New algorithm of formation of the required amplitude distribution under the phase control. Improving the efficiency of a two-mirror adaptive system

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Received July 5, 2007

A possibility of correction of the thermal and turbulent distortions of laser radiation through the amplitude–phase control is considered. The control can be realized in a two-mirror adaptive system. An iteration algorithm is suggested for generating a required amplitude distribution of a corrected beam at the entrance into a distorting medium under the phase control in a plane located at some distance from the output aperture of the system; its accuracy is estimated. Advantages of the method over the phase-conjugation algorithm are shown in numerical experiments.

1. Systems of amplitude–phase radiation control

A violation of the principle of beams reversal in adaptive systems, built on the base of phase-conjugation algorithm, does not allow a full compensation of the distorting effect of a lengthy layer of turbulent atmosphere on laser radiation.[1] To gain the absence of the distortions in the observation plane, it is necessary to generate at the entrance into the medium a beam with the amplitude distribution coinciding with that of the reference radiation and with the phase profile, inversed relative to the reference one.[2]

This can be executed in different ways. For example, using the reference radiation wave front conversion in a nonlinear crystal realizable on the base of the stimulated Brillouin scattering (here the crystal is a so-called SBS-mirror).[3] Disadvantages of the method are known: the presence of power margin, at which the SBS effect appears; distortions in the nonlinear crystal; radiation power loss from 20 to 40%.

Systems of amplitude–phase control are free of such disadvantages as the margin and loss.[4] In such systems, a required amplitude distribution is set under the phase control in a plane somewhat removed from the output aperture.

Construction of such two-mirror systems is of great interest both in Russia[4] and abroad.[5–7] However, despite the reach bibliography on the matter, the main problem, concerning the amplitude–phase control with two mirrors, has not been solved till now, namely, the phase distribution algorithm has not been found, providing for a required amplitude distribution and full correction of distortions. It is necessary to note, that only a partial atmospheric compensation is reported in Refs. 6 and 7, while in Ref. 8 full compensation has been obtained only for a thin layer of turbulent medium.

In this work, an iteration algorithm for generating a required amplitude distribution is described and its accuracy is estimated, as well as results of turbulent and thermal compensation by this algorithm are presented.

2. Parameters of beam and medium

Radiation propagation in atmosphere is described by a parabolic equation with allowance for thermal and turbulent variations of the refractive index on the path.

Nonlinear distortions of radiation are determined by the dimensionless parameter R_v, usually called the parameter of nonlinearity[9]:

$$R_v = \frac{2k^2 a_0^3 \alpha I_0}{n_0 \rho C_p V} \frac{\partial n}{\partial T}, \quad (1)$$

where k is the wave number; I_0 is the power density on the beam axis in the source aperture plane; a_0 is the initial beam radius; V is the medium flow rate; ρ is the medium density; C_p is the heat capacity at a constant pressure; α is the air absorptance; n_0 is the unperturbed value of the refractive index; T is the temperature.

The intensity of turbulent distortions is determined by the Freed radius, connected with the structure constant C_n^2 by the well-known equation[2]

$$r_0 = 1.68\left(C_n^2 k^2 L\right)^{-3/5}. \quad (2)$$

To characterize the light field in the observation plane, the focusing criterion

$$J(t) = \frac{1}{P_0} \iint \rho(x,y) I(x,y,t)\,\mathrm{d}x\mathrm{d}y \quad (3)$$

is used in power-transfer systems. It has a meaning of the relative fraction of luminosity within the limits of an aperture of the radius S_t. Here P_0 is the total radiation power; $\rho(x,y) = \exp\left(-(x^2 + y^2)/S_t^2\right)$ is the aperture function.

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The path length Z in all numerical experiments was normalized to the diffraction wavelength $Z_d = ka_0^2$, where k is the wave number; r_0 is normalized to the initial beam radius a_0.

3. Setting of a required amplitude distribution at the entrance into a distorting medium under the phase control

Amplitude-phase control, realized in a two-mirror adaptive system, which was designed for atmospheric compensation, has been considered in Refs. 10 and 11, where data on compensation of the disturbing effect of a thin, as compared to the path length, layer of the medium are presented (such a layer was modeled with a single screen). To continue related investigations, the algorithm description and the recapitulation of the main results are necessary.

