|r-r'|^2}\times$$

$$\times\delta\left(\omega-\frac{r-r'}{|r-r'|}\right)I(r',\omega')\mathrm{d}r'\mathrm{d}c'\mathrm{d}\psi'+I_0(r,\omega). \qquad (2)$$

In equation (2) we use the following notations: $\omega'=(a',b',c')$, c' is cosine of the zenith angle for direction ω'; ψ' and ψ are the azimuth angles (for instance, with respect to the horizontal OX-axis) for directions ω' and ω, respectively. Cosine of the zenith angle $c'=\langle\omega',e_z\rangle$ is the scalar product of ω' times the unit vertical normal e_z. Below, to simplify the notation, we will neglect the dependence of the functions σ_c, q_c, and $g_{c\psi}$ in Eq. (2) on r'.

The extinction coefficient of the medium (the function σ_c) and the probability of photon survival in a collision (the function q_c) depend only on the cosine of the zenith angle of the direction of radiation propagating. The scattering phase function $g_{c\psi}(c',\psi',c,\psi)$ has the meaning of the probability density of the scattering along the direction set by the cosine of the zenith angle c and azimuth angle ψ, at the direction of photon travel before the collision set by the cosine of the zenith angle c' and azimuth angle ψ'. The normalization condition for the scattering phase function has the form

$$\int_0^{2\pi}\int_{-1}^{1} g(c',\psi',c,\psi)\mathrm{d}c\mathrm{d}\psi=1$$

for all c' and ψ'.

Since we claim the independence of the optical properties of the medium on azimuth angle of the direction of radiation propagating, the scattering phase function $g_{c\psi}(c',\psi',c,\psi)$ virtually depends not on the pair of azimuth angles ψ' and ψ, but rather on their difference $\psi-\psi'$. Moreover, it is natural to assume the symmetry of the scattering phase function with respect to change of the azimuth angle, i.e.,

$$g_{c\psi}(c',\psi',c,\psi)=g_{c\psi}(c',\psi',c,-\psi),$$

which means that the scattering phase function $g_{c\psi}(c',\psi',c,\psi)$ depends, in fact, only on the absolute value of the difference between azimuth angles $\Delta\psi=|\psi-\psi'|$.

Note. In the case of optically isotropic media, when the optical properties of the medium do not depend on the direction of photon propagation, radiative transfer equation (1) can be rewritten as

$$I(r,\omega)=\int_0^{2\pi}\int_{-1}^{1}\int_{R^3} q(r')\sigma(r')g_\mu(\mu,r')\frac{\mathrm{e}^{-\tau(r',r)}}{|r-r'|^2}\times$$

$$\times\delta\left(\omega-\frac{r-r'}{|r-r'|}\right)I(r',\omega')\mathrm{d}r'\mathrm{d}\mu\mathrm{d}\varphi/(2\pi)+I_0(r,\omega).$$

The extinction coefficient and single scattering albedo in this case do not depend on the direction of radiation propagation, while the scattering phase function g_μ depends on the cosine $\mu=<\omega',\omega>$ of the angle between the directions of photon travel before and after scattering, $\int_{-1}^{1} g_\mu(\mu)\mathrm{d}\mu=1$. Let us present the relations between the scattering phase functions $g_\mu(\mu)$ and $g_{c\psi}(c',\psi',c,\psi)$:

$$g_\mu(\mu)\mathrm{d}\mu\mathrm{d}\varphi/(2\pi)=g_{c\psi}(c',\psi',c,\psi)\mathrm{d}c\mathrm{d}\psi,$$

$$\mu=cc'+\sqrt{1-c^2}\sqrt{1-c'^2}\cos(\psi-\psi'), \qquad (3)$$

$$\cos(\varphi)=\frac{c-c'\mu}{\sqrt{1-c'^2}\sqrt{1-\mu^2}}.$$

It is interesting that, the Jacobian of the transformation $(\mu,\varphi)\leftrightarrow(c,\psi)$ turns out to be equal to unity (we have checked this using Mathematica software for symbolic computations).

2. Algorithm of statistical simulation of the radiative transfer through a medium, optically anisotropic with respect to zenith angle of the radiation incidence

Monte Carlo methods are widely used to solve problems in radiative transfer through optically isotropic media (see, e.g., Refs. 6–11). However, for optically anisotropic media, the calculation algorithms require substantial modifications. Below we describe an algorithm of statistical simulation of photon trajectories in the scattering medium, optically anisotropic with respect to zenith angle of radiation incidence, i.e., in the framework of the mathematical model discussed in Section 1.

