Influence of the source spectrum on the accuracy of optical measurements of turbulence

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It is considered how the source spectrum influences the measurement accuracy of optical wave arrival angles, as well as the estimation accuracy of the path-averaged structure parameter of the refractive index fluctuations. Two reasons, which can cause the wavelength dependence of the variance of fluctuations of wave arrival angles, are analyzed. The first one is connected with the fact that phases depend on a wavelength in the approximation of smooth perturbations. The second reason is associated with the wavelength dependence of the refractive index and, consequently, its fluctuations. Strict equations are obtained to take into account the influence of the source spectrum on the measurement accuracy of the variance of arrival angle fluctuations and, indirectly, on the estimation accuracy of the path-averaged refractive index structure parameter. It can be stated that for most radiation sources (even nonmonochromatic) the influence of the source spectral composition can be neglected.

Measurements of optical wave fluctuations are conducted already for a long time to determine the level of atmospheric turbulence.[1] At the same time, an important problem concerning the influence of the source spectrum on the accuracy of such measurements remains poorly studied. This paper considers the influence of the source spectrum on the accuracy of measurements of an optical wave arrival angle and on the errors of estimation of the path-averaged structure parameter fluctuations of the refractive index C_n^2. Two factors causing the dependence of the variance of the arrival angle fluctuations are analyzed. The first one is connected with the fact that, in the approximation of smooth perturbations, fluctuations of eikonal (ratio of the phase to the radiation wave number) depend on the wavelength. The second factor is attributed to the wavelength dependence of the refractive index and, consequently, its fluctuations. We consider sequentially the contributions to the error of the turbulence structure parameter estimation from optical measurements of these factors.

Influence of diffraction parameters of optical beams

Consider the plane and spherical optical waves as limiting cases. It can be shown that in the approximation of smooth perturbations the eikonal variations for the real parameter γ ($\gamma = x/L$ corresponds to the spherical wave, $\gamma = 1$ corresponds to the plane wave) are expressed[1] by the following equation:

$$\Theta(\rho, k) = \int\limits_0^x d\xi \iint d^2 n(\kappa, \xi) \exp(i\kappa\rho\gamma) \cos\left[\frac{\kappa^2 (x-\xi)\gamma}{2k}\right], \quad (1)$$

where $d^2 n(\kappa, \xi)$ is the spectral amplitude of fluctuations of the refractive index $n_1(\mathbf{r})$, which is described by the equation

$$n_1(\mathbf{r}) = \iint d^2 n(\kappa, \xi) \exp(i\kappa\rho);$$

$\mathbf{r} = (\xi, \rho)$; $\rho = (y, z)$; $\kappa = (\kappa_2, \kappa_3)$; $k = 2\pi/\lambda$, λ is the radiation wavelength.

The wave-front tilt in the approximation of smooth perturbations can be calculated as[1]:

$$\nabla_\rho(\rho, k) = = i\int\limits_0^x d\xi\gamma(\xi) \iint d^2 n(\kappa, \xi) \exp(i\kappa\rho\gamma)\kappa \cos\left[\frac{\kappa^2 (x-\xi)\gamma}{2k}\right]. \quad (2)$$

Then we can calculate the wave-front tilt φ as a ratio of the gradient of eikonal $\Theta(\rho, k)$, integrated over the receiving aperture, to the area of this aperture $\Sigma = \iint d^2\rho W(\rho)$:

$$\frac{1}{\Sigma}\iint d^2\rho \nabla_\rho \Theta(\rho, k) = = i\int\limits_0^x d\xi\gamma(\xi) \iint d^2 n(\kappa, \xi)\kappa \exp\left(\frac{-\kappa^2 d^2 \gamma^2}{4}\right)\cos\left[\frac{\kappa^2 (x-\xi)\gamma}{2k}\right].$$

Here $W(\rho)$ is the entrance pupil function of the detector. To simplify calculations, we apply the Gaussian approximation in the form

$$W(\rho) = \exp(-\rho^2/d^2).$$

Now we can pass on to the calculation of the jitter variance of the arrival angle φ for the spherical and plane waves propagating in the turbulent atmosphere.

