Evolution of the effective radius of femtosecond laser beam after its global self-focusing in air

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An equation is derived for the effective radius of a femtosecond laser beam at its nonstationary self-action. For the single filamentation regime, the beam evolution has been qualitatively analyzed, and a model of beam passage through the global nonlinear focus is constructed. The model is based on the results of our numerical investigation. Filamentation of a femtosecond laser beam is considered as a process of formation of many local focuses along the propagation path. The dependence of the laser beam propagation coefficient (averaged effective beam divergence normalized to its initial value) on the initial beam parameters and the light energy absorbed in a medium is determined.

Introduction

Propagation of high-power femtosecond laser radiation in air is accompanied by its nonstationary self-action caused by the Kerr effect and multiphoton ionization. A filament is formed in an ideal unimodal laser beam at the initial power exceeding the critical value of $P_{cr} \approx 3.2$ GW. The filament is a stable energy structure about 100 μm in diameter with the peak intensity up to $5 \cdot 10^{17}$ W/m^2 located near the beam axis and able to contain more than 10% of the pulse energy. For actual laser beams, perturbations are present in the initial intensity profile providing the initial beam power several times exceeds the critical power of self-focusing and causes the formation of multiple filaments distributed in the beam cross section.[1]

The main physical mechanism restricting the growth of filament optical field intensity in gaseous media is usually nonlinear absorption at medium photoionization, although sometimes this role can be played by modulation instability of the beam transversal profile.[3] Every filament exists due to periodic energy inflow from nonfilamented areas surrounding the beam, referred to as an energy reservoir, thus compensating energy losses for nonlinear absorption. The mean length of the filamentation area at a horizontal atmospheric path is usually equal to tens of meters.[4]

Several physical models of ultrashort laser radiation filamentation have been proposed by now: moving focus model,[5] self-induced spatial optical solitons,[6] dynamical moving focus,[4] dynamic replenishment from the energy reservoir.[7] Each of these models describes most thoroughly some or other aspects of this process.

The most universal approach to the study of the single and multiple filamentation is numerical simulation based on the nonlinear Schrödinger equation. However, numerical calculation still fails to predict the behavior of laser beam characteristics at wide varying of initial and boundary conditions of the problem. Therefore, it is important to develop approximate and qualitative methods for analysis of the self-action problem based on solution of the initial equation of wave propagation within the framework of physically justified assumptions.

This paper deals with the following problems of nonstationary self-action of femtosecond laser radiation: derivation of an equation for the laser beam effective radius, finding of regularities in the beam evolution after global self-focusing based on results of numerical calculations, and the use of models of nonstationary self-focusing.

1. Nonlinear Schrödinger equation

Assume that a laser pulse, whose electric field strength has the form:

$$(\xi, \mathbf{R}, t) = U(\xi, \mathbf{R}, t) e^{i(\omega_0 t - k_0 \xi)},$$

interacts with a nonlinear medium. Here U is the slowly varying (in time t and in the direction of pulse propagation ξ) amplitude depending on transversal coordinates $\mathbf{R} = (x, y)$; ω_0 is the radiation central frequency; $k_0 = n_0\omega_0/c$ is the wave number at the radiation central frequency; n_0 is the refractive index of air. For the problem of nonstationary self-action under study, we use the nonstationary Schrödinger equation (NSE). In the "concomitant" coordinate system, $z = \xi - v_g t$, where $v_g = \partial\omega/\partial k$ is the pulse group velocity; ω is the radiation frequency; k is the wave number with allowance for group velocity variance. The nonlinear Schrödinger equation has the form

$$\left\{\frac{\partial}{\partial z} - \frac{i}{2n_0k_0}\nabla^2_\perp + i\frac{k''_\omega \partial^2}{2\partial t^2}\right\}U(\mathbf{R}, t, z) - $$

$$-i\frac{k_0\tilde{\varepsilon}(I)}{2}U(\mathbf{R}, t, z) + \frac{\alpha}{2}(I)U(\mathbf{R}, t, z) = 0. \quad (1)$$

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Here $\nabla_\perp^2 = (\partial^2/\partial x^2 + \partial^2/\partial y^2)$ is the transversal Laplacian; $k''_\omega = \partial^2 k/\partial\omega^2|_{\omega=\omega_0}$ is the coefficient in the expansion of the wave number $k \approx k_0 + v_g^{-1}(\omega - \omega_0) + k''_\omega(\omega - \omega_0)^2/2$ near ω_0; α is the nonlinear absorption coefficient of the medium; $\tilde{\varepsilon}(I)$ is the nonlinear refraction coefficient. As an initial condition, we take the Gaussian beam having the following form in space and time:

$$U(R,t,z)\big|_{z=0} = U_0 \exp\left\{-\frac{R^2}{R_0^2}\left(1+\frac{ik_0}{F}\right) - \frac{t^2}{t_p^2}\right\}, \quad (2)$$