For simplicity of presentation, we assume that the optical medium is homogeneous (the single scattering albedo, the extinction coefficient, and the scattering phase function do not depend on the spatial variable) and the functions σ_c and q_c, in equation (2), depend only on the cosine of the zenith angle of the direction of a photon travel. The algorithm of statistical simulation of photon trajectory involves the following steps.

Step 1. We simulate the initial coordinates $r_0 = (x_0, y_0, z_0)$ and the initial direction $\omega_0 = (a_0, b_0, c_0)$, $|\omega_0| = 1$ of the photon in accordance with the distribution of sources and assume that $n = 0$.

Step 2. We simulate the free path l of a photon according to the distribution with the density

$$p(l) = \sigma_c(c_n)\exp\{-l\sigma_c(c_n)\}, \quad l > 0,$$

where $\sigma_c(c)$ is the extinction coefficient of the medium along the direction with the cosine of the zenith angle c.

Step 3. We assume that $n' = n + 1$ and calculate the coordinates of the next photon collision with a particle of the medium:

$$x_n = x_{n-1} + a_{n-1}l; \quad y_n = y_{n-1} + b_{n-1}l;$$
$$z_n = z_{n-1} + c_n l; \quad r_n = (x_n, y_n, z_n).$$

Step 4. We simulate the type of the collision: scattering with the probability $q_c(c_n)$ and absorption with the probability $1 - q_c(c_n)$, where $q_c(c)$ is the single scattering albedo for the cosine of zenith angle c of the direction of photon travel before the collision. If absorption takes place, the trajectory is interrupted.

Step 5. If at the step 4 the scattering occurred, the new direction of the photon ω_n travel is simulated using the scattering phase function $g(\omega_{n-1}, \omega_n)$, followed by returning to the step 2.

It is a specific feature of the algorithm of statistical simulation for optically anisotropic media that the single scattering albedo, extinction coefficient, and scattering phase function are now dependent on the direction of a photon travel. The key issue is the simulation of new direction at the step 5. Let us describe this step in a more detail. The direction of photon travel before the scattering will be denoted as $\omega' = (a', b', c') = \omega_{n-1}$, and the direction of travel after scattering as $\omega = (a, b, c) = \omega_n$. Here (a', b', c') and (a, b, c) are the decompositions of the corresponding direction vectors into components with respect to the OX-, OY-, and OZ- axes. First, we simulate the cosine of the zenith angle c. For this, we use the distribution $P(c', c)$ over $c \in [-1, 1]$ at a fixed value c'. Then, we simulate the change of the azimuth angle $\Delta\psi$ (in the horizontal plane OXY). For this, it is necessary to know the distribution $Q(c', c, \Delta\psi)$ over $\Delta\psi \in [-\pi, \pi]$, when c' and c values are fixed. The components a and b of the vector of the new direction ω are calculated by the following formulas:

$$a = [a'\cos(\Delta\psi) - b'\sin(\Delta\psi)](1 - c^2)^{1/2}[1 - (c')^2]^{-1/2},$$
$$b = [a'\sin(\Delta\psi) - b'\cos(\Delta\psi)](1 - c^2)^{1/2}[1 - (c')^2]^{-1/2}.$$

Here it is assumed that $|c'| \neq 1$ (if $|c'| = 1$, then (a, b) is isotropic random vector). Thus, for simulation of scattering in the considered anisotropic medium, it is necessary to know the families of the distributions $P(c', c)$ and $Q(c', c, \Delta\Psi)$ for the parameters $c' \in [0, 1]$, $c \in [-1, 1]$. This corresponds to the representation of the scattering phase function in terms of the product of marginal distribution over c and the conditional distribution over $\Delta\psi = |\psi - \psi'|$:

$$g_c(c', \psi', c, \psi)\mathrm{d}c\mathrm{d}\psi = p_0(c', c)\mathrm{d}c\, q_0(c', c, |\psi - \psi'|)\mathrm{d}\psi,$$

where p_0 and q_0 are the densities of the distributions P and Q, respectively.