Propagation of reference and corrected radiations ("direct beam") in the system is shown schematically in Fig. 1.

Phase control is carried out in the planes $M1$ and $M2$, separated by the free diffraction zone Z_1 ($M2$ is, in fact, the plane of exit aperture). The distorting screen is Z_2-distanced from $M2$.

Reference radiation (Gaussian beam) propagates from the far right plane toward the adaptive system, passes through the screen, thus becoming distorted, and falls to sensors in $M2$, which register its amplitude and phase distributions. The phase profile is computed when processing the obtained data; its assignment to Gaussian-profile emission in the plane $M1$ provides for a required amplitude distribution of the corrected beam at the entrance into the distorting medium.

The phase is computed in the following way. A specially purposed program simulates the propagation of the reference beam with amplitude distribution, registered in $M2$, with the phase profile, faced to the plane $M1$ relative to those, registered in $M2$. If $Z_2 = Z_1$, the radiation amplitude distribution in $M1$ is the same as those of the reference beam incident on the distorting screen, i.e., Gaussian. Assigning a phase profile, inverse to the computed one, to a Gaussian beam in the plane $M1$ at an entrance into the distorting medium (after the beam has passed the distance Z_1), we obtain the amplitude distribution, which precisely coincides with the reference beam distribution.

Amplitude change of the reference (top row) and corrected (bottom row) radiation while propagating are shown in Fig. 1. Each figure is located under the path plane, to which it corresponds. It is seen that the amplitude of reference radiation in the plane $M1$ is precisely the amplitude of a beam incident on the distorting screen; propagation of the corrected beam in the plane $M2$ actually results in a required intensity distribution. Note, that the absence of constraints on the adaptive system, namely, the infinitely high operating speed, the absolutely precise phase definition and assignment, as well as the infinite aperture radius, yields a full compensation independently of the screen parameters, i.e., the quality of system operation is independent of the turbulent intensity.

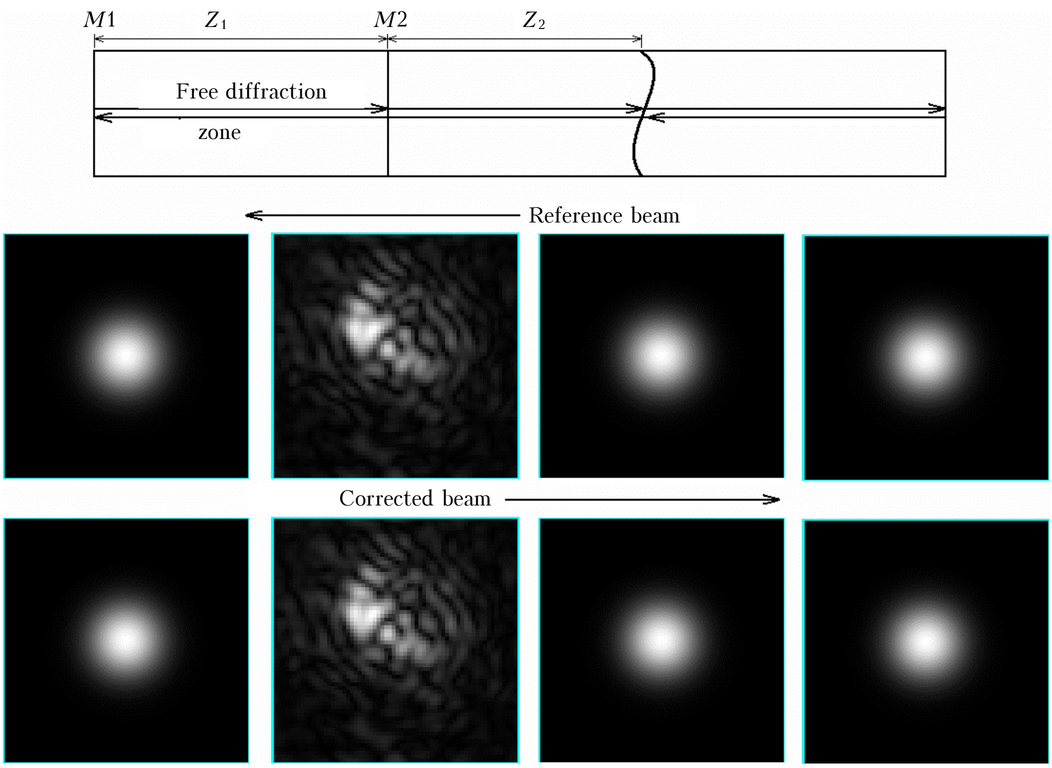


Fig. 1. Amplitude–phase control algorithm for compensation of a thin, as compared to the path length, layer of a distorting medium.