where R_0 and F are the initial radius of the beam and the curvature of its wave front; t_p is the pulse duration. We use the model from Ref. 8, which takes into account the instantaneous and delayed Kerr effects, multiphoton and cascade ionization, and plasma nonlinearity. This allows equation (1) to be written in the form

$$\left\{\frac{\partial}{\partial z} - \frac{i}{2n_0k_0}\nabla_\perp^2 + i\frac{k''_\omega\partial^2}{2\partial t^2}\right\}U(\mathbf{R},t,z) -$$
$$-ik_0n_2\left\{(1-f_R)|U|^2 + f_R\int_{-\infty}^{\infty}dt'\mu(t-t')|U(t')|^2\right\}U(\mathbf{R},t,z) +$$
$$+\frac{\eta_{cas}}{2}(i\omega_0\tau_c)\rho_e(t)U(\mathbf{R},t,z) = -\frac{\eta_{MPA}^{(m)}}{2}|U|^{2m-1}U(\mathbf{R},t,z) -$$
$$-\frac{\eta_{cas}}{2}\rho_e(t)U(\mathbf{R},t,z). \quad (3)$$

Here n_2 is the coefficient of the nonlinear addition to the refractive index of gas n_0; f_R is the specific fraction of the delayed Kerr effect with the response function $\mu(t - t')$ in the summarized change of the refractive index; τ_c is the characteristic time of electron collision; $\eta_{MPA}^{(m)}$ and η_{cas} are the rates of m-photon and cascade ionization of gas, respectively. As $\mu(t - t')$, we use the equation following from the model of a damped oscillator: $\mu(t) = \sin\Omega t\exp(-t/\tau_d)$, where $\Omega \simeq 20$ THz is the frequency of oscillations; $\tau_d \simeq 70$ ns is the characteristic time of damping.

For the concentration of free electrons, we use the following evolutional equation:

$$\frac{\partial\rho_e}{\partial t} = \frac{\eta_{MPA}^{(m)}}{m\hbar\omega_0}|U|^{2m} + \frac{\eta_{cas}}{\Delta E_i}\rho_e|U|^2, \quad (4)$$

where ΔE_i is the effective ionization potential of air molecules.

2. Effective laser beam parameters and equations of their evolution at nonstationary self-action

Once the beam has passed a distance z, the energy transfer coefficient (transmission function) is determined as

$$T_e(z) = E(z)/E(0), \quad (5)$$

where $E(z) = \int_{-\infty}^{\infty} I(\mathbf{R},z,t)\,d\mathbf{R}dt$ is the total energy of the light pulse; $I(\mathbf{R},t,z) = \frac{cn_0}{8\pi}|U|^2$ is the intensity of the light wave.

The effective beam radius has the following form:

$$R_e(z) = \left(\tilde{R}^2(z)T_e^{-1}\right)^{1/2}/R_e(0), \quad (6)$$

where $\tilde{R}^2(z)$ is the "beam moment of inertia" normalized to the initial energy:

$$\tilde{R}^2(z) = \frac{1}{E(0)}\int_{-\infty}^{\infty} I(\mathbf{R},z,t)\mathbf{R}^2 d\mathbf{R}dt. \quad (7)$$

Hereinafter, the following designation is used:

$$\int_{-\infty}^{\infty} f(\mathbf{R},t)\,d\mathbf{R}dt = \int_{-\infty}^{\infty}dx\int_{-\infty}^{\infty}dy\int_{-\infty}^{\infty}f(x,y,t)\,dt\,.$$

With the use of Eq. (1) and the effective parameters (5) and (7), we obtain equations for T_e and $\tilde{R}_e^2$. For the radiation energy transfer coefficient, the equations have the following form:

$$\frac{dT_e(z)}{dz} = -\frac{1}{E(0)}\int_{-\infty}^{\infty}\alpha(I)I\,d\mathbf{R}dt. \quad (8)$$

For the parameter $\tilde{R}^2(z)$, the following equation is formulated:

$$E(0)\frac{d^2}{dz^2}\tilde{R}_e^2 = E(0)\tilde{\theta}_e^2 + \left(\frac{1}{4n_0}\int_{-\infty}^{\infty}\nabla_\perp\tilde{\varepsilon}I\mathbf{R}d\mathbf{R}dt -\right.$$
$$\left.-\frac{k''_\omega k_0}{4}\int_{-\infty}^{\infty}\frac{\partial}{\partial t}\tilde{\varepsilon}It + 2\left|\frac{\partial}{\partial t}U\right|^2 d\mathbf{R}dt\right) + \omega_0\tau_c\eta_{cas}\int_{-\infty}^{\infty}\rho_e(t)Id\mathbf{R}dt +$$
$$+2\frac{\partial}{\partial z}\int_{-\infty}^{\infty}I\alpha(I)\mathbf{R}^2d\mathbf{R}dt + 2\int_{-\infty}^{\infty}\alpha(I)(S_\perp\mathbf{R})\,d\mathbf{R}dt +$$
$$+\left\{E(0)\frac{1}{k''_\omega n_0k_0}\frac{d^2}{dz^2}\tilde{t}_{pe}^2 - 2\frac{\partial}{\partial z}\int_{-\infty}^{\infty}\alpha(I)It^2d\mathbf{R}dt -\right.$$
$$\left.-2\int_{-\infty}^{\infty}\alpha(I)(S_t t)\,d\mathbf{R}dt\right\}. \quad (9)$$