Note. For an optically isotropic medium, the vector of the new direction of photon travel after scattering can be calculated by the following formulas (see, e.g., Ref. 6):

$$a = a'\mu - [b'\sin(\varphi) + a'c'\cos(\varphi)]\left(\frac{1-\mu^2}{1-(c')^2}\right)^{1/2};$$

$$b = b'\mu - [a'\sin(\varphi) - b'c'\cos(\varphi)]\left(\frac{1-\mu^2}{1-(c')^2}\right)^{1/2};$$

$$c = c'\mu + \cos(\varphi)\left([1-\mu^2][1-(c')^2]\right)^{1/2}$$

(or by the formulas of Subbotin–Chentsov, which have certain advantages in calculations, see Ref. 12). Here $\mu = a'a + b'b + c'c$ is cosine of the angle between directions before and after scattering, while φ is the angle between the planes (ω', OZ) and (ω', ω). The value μ is simulated according to the scattering phase function g_μ for the optically isotropic medium. As we have already noted above, the Jacobian of passing from (μ, φ) to (c, ψ) variables [see formulas (3)] equals unity. This fact was used to test the algorithm of statistical simulation of radiative transfer developed for the optically anisotropic medium, by applying it to the simulation of radiative transfer through the optically isotropic medium.

3. Results of numerical experiment

Let us describe the numerical experiment, carried out to study the influence of ice crystal orientation on the optical characteristics of the cloud layer. The calculations were made for the plane parallel source and visible wavelength range (in the region near 550 nm) with the refractive index of ice being equal to 1.311. The absorption of radiation in the cloud was not considered. For the ice cloud we used the following simplified models.

It was assumed that the scattering medium consists of ice crystals all having the shape of hexagonal cylinders with the ratio (height of cylinder)/(side of hexagon) = 5. We considered three "cloud" models for different crystal orientations in the cloud. Model I: all crystals are isotropically oriented in the space. Model A1: all crystals are isotropically oriented in a horizontal plane (meaning that the axes of the cylinders lie horizontally) with random isotropic rotation of the crystal around the axis of the cylinder (Fig. 1*a*).

Model A2: as in the model A1, all crystals are oriented isotropically in the horizontal plane but, at the same time, two of the side faces lie strictly in the horizontal plane (Fig. 1*b*). For the model I the scattering medium is optically isotropic, whereas for the models A1 and A2 the medium is optically anisotropic.

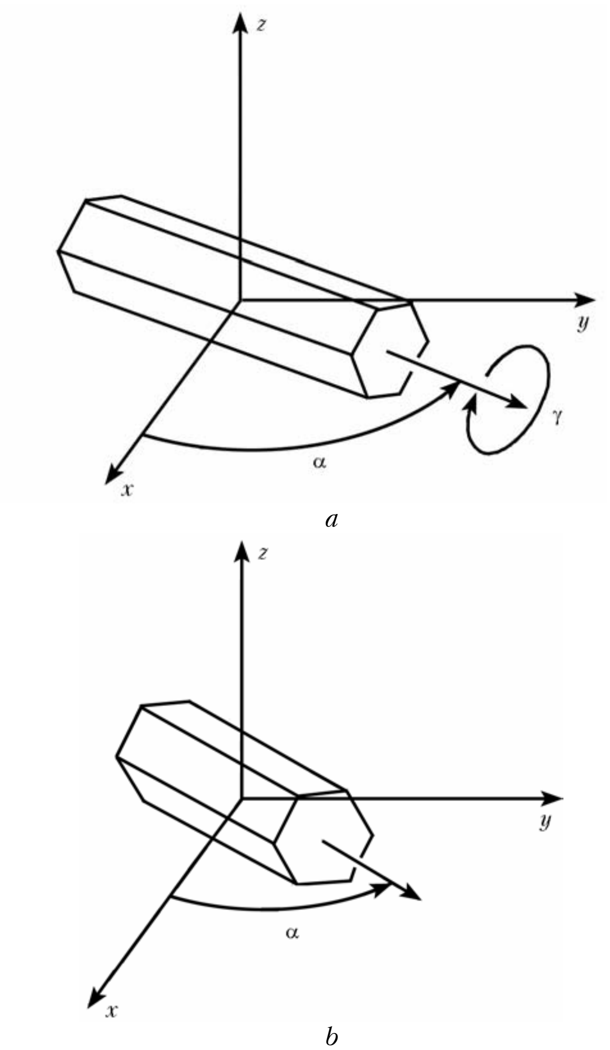


Fig. 1. Arrangement of ice crystals for model A1 (*a*) and model A2 (*b*). The random values of the angles α and γ are uniformly distributed.