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In the approximation of smooth perturbations, we obtain

$$<\varphi^2> = \int_0^x d\xi_1\gamma(\xi_1)\int_0^x d\xi_2\gamma(\xi_2)\times$$

$$\times\iint <d^2 n(\kappa_1,\xi_1)d^{2^*}n(\kappa_2,\xi_2)>\kappa_1\kappa_2\exp\left[\frac{-\kappa_1^2 d^2\gamma^2(\xi_1)}{4}\right]\times$$

$$\times\exp\left[\frac{-\kappa_2^2 d^2\gamma^2(\xi_2)}{4}\right]\cos\left[\frac{\kappa_1^2(x-\xi_1)\gamma(\xi_1)}{2k}\right]\times$$

$$\times\cos\left[\frac{\kappa^2(x-\xi_2)\gamma(\xi_2)}{2k}\right], \quad (3)$$

where the angular brackets <…> denote the averaging over the ensemble of realizations of a turbulent medium.

To perform calculations, we apply the properties of the spectral expansion

$$<d^2 n(\kappa_1,\xi_1)d^{2^*}n(\kappa_2,\xi_2)> =$$

$$= 2\pi\delta(\xi_1-\xi_2)\delta(\kappa_1+\kappa_2)\Phi_n(\kappa_1,\xi_1)d\kappa_1 d\kappa_2 \quad (4)$$

and the Kolmogorov spectrum of turbulence

$$\Phi_n(\kappa,\xi) = 0{,}033C_n^2(\xi)\kappa^{-11/6}. \quad (5)$$

Finally, we obtain

$$<\varphi^2> = 2\pi\int_0^x d\xi\gamma^2(\xi)\iint d^2\kappa\kappa^2\Phi_n(\kappa,\xi)\times$$

$$\times\exp\left[\frac{-\kappa^2 d^2\gamma^2(\xi)}{2}\right]\cos^2\left[\frac{\kappa^2(x-\xi)\gamma(\xi)}{2k}\right]. \quad (6)$$

Here $\delta(\xi_1-\xi_2)$ is the Dirac delta function.

Then we use the expansion into the Taylor series for the cosine term in the integrand of Eq. (6)

$$\cos^2\left[\frac{\kappa^2(x-\xi)\gamma(\xi)}{2k}\right] = \frac{1+\cos\left[\kappa^2(x-\xi)\gamma(\xi)/k\right]}{2}\approx$$

$$\approx 1-\frac{\kappa^4(x-\xi)^2\gamma^2}{2k^2}.$$

To calculate the integral in Eq. (6), it is necessary to calculate the following two integrals:

$$\int_0^\infty d\kappa\kappa^{3-11/3}\exp\left(\frac{-\kappa^2 d^2\gamma^2}{2}\right) = 2^{-5/6}\Gamma\left(\frac{1}{6}\right)d^{-1/3}\gamma^{-1/3},$$

$$\frac{(x-\xi)^2\gamma^2}{4}\int_0^\infty d\kappa\kappa^{7-11/3}\exp\left(\frac{-\kappa^2 d^2\gamma^2}{2}\right) =$$

$$= 2^{-5/6}\Gamma\left(\frac{1}{6}\right)d^{-1/3}\gamma^{-1/3} = \frac{2^{1/6}\Gamma(13/6)}{4k^2}(x-\xi)^2\gamma^{-7/3}d^{-13/3}.$$

Summing up the two last integrals, for the variance of jitter we obtain

$$<\varphi^2> = 2^{7/6}\pi^2 0.033\Gamma\left(\frac{1}{6}\right)d^{-1/3}\int_0^x d\xi\gamma^{5/3}C_n^2(\xi) -$$

$$-2^{1/6}\pi^2 0.033\Gamma\left(\frac{13}{6}\right)d^{-13/3}xk^{-2}\int_0^x d\xi\gamma^{-1/3}C_n^2(\xi)(1-\gamma)^2. \quad (7)$$

The analysis of Eq. (7) shows that the variance of the arrival angle of an optical wave depends on the wavelength of this wave, when calculating in the approximation of smooth perturbations (compared to the calculations in the geometric optics approximation). There appears the second term, giving the explicit wavelength dependence of the measured variance. This second term has the order of smallness x^2/k^2d^4 as compared to the first term, being the squared inverted wave parameter for the receiving aperture. It should be noted that this wavelength dependence of the variance of the arrival angle is true for both plane and spherical waves.