Here

$$S_t = \left(U^*\frac{\partial}{\partial t}U - U\frac{\partial}{\partial t}U^*\right)/2,$$
$$S_\perp = \left(U\nabla_\perp U^* - U^*\nabla_\perp U\right)/2i$$

is the transversal component of the Poynting vector of radiation;

$$\tilde{\theta}_e^2 = \theta_e^2T_e = \frac{1}{2n_0k_0}\int_{-\infty}^{\infty}\left(|\nabla U|^2\right)d\mathbf{R}dt/E(0),$$

θ_e is the effective angular divergence of the beam;

$$\tilde{t}_{pe}^2(z) = 1/E(0)\int_{-\infty}^{\infty} t^2 I \mathrm{d}\mathbf{R}\mathrm{d}t\,,\ \ t_{pe} = \sqrt{T_e^{-1}\tilde{t}_{pe}^2(z)}$$

is the effective pulse duration.[2]

Below we consider the single filamentation, when the ratio P_0/P_{cr} is not too large (P_0 is the initial power in a femtosecond pulse).

Based on numerical simulation of the problem of nonstationary self-action, it was found in Ref. 9 that the effective radius of the beam during the evolution in the regime of single filamentation passes three spatial regions (zones), each characterizing a specific stage of nonstationary self-focusing of radiation.

In the first zone, the beam energy almost does not change $T_e \approx 1$. Later the beam contraction occurs, and the beam intensity increases due to Kerr nonlinearity. This corresponds to the situation, when the right-hand side of Eq. (9) is constant in this region, namely, $\mathrm{d}^2\tilde{R}^2/\mathrm{d}z^2 = \mathrm{const}$. Consequently, the square effective radius varies by the parabolic law.[1] In Ref. 9, the approximation equation was obtained for the effective radius in the first zone:

$$R_e^2(z) = R_0^2\left[\left(1-\frac{\eta}{b}\right)\left(\frac{z}{2L_R}\right)^2 + \left(1-\frac{z}{F}\right)^2\right],$$
$$b = 1.62,\quad \eta = \frac{P_0}{P_{cr}},\quad z \in \left[0; z_f\right], \tag{10}$$

where $L_R = k_0 R_0^2/2$ is the Rayleigh length of the beam. Equation (10) is valid before the filamentation beginning. The coordinate of the filamentation beginning z_f (local nonlinear focus) is determined by the equation[1]:

$$z_f = \frac{2L_R}{2.725\sqrt{\left[(\eta/b)^{1/2} - 0.852\right]^2 - 0.022}}.$$

The second zone begins from the vicinity of the global nonlinear focus of the beam z_w, in which $\mathrm{d}R_e^2/\mathrm{d}z\big|_{z=z_w} = 0,$ and is characterized by the propagation of the laser pulse most intense part in the form of a light filament having the length l_f and terminating at the point z_{NL}. The presence of the filament decreases the beam energy. The mechanism restricting the energy is multiphoton ionization of the medium. It ceases the growth of the beam intensity, stopping it at some maximal value $I_{cr} \approx 5\cdot 10^{17}$ W/m^2 lying near the air breakdown threshold at multiphoton mechanism of ionization.

Finally, the third zone is the zone of linear propagation of radiation having passed the zone of nonlinear interaction. After the passage of the filamentation zone, the light field acquires a complex spatiotemporal profile due to self-modulation. Dispersion along with Kerr nonlinearity still significantly affects the evolution of the effective beam radius. As a result, a bend (sharp decrease in the rate of growth of the effective radius) arises in the second zone after the termination of filamentation. Actually, the possibility of this effect follows from the equation for the effective radius (9), which takes the following form in the absence of absorption:

$$\frac{\mathrm{d}^2}{\mathrm{d}z^2}R_e^2 - \frac{1}{k''_\omega n_0 k_0}\frac{\mathrm{d}^2}{\mathrm{d}z^2}t_{pe}^2 =$$
$$= \left(\theta_e^2 + \frac{1}{4n_0}\int_{-\infty}^{\infty}\nabla_\perp\tilde{\varepsilon} I\mathbf{R}\mathrm{d}\mathbf{R}\mathrm{d}t - \frac{k''_\omega k_0}{4}\int_{-\infty}^{\infty}\frac{\partial}{\partial t}\tilde{\varepsilon} It\mathrm{d}\mathbf{R}\mathrm{d}t\right) = \mathrm{const}.$$

It follows from this equation that in the presence of dispersion and Kerr nonlinearity, the second derivative of the square effective interval of the pulse $L^2 = R_e^2 - (t_{pe}^2/k''_\omega n_0 k_0)$ is an invariant of the problem in this case. During the following propagation (third zone) of the expanding beam, the Kerr nonlinearity becomes negligibly small. As a result, the invariant, associated with the effective interval, breaks into two independent invariants: $\mathrm{d}^2R_e^2/\mathrm{d}z^2$ and $\mathrm{d}^2t_{pe}^2/\mathrm{d}z^2$.