The local optical characteristics for the three above-mentioned models of the scattering medium were calculated based on the geometric optics method (see Refs. 13 and 14). For approximation of the function σ_c and distributions P and Q (see step 2 and step 5 in description of the simulation algorithm) we used the arrays of dimensionalities 101, 101×201, and $101\times201\times91$, respectively. It should be noted that calculation of the distributions P and Q for real ice clouds in the atmosphere is too complicated problem and that no techniques have yet been developed for a comprehensive solution of this problem.

The numerical experiment conducted assumed that the ice cloud is a homogeneous plane layer with the distance between the top and bottom boundaries of 150 m. The cloud optical depth for the isotropic model I was 3 ($\sigma = 0.02\ \mathrm{m}^{-1}$). Two other cloud models assumed that the concentration of particles in the cloud is the same as in the model I.

The extinction coefficient for optically anisotropic models A1 and A2 depends on the direction of radiation propagation: the medium is optically denser along the vertical than along a horizontal direction. Figure 2 presents the ratio of the extinction coefficient for the optically anisotropic models A1 and A2 to the extinction coefficient for the optically isotropic model I with the identical particle concentration in the scattering medium.

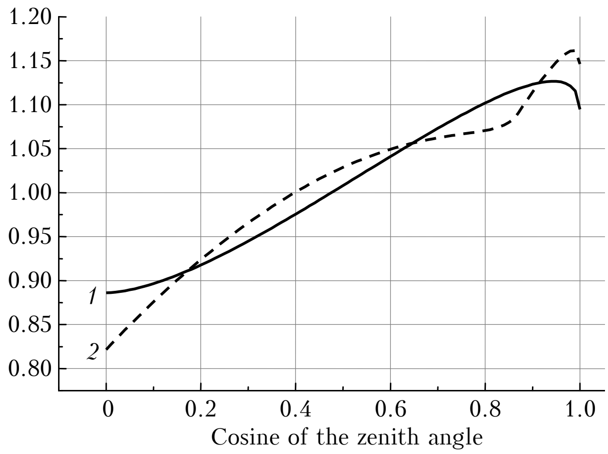
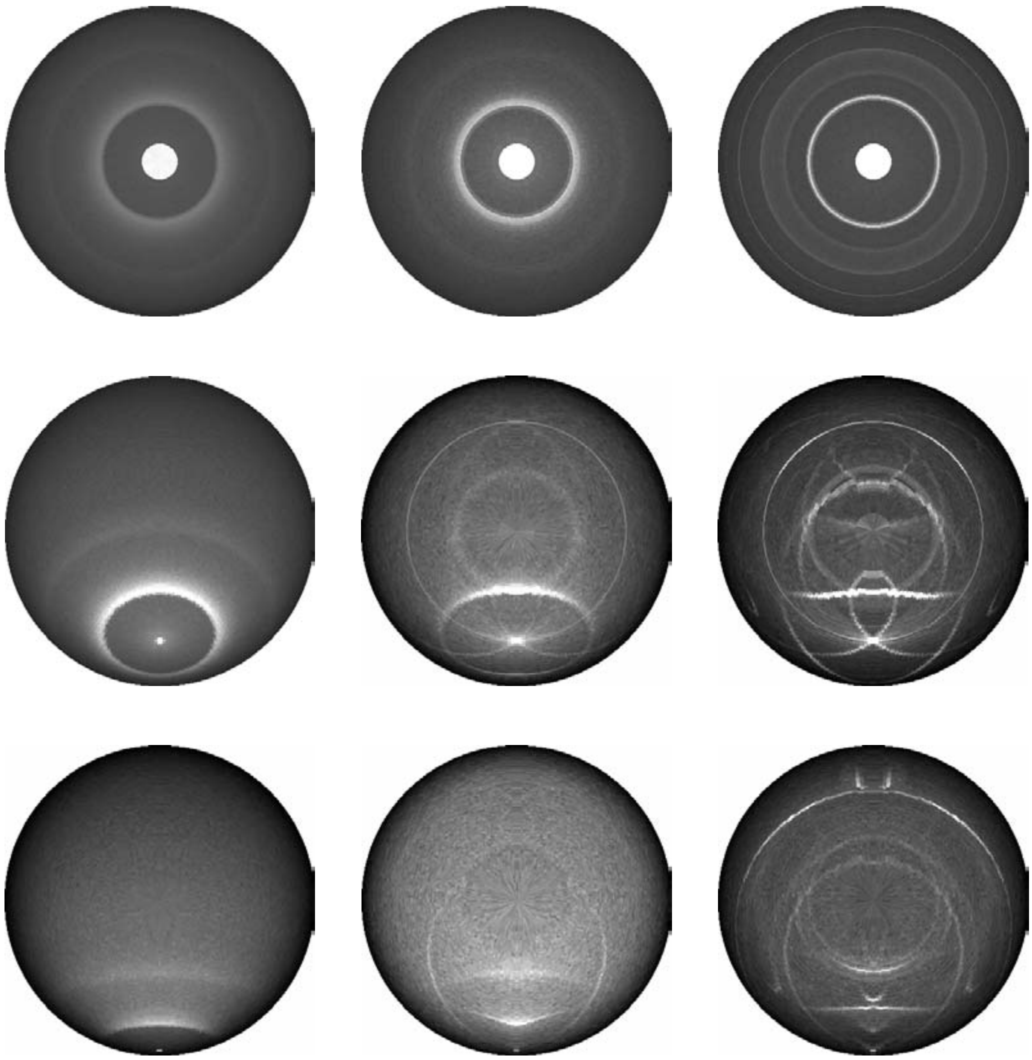