All the above arguments are valid only for the case of a single distorting screen. Therefore, mechanical extension of the constructed algorithm to a compensation system for distributed distorting layer results in a loss of control efficiency.[10] In this case, a reference beam in the plane $M1$ differs from the Gaussian one, and the amplitude distribution of the corrected beam in the plane $M2$ differs from the required one.

To solve the problem on the base of the above method, the following iterative algorithm is suggested. As in the previous case, parameters are registered in $M2$ and then used by the program. In the model, the phase is conjugated, the beam passes to the plane $M1$, the reference beam amplitude changes there for the Gaussian amplitude of the "direct" beam, the phase is conjugated, and propagation toward $M2$ is simulated again. In $M2$, we set the propagation coinciding with the required amplitude, conjugate the phase, and again simulate beam passage to the plane $M1$. The procedure is repeated several times, resulting at an entrance into a distorting medium in the beam with the profile, close to the preset one.

The algorithm convergence has not been rigorously proved up to now. Nevertheless, it can be said with certainty that it provides for a high accuracy of a required amplitude in all situations typical for atmospheric optics. This is illustrated by Table 1, where light field distributions of reference beam are given, obtained when propagating under self-acting conditions and in the presence of turbulent distortions.

The parameters in the left column characterize the medium distorting effect, next columns shows the amplitude distribution of reference radiation in the plane $M2$, i.e., the required distribution, and of radiation, generated in the adaptive system. The last column consists the squared error

$$\varepsilon = \frac{\iint\sqrt{\left[A(x,y) - A_{\mathrm{ref}}(x,y)\right]^2}\,\mathrm{d}x\mathrm{d}y}{\iint A(x,y)\,\mathrm{d}x\mathrm{d}y},$$

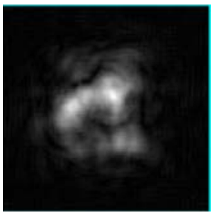
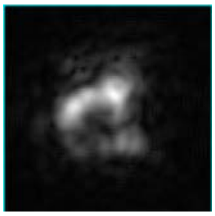
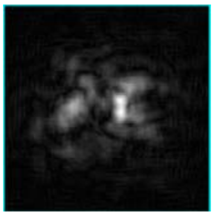
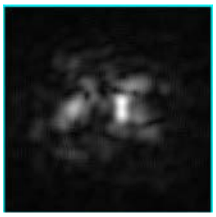
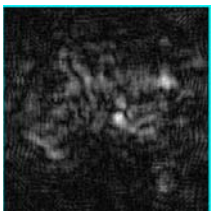
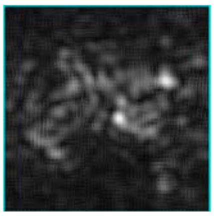
characterizing the deviation of the obtained beam from the preset one. Here $A_{\mathrm{ref}}(x,y)$ and $A(x,y)$ are the amplitude distributions of reference and corrected radiations.

The squared error is evidently small in conditions, when the required distribution is of sufficiently simple form (11% for $R_v = -20$ and $r_0 = 0.001$, the first column in Table 1). The accuracy decreases with complication of the relief of amplitude distribution of the reference beam. Thus, at a high turbulence intensity ($r_0 = 0.002$) ε increases to 28% (bottom row of Table 1).

4. Self-action and turbulence compensation with the amplitude–phase control algorithm

The quality of adaptive correction for the case of distortions due to highly intensive turbulence is shown in Fig. 2 (hereinafter the path length Z is normalized to the diffraction length). The recorded values of focusing criterion J are given for every numerical experiment.