3. Diffraction model of evolution of the effective radius of femtosecond laser beam in the regime of filamentation

Let us reveal the role of effects associated with multiphoton ionization, absorption, and refraction of radiation in plasma resulted from air ionization. For simplicity, we neglect the influence of dispersion on formation of the beam effective radius immediately after its global focusing.

The multifocus model of nonstationary self-action[5] is taken as a model of filamentation. According to this model, a filament is a set of local focuses formed as a result of consecutive focusings of temporal "sections" of the light beam, which have the power higher than P_{cr}. We neglect the transversal dimensions of local focuses as compared to the beam effective radius. Assume also that the focus sizes are much smaller than the distance between focuses. In this case, the dependence of the absorption coefficient on coordinates can be replaced with delta functions located at the beam axis at the points z_i, where i is a number of a local focus. Between these points, absorption is absent. Introduce the following function:

$$H(z) = \tilde{\theta}^2 - \tilde{\theta}_{NL}^2,$$
$$\tilde{\theta}^2 = \left(\int_{-\infty}^{\infty}\left(\left|\nabla_\perp U\right|^2\right)\mathrm{d}\mathbf{R}\mathrm{d}t/E(0)2n_0k_0\right),$$
$$\tilde{\theta}_{NL}^2 = \left(\int_{-\infty}^{\infty}\tilde{\varepsilon} I\mathrm{d}\mathbf{R}\mathrm{d}t/E(0)4n_0k_0\right). \tag{11}$$

It is well-known that in the absence of absorption $H = \mathrm{const}$ for a nonlinear medium of the Kerr type.[1] Upon the passage of the ith local focus, the function H changes its value from H_i to H_{i+1}

stepwise, remaining constant between focuses. In this approximation, Equations (8) and (9) for the effective parameters of the beam take the following form:

$$T_e(z) = 1 - \sum_{i=1}^{N(z)} \Delta T_{ei}, \tag{12}$$

where $\Delta T_{ei} = T_{ei+1} - T_{ei}$;

$$\frac{d^2}{dz^2}\tilde{R}_e^2 = 2H = 2\left(H_0 + \sum_{i=1}^{N(z)} \Delta H_i\right), \tag{13}$$

where $\Delta H_i = H_{i+1} - H_i$.

Taking into account Eq. (12), Equation (13) can be written in a different form (the sense of this representation will be clear later):

$$\frac{d^2}{dz^2}\tilde{R}_e^2 = 2H = 2\left(H_0 + \sum_{i=1}^{N(z)} \gamma_i \Delta T_i\right), \quad \gamma_i = \tilde{\gamma}_i(1-\mu_i),$$

$$\tilde{\gamma}_i = \frac{\Delta\tilde{\theta}_i^2}{\Delta T_{ei}}, \quad \mu_i = \frac{\left|\Delta\tilde{\theta}_{NLi}^2\right|}{\Delta\tilde{\theta}_i^2 \Delta T_{ei}}, \tag{13a}$$

$$\Delta\tilde{\theta}_i^2 = \tilde{\theta}_{i+1}^2 - \tilde{\theta}_i^2.$$

Here $N(z)$ is the number of local focuses after the passage of the distance z; $H_0 = H(0)$.

The passage of the laser beam through each focus can be considered as a scattering of a light wave at an inhomogeneity having the complex dielectric permittivity, whose imaginary part is determined by the absorption coefficient $\alpha(I)$, while the real part is associated with the dielectric inhomogeneity of plasma.

Within the multifocus model, the problem of propagation of a femtosecond laser pulse through a nonlinear refracting and absorbing medium can be divided into two problems. The first problem reduces to the study of radiation propagation through a nonlinear medium of the Kerr type, while the second problem can be reduced to the study of light scattering at localized inhomogeneities. To formalize the consideration, we introduce the evolution operator $\hat{S}$, defined as follows:

$$U(z,\mathbf{R},t) = \hat{S}(z,\mathbf{R},t)U_0(\mathbf{R},t),$$
$$U(z,\mathbf{R},t)\big|_{z=0} = U_0(\mathbf{R},t).$$