Then we consider the case of a homogeneous path, that is, $C_n^2(\xi) = C_n^2(0)$. In this case, Equation (7) for the spherical wave can be written in the form

$$<\varphi^2>_{\rm sph} = 2^{7/6}\pi^2 0.033\Gamma\left(\frac{1}{6}\right)d^{-1/3}C_n^2(0)\frac{3}{8}x -$$

$$-2^{1/6}\pi^2 0.033\Gamma\left(\frac{13}{6}\right)d^{-13/3}x^2k^{-2}C_n^2(0)\int_0^x d\xi\gamma^{-1/3}(1-\gamma)^2 =$$

$$= 2^{7/6}\pi^2 0.033\Gamma\left(\frac{1}{6}\right)d^{-1/3}C_n^2(0)\frac{3}{8}x[1-0.175\Omega^{-2}], \quad (8)$$

where $\Omega = kd^2/x$ is the wave parameter for the receiving aperture.

At a first glance, we have obtained rather strong wavelength dependence of the variance of fluctuations of the spherical wave arrival angle. For two different wavelengths (λ_1,λ_2), the ratio of the variances is

$$\frac{<\varphi^2(\lambda_1)>_{\rm sph}}{<\varphi^2(\lambda_2)>_{\rm sph}} = \frac{1-0.175\left(x^2/k_1^2d^4\right)}{1-0.175\left(x^2/k_2^2d^4\right)}. \quad (9)$$

It can be easily shown that if the receiving aperture is small in the diffraction meaning, that is, $\Omega = kd^2/x < 1$, then we have a strong wavelength dependence. If the receiver is large for a given path, $(\Omega = kd^2/x \gg 1)$, then the second term depending on λ can be neglected due to its smallness.

For the initial plane wave, the variance of fluctuations of the arrival angle is described by the following equation:

$$<\varphi^2>_{\rm pl} = 2^{7/6}\pi^2 0.033\Gamma\left(\frac{1}{6}\right)d^{-1/3}\int_0^x d\xi C_n^2(\xi) -$$

$$-2^{1/6}\pi^2 0.033\Gamma\left(\frac{13}{6}\right)d^{-13/3}k^{-2}\int_0^x d\xi C_n^2(\xi)(x-\xi)^2. \quad (10)$$

For the homogeneous path

$$<\varphi^2>_{\rm pl} = 2^{7/6}\pi^2 0.033\Gamma\left(\frac{1}{6}\right)d^{-1/3}C_n^2(0)x\left[1-\frac{\Gamma\left(\frac{13}{6}\right)}{\Gamma\left(\frac{1}{6}\right)6}\frac{x^2}{k^2d^4}\right] =$$

$$= 2^{7/6}\pi^2 0.033\Gamma\left(\frac{1}{6}\right)d^{-1/3}C_n^2(0)x(1-0.03\Omega^{-2}). \quad (11)$$

We have obtained that for the initial plane wave the wavelength dependence of the variance of fluctuations of the arrival angles is very weak. Even for small wave numbers of the receiving aperture, when $\Omega = kd^2/x < 1$, the wavelength dependence of jitter can be neglected. As compared to the case of the spherical wave, the wavelength dependence is weaker.

Now we can estimate the wavelength influence in a standard experiment. Let the initial radiation be closer to the divergent spherical wave, since the size of the emitting area a is about 7 mm, and therefore $\Omega_{\rm em} = ka^2/x = 1$. So, we use Eq. (8) and thus obtain

$$<\varphi^2>_{\rm sph} = 2^{7/6}\pi^2 0.033\Gamma\left(\frac{1}{6}\right)d^{-1/3}C_n^2(0)\frac{3}{8}x[1-0.175\Omega^{-2}]. \quad (12)$$

For example, at $\lambda = 0.6$ μm, belonging to the visible region, the path length $x = 500$ m, and receiving aperture of 40 mm, the wave parameter for the receiving aperture turns out to be $\Omega_{\rm rec} = kd^2/x = 32$. Thus, in Eq. (12) the term in the square brackets is $[\ldots] = (1 - 0.00017)$, that is, nearly equal to unity. Thus, the wavelength dependence of the variance of the arrival angle (from the viewpoint of radiation diffraction parameters) is nearly absent, and the mean level of turbulence for the homogeneous path can be calculated by the following equation:

$$<\varphi^2>_{\rm sph} = 2^{7/6}\pi^2 0.033\Gamma\left(\frac{1}{6}\right)d^{-1/3}C_n^2(0)\frac{3}{8}x. \quad (13)$$

Influence of the wavelength dependence of the atmospheric refractive index

When using Eq. (13), it is necessary to estimate the wavelength influence caused by the wavelength dependence of the structure parameter $C_n^2(0) = f(\lambda)$.