Fig. 2. Dependence of the extinction coefficient on the cosine of the zenith angle of direction of photon travel for the optically anisotropic models A1 (curve *1*) and A2 (curve *2*).

Figure 3 presents the results on halo statistically simulated taking into account the multiple scattering using three models of ice cloud and different zenith angles of the source. Circles are the projections of sky hemisphere onto the horizontal plane and reflect the brightness of the corresponding regions of the sky. The center of the circle corresponds to the zenith, while the points of the circle periphery to the horizon. As seen from the figure, the shape of halo significantly depends on the ice crystal orientation in the cloud.

Figure 4 presents the albedos of the cloud layer for three models of the cloud medium as functions of the zenith angle of the source. The albedo values for cloud models with crystal orientation in the horizontal plane (models A1 and A2) practically coincide, substantially exceeding albedo values for the optically isotropic model I (primarily due to the significant difference between the scattering phase functions).

Note that the dependence of extinction coefficient on the zenith angle for optically anisotropic models does not play any significant role here: the results presented in Fig. 4 were obtained for the case when the maximum (!) values of the extinction coefficient for optically anisotropic models coincided with that for model I of an optically isotropic cloud.



a *b* *c*

Fig. 3. Results on halo simulated for three ice cloud models with different particle orientations in the space: optically isotropic model I (*a*), model A1 (*b*), and model A2 (*c*). Also different zenith angles of the source were used (upper row is for 0°, middle row is for 45°, and lower row is for 75° zenith angle).

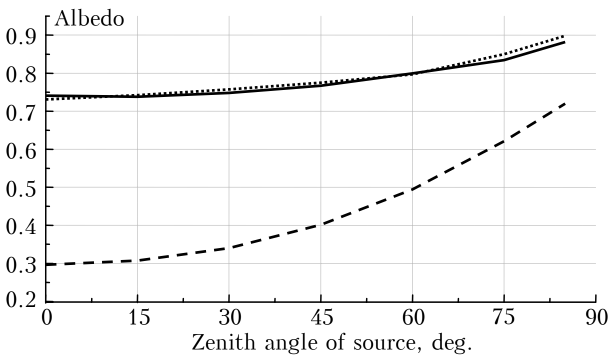


Fig. 4. Albedo of the cloud layer as a function of zenith angle of the source for three models of the cloud medium: dashed line is for model I, solid line for model A1, and dotted line for A2 model.

Thus, the results of the numerical experiment conducted confirm that orientation of particles of the scattering medium can critically influence the cloud radiation characteristics.

The software that we have developed for simulation of radiative transfer through optically anisotropic media and the model calculations are partly available at the website http://osmf.sscc.ru/~smp/intas.html and http:// alpha. emitter. googlepages.com.

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