Taking this into account, within the framework of the multifocus model the operator $\hat{S}$ can be represented in the form

$$\hat{S}(z,\mathbf{R},t)U_0 = \prod_{i=1}^{N(z)} \hat{S}_\alpha^i \hat{S}_{\text{ker}} U_0, \quad \hat{S}_\alpha^i = \hat{S}_{\text{ker}}\hat{S}_\alpha(z_i).$$

Here $\hat{S}_{\text{ker}}$ is the evolution operator in a medium with noninertial Kerr nonlinearity, and $\hat{S}_\alpha(z_i)$ is the operator of scattering at a dielectric inhomogeneity in the ith local focus defined as

$$\hat{S}_\alpha(z_i)U(z_i - 0,\mathbf{R},t) = U(z_i + 0,\mathbf{R},t).$$

Then the parameter $\tilde{\gamma}_i$, defined in Eqs. (13) can be represented as

$$\tilde{\gamma}_i = \left\{\int_{-\infty}^{\infty}\left|\nabla_\perp\left[\hat{S}(z_i+0)U_0\right]\right|^2 d\mathbf{R}dt - \int_{-\infty}^{\infty}\left|\nabla_\perp\left[\hat{S}(z_i-0)U_0\right]\right|^2 d\mathbf{R}dt\right\} \Big/ \left\{\int_{-\infty}^{\infty}\left|\hat{S}(z_i+0)U_0\right|^2 d\mathbf{R}dt - \int_{-\infty}^{\infty}\left|\hat{S}(z_i-0)U_0\right|^2 d\mathbf{R}dt\right\}.$$

Since the light wave scatters at a spatially localized inhomogeneity having complex permittivity, both refraction and diffraction are physical mechanisms of scattering. To understand the physical meaning of $\tilde{\gamma}_i$, defined as a ratio of the increment of the square effective beam divergence to the absorbed energy at every local focus, we consider a physical example. Assume that a plane wave with the amplitude A_{0i} and phase φ_{0i}: $U_i = A_{0i}\exp(i\varphi_{0i})$ is incident on a partly absorbing (gray) round screen of radius a_i. Immediately after the passage through the ith screen, the field takes the form

$$U_i^{\text{out}}(|\mathbf{R}|) = A_{0i}\left[1 - \beta_i^I \vartheta(a_i - |\mathbf{R}|)\right]\times$$
$$\times\exp\left[i(\varphi_{0i} + \beta_i^\varphi \vartheta(a_i - |\mathbf{R}|)\right],$$

where ϑ is the unit Heaviside function, and the coefficients β_i^I, β_i^φ determine the changes in the amplitude and phase of the field upon the passage of the ith local focus. In this case, we obtain the following equation for $\tilde{\gamma}_i$:

$$\tilde{\gamma}_i \approx \frac{1}{2n_0k_0}\frac{\int_{-\infty}^{\infty}\left|\nabla_\perp U_i^{\text{out}}\right|^2 d\mathbf{R}dt}{A_{0i}^2\int_{-\infty}^{\infty}\left[1-(1-\beta_i^I\vartheta(a_i-|\mathbf{R}|))^2\right]d\mathbf{R}dt} = \frac{1}{2k_0n_0a_i}\frac{2\beta_i^I + \beta_i^\varphi}{2-\beta_i^I}. \tag{14}$$

It is seen from Eq. (14) that $\tilde{\gamma}_i$ are determined only by parameters of the induced scatterer. For above-critical powers ($P_0 > P_{\text{cr}}$), the intensity of the light field near a focus is virtually independent of the initial laser beam characteristics [Ref. 1, p. 152]. Therefore, we assume that the form of a localized scatterer is independent of the focus number, that is, $\tilde{\gamma}_i = \tilde{\gamma}$. Thus, the value of $\tilde{\gamma}$ for above-critical beams is determined only by the parameters of the medium $\tilde{\gamma} = \tilde{\gamma}(\eta_{\text{MPA}}^{(m)}, n_2, f_{\text{R}}, \eta_{\text{cas}})$. The value of μ (see Ref. 1), as well as $\tilde{\gamma}$, depends only on the properties of the medium. Thus, it follows from Eqs. (12) and (13a) that

$$\frac{d^2}{dz^2}\tilde{R}_e^2 \approx 2\left(H_0 + \gamma\sum_{i=1}^{N(z)}\Delta T_i\right) = 2\left[H_0(\eta,F) + \gamma D_a\right],$$
$$H_0 = \left(1 - \frac{\eta}{2b}\right)\Big/(k_0R_0)^2 + \frac{1}{F^2}, \quad \gamma = \tilde{\gamma}(1-\mu), \tag{15}$$

where $D_a = [1 - T_e(z, \eta, F)]$ is the function of absorption of light energy in the medium.