Consider a nonmonochromatic radiation source with a wavelength band in the range (λ_1, λ_2). The task is to calculate the mean reduced value

$$\left[\int_{\lambda_1}^{\lambda_2} d\lambda K(\lambda)\right]^{-1}\int_{\lambda_1}^{\lambda_2} d\lambda K(\lambda)C_n^2(\lambda) \approx C^2_{n\,\rm mean}, \quad (14)$$

where $K(\lambda)$ is the spectral energy density of the source radiation; λ_1 and λ_2 are the limiting wavelengths in the source spectrum.

As a result, for the case of the source homogeneously emitting in the range (λ_1, λ_2), we obtain:

$$\int_{\lambda_1}^{\lambda_2} d\lambda C_n^2(\lambda)/(\lambda_2-\lambda_1) \approx C^2_{n\,\rm mean}. \quad (15)$$

For a monochromatic source $K(\lambda) = \delta(\lambda - \lambda_0)$, where λ_0 is the radiation wavelength.

Then in our calculations we apply the equation describing the wavelength dependence of the refractive index (and its fluctuations). It can be shown[1] that for the wavelengths in a range from 0.2 to 20 μm the formula:

$$N_\lambda = (n_\lambda - 1)\cdot 10^6 = \frac{77.6P}{T} + \frac{0.584P}{T\lambda^2} - 0.06P_{\rm w.v} \quad (16)$$

can be used.

Here P is the pressure, in mbar; T is the temperature, in K; λ is the wavelength, in μm; $P_{\rm w.v}$ is the water vapor pressure. For long waves at a pressure of 1013 mbar and $T = 288$ K we obtain $N_{\lambda\to\infty} = 77.6P/T = 273$. Derive the explicit wavelength dependence of the refractive index

$$N_\lambda = N_{\lambda\to\infty}\left(1+\frac{0.584}{77.6\lambda^2}\right), \quad (17)$$

finally,

$$C_n^2(\lambda) = C_n^2(\lambda\to\infty)\left(1+\frac{0.584}{77.6\lambda^2}\right)^2. \quad (18)$$

As a result, the structure characteristic for an arbitrary wavelength in the optical wavelength range is determined by the equation of the form (17). The calculations show that Equation (14) yields the following equation:

$$\int_{\lambda_1}^{\lambda_2} d\lambda C_n^2(\lambda)/|\lambda_2-\lambda_1| = C^2_{n\,\rm mean}/|\lambda_2-\lambda_1| \times$$

$$\times \int_{\lambda_1}^{\lambda_2} d\lambda(1+0.0075\lambda^{-2})^2 \approx C^2_{n\,\rm mean}(1+0.015/\lambda_1\lambda_2). \quad (19)$$

Thus, the deviation from the monochromatic dependence corresponding to Eq. (14) leads to Eq. (19), and the maximal resulting error cannot be higher than 5%.

Consider the following example. Let two wavelengths be given: 0.5 and 0.6 μm. If we calculate the term in parenthesis in Eq. (17), characterizing the difference from a monochromatic source, then we obtain for 0.5 μm $(\ldots) = (1 + 0.0075/0.25)^2 = 1 + 0.06$, and for 0.6 μm we have $(\ldots) = (1 + 0.0075/0.36)^2 = 1 + 0.042$. Thus, the difference is about 1.8%. Such a difference can be neglected.

Conclusions

As a result of this study, we have derived the strict equations taking into account the influence of the source spectrum on the error of measuring the variance of the arrival angle fluctuations and, indirectly, on the error of estimation of the path-averaged structure parameter of the refractive index. We can state that for most radiation sources (even nonmonochromatic) the influence of the spectral composition can be neglected.

The use of a wide collimated beam should be considered as the most efficient from the viewpoint of energy. The source energy is closely connected with the possibility to obtain an image of a separate frame with a rather short exposure, that is, with the provision of operation of a receiving device with a high frequency of detection of a series of optical beam images.

The correct estimation of the optical turbulence parameters from optical measurements can be reached only in the case that the prevalent wind direction is perpendicular to the direction of optical beam propagation along the measurement path.

References

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