Table 1. Accuracy of a required amplitude distribution at the entrance into a distorting medium in different conditions

Parameter	Amplitude distribution		Squared deviation
	of reference radiation	of corrected beam	
$R_v = -20$, $r_0 = 0.001$			ε = 0.112
$R_v = 0$, $r_0 = 0.004$			ε = 0.189
$R_v = 0$, $r_0 = 0.002$			ε = 0.279

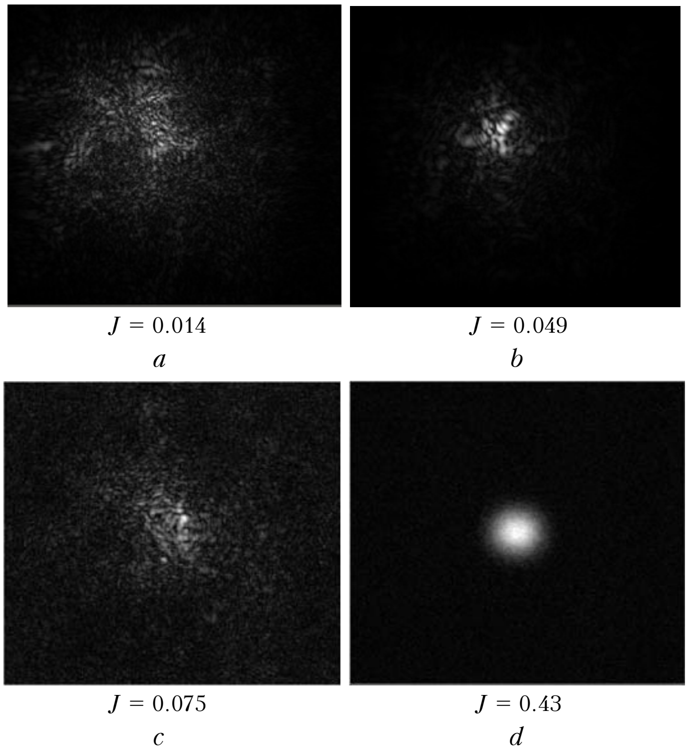


Fig. 2. Amplitude distribution for a beam passing a layer of distorting medium with the length $Z = 0.5$ at $r_0 = 0.001$: without compensation for a collimated (*a*) and optimally focused (*b*) beams, with the use of phase conjugation (*c*) and amplitude–phase control (*d*).

The amplitude-phase correction evidently allows more than 10-fold increase of the distortion compensation efficiency, as compared to the phase control, and generation in the observation plane of a beam with amplitude distribution, close to Gaussian.

Quantitative data characterizing system efficiency in atmospheric turbulence compensation are given in Figs. 3 and 4.

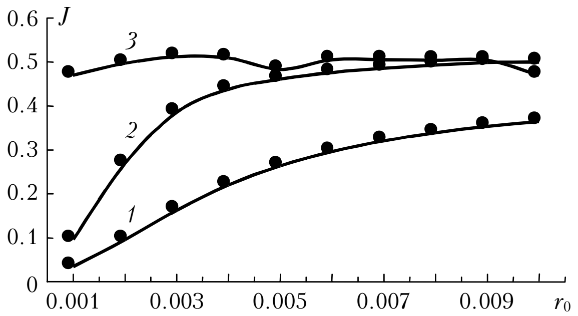


Fig. 3. Efficiency of atmospheric turbulence compensation as a function of distortion intensity for one of realizations in a system without control (*1*), phase conjugation system (*2*), and ideal one (*3*).

The turbulence is modeled by five screens equally spaced on a path of the length $Z = 0.5$. The use of amplitude–phase correction allows virtually constant values of focusing criterion to be obtained in the whole r_0 variation range. Practically, we obtain J values corresponding to a collimate beam propagating in an undistorted medium.

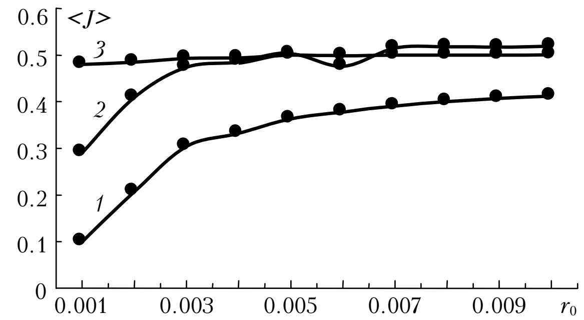
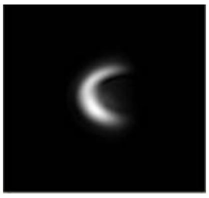
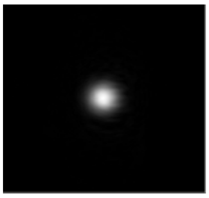
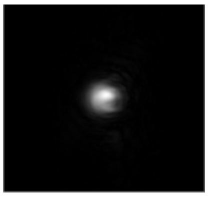