Based on definition (6) and taking into account Eq. (15), for the spatial zone of filamentation $z \in (z_f, z_{NL})$, where z_{NL} is the point of filamentation termination, we obtain

$$R_e^2(z) \approx \tilde{R}^2\Big|_{z=z_f} T_e^{-1}(z) + (z - z_f)\frac{d}{dz}\tilde{R}_e^2\Big|_{z=z_f} T_e^{-1}(z) + (z - z_f)^2 H_0 T_e^{-1}(z) + 2\gamma\left(\int_0^{\infty} G(z - z')D_a(z')\,dz'\right)T_e^{-1}(z). \quad (16)$$

Here $G(\xi) = \xi\vartheta(-\xi)$ is the Green's function of Eq. (15) with the Heaviside function ϑ.

If there is a point of global focus of the beam defined by the condition

$$dR_e^2/dz\Big|_{z=z_w} = 0, \quad (17)$$

then after its passage by the light pulse equation (16) can be written in the form

$$R_e^2(z) \approx R_{ew}^2 + (z - z_w)^2 H_0 T_e^{-1}(z) + 2\gamma\left(\int_0^{\infty} G(z - z')D_a(z', \eta, F)\,dz'\right)T_e^{-1}(z), \quad (18)$$

where R_{ew} is the effective radius of the beam at a waist determined by Eq. (16) taking into account condition (17).

Equation (18) provides a simple relation between the effective parameters of the beam R_e^2 and T_e. However, γ remains undetermined.

To find γ, we use the results of numerical solution of the problem given by Eqs. (2)–(4). For laser pulses with the initial Gaussian spatiotemporal profile and the following parameters: wavelength $\lambda_0 = 810$ nm, duration $t_p = 80$ fs, radius $R_0 = 1$ mm, and peak power $P_0/P_{cr} = 5$, 10, 15, and the initial curvature radius of the phase front $F = 2L_R$, it was found that $\gamma \approx 1.3 \cdot 10^{-5}$ and does not depend on P_0.

Figures 1 and 3 show the functions $T_e(z)$ obtained in the numerical experiment at a different choice of model parameters and initial conditions. For these dependences, the corresponding approximation functions were constructed and then used to obtain $R_e^2(z)$ (Figs. 2 and 4) according to Eq. (16). Comparison of these functions with the result of numerical solution of NSE (see Figs. 2 and 4) is indicative of validity of Eq. (16).

We have conducted numerical experiments to study the influence of plasma inhomogeneities on the behavior of the beam effective radius. It has been found that if the real part of plasma inhomogeneities of permittivity in NSE (3) is "excluded," the dependence of the effective radius of the femtosecond laser pulse on the distance does not change significantly.

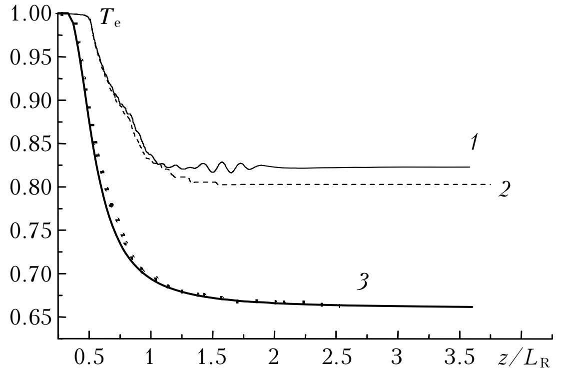

Fig. 1. Coefficient of energy transfer as a function of the propagation distance: complete model with $f_R = 0.5$ (*1*); model neglecting the dispersion with $f_R = 0.5$ (*2*); model neglecting the dispersion with $f_R = 0$ (*3*).

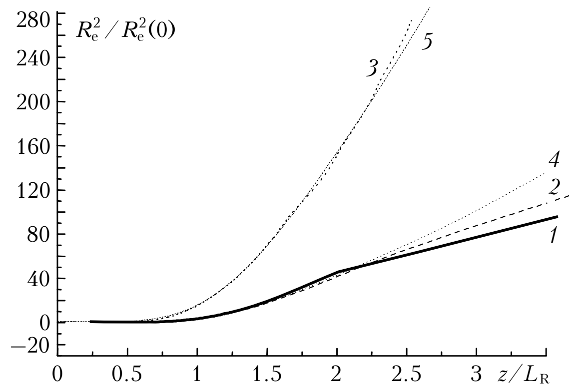

Fig. 2. Square normalized effective radius during the beam propagation in air: numerical calculation, complete model (curve *1*); numerical calculation, model neglecting the dispersion with $f_R = 0.5$ (curve *2*) and 0 (*3*); function (16) with $\tilde{\varepsilon} = k_0 n_2\{1/2|U|^2\}$ (*4*); and with $\tilde{\varepsilon} = k_0 n_2\{|U|^2\}$ (*5*).

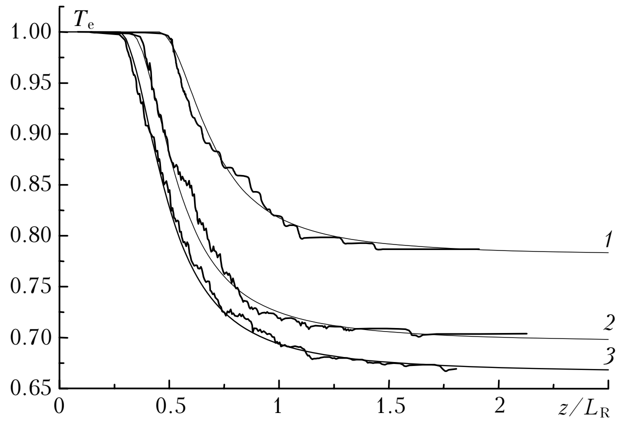

Fig. 3. Coefficient of energy transfer as a function of the propagation distance: complete model; (bold curves) numerical calculation by the model (3)–(4); (light curves) calculation by the equation $T_e(z) = T_e(z_{NL}) + D_a(z_{NL})\vartheta(z - z_f) \times [1 - (1 + q(z - z_f)^2)^{-1}]$ at $P_1(0) = 5P_{cr}$, $q_1 = 16$, $D_a(z_{NL})_1 = 0.22$ (*1*); $P_2(0) = 10P_{cr}$, $q_2 = 20$, $D_a(z_{NL})_2 = 0.30$ (*2*); $P_3(0) = 15P_{cr}$, $q_3 = 20$, $D_a(z_{NL})_3 = 0.34$ (*3*).

This indicates that the diffraction mechanism prevails over the refraction in light scattering at local focuses.

At such self-action regime, it was determined that the rate of growth of the effective beam radius, after leaving the nonlinear beam waist at a global self-focusing, significantly depends on the light energy used for creation of plasma and absorbed in plasma

$$D_{\mathrm{a}}(z) = [E(0) - E(z)]/E(0).$$

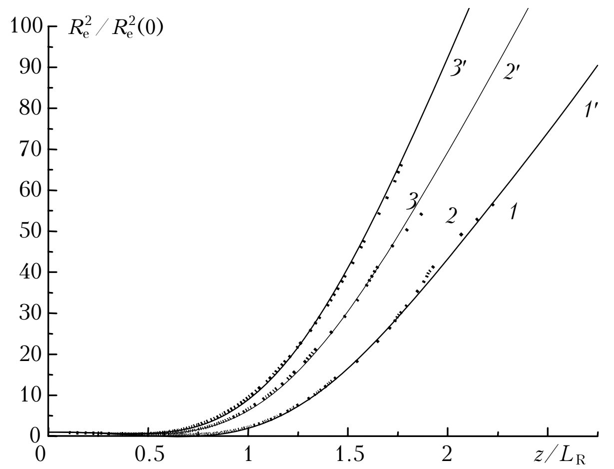

Fig. 4. Square normalized effective radius during the beam propagation in air: (*1–3*) complete model, numerical calculation; (*1′–3′*) plots drawn based on Eq. (16) by the function $D_{\mathrm{a}}(z')$ (see Fig. 3).

The important notes should be made. The right-hand side in Eq. (15) is determined by the function of absorption of light energy. The only significant assumption in derivation of Eq. (15) was independence of γ of the focus number, which is valid for laser beams with supercritical power and is not connected with the beam structure in spatial coordinates. *Therefore, within the proposed model equation (15) is valid for a wide class of laser beams and self-action regimes, in particular, for beams with the multifilamentation structure.*

4. Propagation coefficient

In Ref. 10, for the square effective radius R_{e}^2 in the second zone, the approximate equation with the square dependence on z was obtained. The coefficient of z^2 is the square propagation coefficient $M^2 \geq 1$ [Ref. 11]. In dimensionless coordinates, the approximate equation has the form

$$\bar{R}_{\mathrm{e}}'^2 = R_{\mathrm{ew}}'^2 + \frac{(M^2)^2}{4}(z' - z_{\mathrm{w}}')^2, \qquad (19)$$

where

$$R_{\mathrm{e}}' = R_{\mathrm{e}}/R_0, \ \ R_{\mathrm{ew}}' = R_{\mathrm{ew}}/R_0, \ \ z' = 2z/\left(k_0 R_0^2\right).$$

In the linear medium, the Gaussian beam has the minimal value of M^2 ($M^2 = 1$). That is why the factor M^2 is often referred to as a criterion of beam quality in the sense that the higher is M^2 value, the wider is the difference of the beam divergence from the diffraction divergence of the Gaussian beam $\theta_0 = (k_0 R_0)^{-1}$. Assuming that the approximation condition is fulfilled at the point of filamentation termination z_{NL} determined from the condition of absorption termination:

$$\bar{R}_{\mathrm{e}}'^2\Big|_{z=z_{NL}} = R_{\mathrm{e}}'^2\Big|_{z=z_{NL}}, \qquad (20)$$

and taking into account Eq. (18) for M^2 in the nonlinear medium, we obtain the following equation:

$$M^2(\eta, F) \approx \theta_0^{-1} \times \sqrt{\left[H_0 + 2\gamma \frac{\int_0^\infty G(z_{NL}' - z') D_{\mathrm{a}}(z', \eta, F)\,\mathrm{d}z'}{l_{\mathrm{f}}'^2}\right] T_{\mathrm{e}}^{-1}(z_{NL}')}, \qquad (21)$$

where l_{f}' is the normalized length of the laser beam filamentation: $l_{\mathrm{f}}' = z_{NL}' - z_f'$, which is understood as the normalized distance from the beginning of beam filamentation to the point of termination of light energy absorption in the medium.