Fig. 4. Efficiency of atmospheric turbulence compensation as a function of distortion intensity at averaging over 50 realizations. Curve numbers and the parameter value correspond to those in Fig. 3.

Note the difference of these data from the results of phase conjugation, at which criterion values decrease with an increase in distortion intensity. A two-mirror system provides for the 1.8-fold increase of efficiency for minimal r_0, using in the applying computation grid, as compared to the phase conjugation (Fig. 4, averaged results); this difference can attain 5–6 times for individual turbulence realizations (Fig. 3, curves *2* and *3*).

The system of amplitude–phase control operates stably in thermal self-acting as well. The correction process is qualitatively illustrates by Table 2. Here generation of a beam with intensity distribution, coinciding with the Gaussian profile of reference radiation, is shown at a moderate radiation power ($R_v = -30$), i.e., distortions are fully compensated.

Table 2. The use of the amplitude–phase control algorithm for thermal self-action compensation

Parameter of nonlinearity	Uncorrected beam	Compensation result
$R_v = -30$		
$R_v = -50$		

Insignificant deviation of the resulted distribution from the reference one is observed with a power increase ($R_v = -50$) (some irregularity remains), but the major part of energy is concentrated on the axis of radiation propagation in this case as well.

Data, characterizing self-action compensation quantitatively, are shown in Fig. 5; here values of focusing criterion are given, obtained in using amplitude–phase control, phase conjugation, and for a system with open feedback (free of control). In a two-mirror system, the focusing criterion equals to 0.5 throughout the variation range of the nonlinearity parameter (from $|R_v| = 10$ to $|R_v| = -50$), i.e., to the value obtained for a collimated beam (a collimated beam was taken as a reference, hence, it is impossible in principle to obtain large values of the criterion).

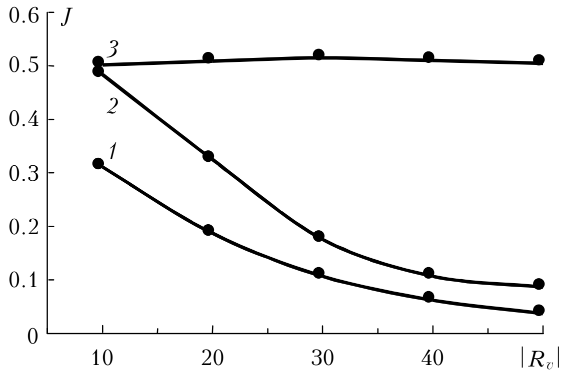


Fig. 5. Compensation of thermal self-action with the use of a two-mirror system ($Z = 0.5$) in a system without control (*1*), phase conjugation one (*2*), and ideal system (*3*).

Essentially lower efficiency is observed in the phase conjugation, for which the criterion value decreases from 0.49 ($|R_v| = 10$) to 0.1 ($|R_v| = 50$). It can be concluded, that amplitude–phase control allows a 5-fold increase in the correction efficiency, as compared to the phase conjugation.

Advantages of a two-mirror system are also evident in situations, when distortions are caused by a combined effect of the turbulence and self-action. The corresponding data are shown in Figs. 6–9.

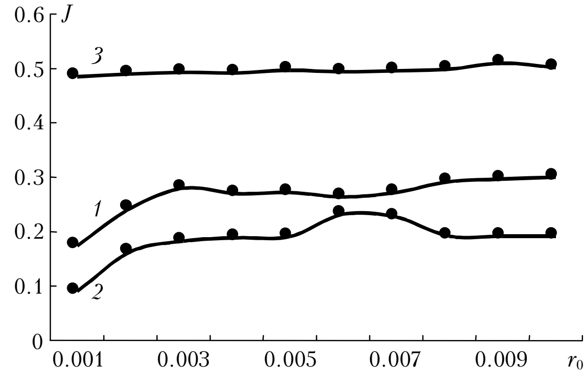


Fig. 6. Efficiency of atmospheric turbulence and thermal self-action compensation as a function of medium distortion intensity for one of realizations in a system without control (*1*), phase conjugation system (*2*), and ideal one (*3*); $R_v = -20$.