The value of M^2 can be determined from experimental data. Thus, for the class of self-acting beams having a global focus in the regime of single and multiple filamentation according to Eqs. (19) and (21), the set of parameters ($\bar{R}_{\mathrm{ew}}$, $\bar{z}_{\mathrm{w}}$, M^2) formed in the process of beam evolution, universally determines the behavior of the beam after its global self-focusing.

To estimate the value of M^2 in the case of single filamentation, that is, in the regime of laser beam propagation, when one filament is formed, we approximate D_{a} by the function of the form

$$D_{\mathrm{a}}(z') = D_{\mathrm{a}}(z_{NL}')\vartheta(z' - z_f') \times$$
$$\times \left\{1 - \left[1 + q(z' - z_f')^2\right]^{-1}\right\}\left[\vartheta(z_{NL}' - z')\right]. \qquad (22)$$

Figures 1 and 3 show $T_{\mathrm{e}}(z')$ drawn based on Eq. (22) with the corresponding parameters $D_{\mathrm{a}}(z_{NL}')$ and q selected based on the results of numerical calculation. It is seen from Figs. 1 and 3 that approximation (22) is in a good agreement with the results of numerical calculation. Using Eq. (22) and according to Eq. (21), we obtain the following approximate equation for M^2:

$$M^2(D_{\mathrm{a}}(z_{NL}')) \approx \theta_0^{-1} \times \sqrt{\frac{H_0 + \frac{\gamma D_{\mathrm{a}}(z_{NL}')}{D_{\mathrm{a}}(z_{NL}') - \Delta D_{\mathrm{a}}}\left(D_{\mathrm{a}}(z_{NL}') - \Delta D_{\mathrm{a}}\left\{1 + \ln\left[\frac{D_{\mathrm{a}}(z_{NL}')}{\Delta D_{\mathrm{a}}}\right]\right\}\right)}{1 - D_{\mathrm{a}}(z_{NL}')}}, \qquad (23)$$

where $\Delta D_{\mathrm{a}} = -\Delta T_{\mathrm{e}} \approx 0.04$ is the relative increment of the energy stored in the medium during the passage of each local focus. Neglecting $\Delta D_{\mathrm{a}}/D_{\mathrm{a}}(z_{NL}') \ll 1$, which corresponds to the condition $P_0/P_{cr} \gg 1$, Eq. (23) takes the following form:

$$M^2(D_a(z'_{NL})) \approx \theta_0^{-1}\sqrt{\frac{H_0 + \gamma D_a(z'_{NL})}{1 - D_a(z'_{NL})}} \approx$$
$$\approx \theta_0^{-1}\sqrt{\gamma}\sqrt{0.814 D_a(z'_{NL}) + 2D_a(z'_{NL})^2}. \quad (23a)$$

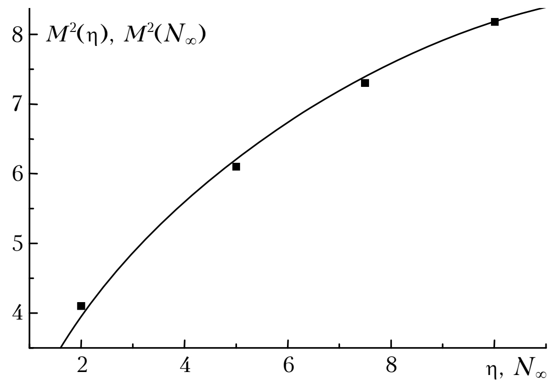

Fig. 5. Propagation coefficient M^2 in the second spatial self-action zone as a function of normalized initial pulse power $\eta = P_0/P_{cr}$ (numerical calculation in Ref. 9, squares) and of N_∞ at $F = 2L_R$ (curve drawn by Eq. (23a)).

Let us introduce the concept of the total number of focuses formed after the termination of filamentation as $N_\infty = D_a(z'_{NL})/\Delta D_a$. Figure 5 shows the dependence of M^2 on N_∞. One can see an approximate relation between the number of focuses N_∞ and the initial power of radiation: $N_\infty \approx \eta$.

Thus, in the regime of single filamentation, the propagation coefficient M^2 is determined by the initial parameters H_0 and F and the amount of energy stored in the medium $D_a(z'_{NL})$.

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