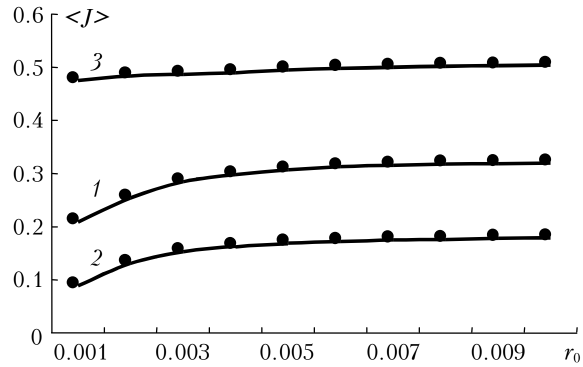


Fig. 7. Efficiency of atmospheric turbulence and thermal self-action compensation as a function of medium distortion intensity at averaging over 50 realizations. Curve numbers and the parameter value correspond to those in Fig. 6.

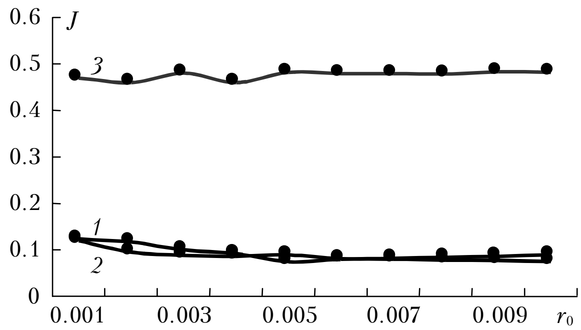


Fig. 8. Efficiency of atmospheric turbulence and thermal self-action compensation as a function of medium distortion intensity for one of realizations in a system without control (*1*), phase conjugation system (*2*), and ideal one (*3*); $R_v = -40$.

The Fried's radius is a variable parameter in numerical experiments. The parameter of nonlinearity was equal to -20 ($|R_v| = 20$, moderate radiation power) in Figs. 6 and 7 and its modulus was increased to 40 in Figs. 8 and 9. The resulting values of focusing criterion were close to 0.5 in the amplitude–phase control in all cases.

It is also necessary to note that the registered compensation quality weakly depends on the medium and beam parameters. This means that we obtained the beam with intensity distribution close to the Gaussian profile or, in other words, almost full compensation of distortions in the whole variation range of the problem parameters.

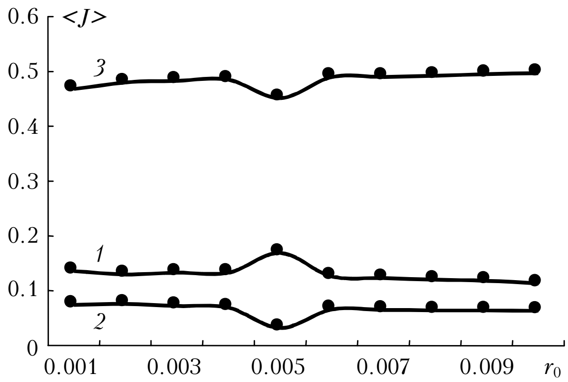


Fig. 9. Efficiency of atmospheric turbulence and thermal self-action compensation as a function of medium distortion intensity at averaging over 50 realizations. Curve numbers and the parameter value correspond to those in Fig. 8.

Essentially lower correction efficiency is observed in the use of phase conjugation. The obtained resulting criterion values weakly depend on the Fried's radius, i.e., thermal self-action mainly contributes in combined distortions in both cases (at $|R_v| = 20$ and 40). However, the difference of the criterion from data, obtained without correction, is also insignificant. This especially appears at $|R_v| = 40$, when the criterion values under control (Fig. 9, curve *2*) exceed those in the case without control (curve *1*) not more than by 0.05 and are almost equal for certain realizations (corresponding curves in Fig. 8).

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