# Consideration of chromatic aberrations in measurements of spectral characteristics of radiation 

A.A. Zemlyanov, ${ }^{1}$ A.M. Kabanov, ${ }^{1}$ A.N. Stepanov, ${ }^{2}$ S.B. Bodrov, ${ }^{2}$ N.S. Zakharov, ${ }^{3}$ and S.V. Kholod ${ }^{3}$<br>${ }^{1}$ V.E. Zuev Institute of Atmospheric Optic, Siberian Branch of the Russian Academy of Sciences, Tomsk ${ }^{2}$ Institute of Applied Physics, Russian Academy of Sciences, Nizhny Novgorod<br>${ }^{3} 12$ Central Scientific Research Institute, Ministry of Defence of the Russian Federation, Sergiev Posad

Received July 1, 2008


#### Abstract

The influence of chromatic aberrations on measurements of spectral characteristics of broadband radiation excited in condensed media by femtosecond laser pulses has been investigated.


## Introduction

In many experiments involving measurements of spectral characteristics of radiation, a significant problem is to direct the laser beam into the measurement device, for example, into the aperture of a light guide going to the spectrometer. This problem becomes especially urgent, when radiation comes to aperture at large angles and at a wide wavelength range. In this case, it is difficult to focus radiation at the lightguide aperture because of significant chromatic aberrations in the optical system. Such problems can be solved in two ways: by using specialized optics, compensating aberrations, or by calculating corrections for chromatism for the obtained measurements.

## Schematic of measurements

The simplest case, in which we can face this problem, is the measurement of a spectrum from a nearly point white-light source (source dimensions are smaller than the lens focal length) in the cylindrical formulation of the problem (Fig. 1).


Fig. 1. Optical arrangement of measurements: source 1; collecting lens 2; lightguide aperture 3; fiber lightguide 4; spectrometer 5 , and shutter 6 .

This arrangement was used in experiments at a laser setup at the Institute of Applied Physics RAS (Nizhny Novgorod). In those experiments, the spectral-energy characteristics of radiation, emitted at the interaction of filaments of Ti:Sa laser radiation with a K8 glass plate, were studied. ${ }^{1}$ This interaction resulted in generation of the supercontinuum or the formation of a wide continuum spectrum upon propagation of high-power radiation pulses through a nonlinear medium. The supercontinuum radiation spot on the screen, installed in front of a collecting lens, and the radial distribution of the color balance from the center to the edge of the image of this spot are shown in Fig. 2.

The presence of a wide blue-violet edge in the radiation spot is indicative of the higher divergence of the shortwave share of the supercontinuum radiation. The spectral density of the radiation energy $w_{\lambda}$ recorded by the spectrometer and normalized to the lightguide transfer function, is shown in Fig. 3.

A bright spot of laser radiation passed through a sample was observed in the central part of the image (Fig. $2 a$ ). To separate the supercontinuum radiation from the laser radiation, the shutter 6 was set at the optical axis in front of the collecting lens during measurement of spectral characteristics (see Fig. 1).

It is seen that the radiation from the sample has a pronounced blue-violet component both visually and in the photography (Fig. 2). At the same time, the intensity of the blue-violet spectral region (380490 nm ) of the recorded radiation (see Fig. 3) turnes out to be much lower than that of the red region ( $630-760 \mathrm{~nm}$ ). This indicates that quantitative results do not correspond to qualitative observations. Thus, it is necessary to perform calculations and to obtain the correction dependence for adequate consideration of chromatic aberrations.


Fig. 2. Radiation spot on the screen installed in front of the collecting lens: ( $a$ ) photography and ( $b$ ) radial distribution of the color balance ( $R$ is red, $G$ is green, and $B$ is blue color channels).


Fig. 3. Recorded spectrum; an energy density of the incident radiation is (1) 0.56 and (2) $6.09 \mathrm{~J} / \mathrm{cm}^{2}$; a part of radiation is cut off by the SZS21 GOST 9411-91 filter.

## Calculation of aberrations

In the general case, the cylindrically symmetric radiation source is an object with the spectral energy density $w_{\lambda}(r, \lambda)$, in $J /\left(\mathrm{cm}^{2} \cdot \mathrm{~nm}\right)$, where the total pulse energy is

$$
\begin{equation*}
W=\int_{0}^{\infty} \int_{0}^{\infty} w_{\lambda} 2 \pi r \mathrm{~d} r \mathrm{~d} \lambda \tag{1}
\end{equation*}
$$

and the radiation from a source propagates at an angle $\theta$ toward a lens by some law $W_{\theta}(\theta)$, in $\mathrm{J} / \mathrm{rad}$, where the total pulse energy is

$$
\begin{equation*}
W=\int_{0}^{\pi / 2} W_{\theta} \sin \theta \mathrm{d} \theta \tag{2}
\end{equation*}
$$

Therefore, we represent the source as an object with the spectral-angular energy density $w_{\lambda \theta}(r, \lambda, \theta)$, in $\mathrm{J} /\left(\mathrm{cm}^{2} \cdot \mathrm{~nm} \cdot \mathrm{rad}\right)$, where the total pulse energy is

$$
\begin{equation*}
W=\int_{0}^{\pi / 2} \int_{0}^{\infty} \int_{0}^{\infty} w_{\lambda \theta} 2 \pi r \mathrm{~d} r \mathrm{~d} \lambda \sin \theta \mathrm{~d} \theta \tag{3}
\end{equation*}
$$

We assume that the radiation has a continuum spectrum peaking at the wavelength $\lambda_{0}$. Then for the maximal sensitivity, the lightguide aperture should be at the place of the real image of the source obtained with the use of the collecting lens at the wavelength $\lambda_{0}$.

In the experiments, the source was located at the double focal length $S_{0}=2 f_{0}$ from the collecting lens and the real image of the source was also formed at the double focal length from the lens.

The shift of the real image of the source of radiation with the wavelength $\lambda$ relative to the image with the wavelength $\lambda_{0}$ can be expressed $\mathrm{as}^{2}$ :

$$
\begin{equation*}
\Delta s(\lambda)=S_{0}^{2} /\left[f_{0} v(\lambda)\right] \tag{4}
\end{equation*}
$$

where $f_{0}$ is the focal length of the lens at the wavelength $\lambda_{0} ; S_{0}$ is the conjugate focal length of the lens at the wavelength $\lambda_{0}$ equal to the double focal length;

$$
\begin{equation*}
v(\lambda)=\left(n_{0}-1\right) /\left[n(\lambda)-n_{0}\right] \tag{5}
\end{equation*}
$$

is the dispersion of the lens at the wavelength $\lambda ; n(\lambda)$ and $n_{0}$ are the refractive indices of the lens material at the wavelengths $\lambda$ and $\lambda_{0}$, respectively.

It follows from Eqs. (4) and (5) that

$$
\begin{equation*}
\Delta s(\lambda)=\frac{S_{0}^{2}}{f_{0}} \frac{n(\lambda)-n_{0}}{n_{0}-1} \tag{6}
\end{equation*}
$$

For different optical materials, the value of chromatic aberrations becomes significant (Fig. 4).

Thus, the near-UV edge of the spectrum (350 nm) is focused nearly by 2 cm closer than the lightguide aperture, while the IR edge of the spectrum $(1.05 \mu \mathrm{~m})$ is focused by 5 mm farther.

The radiation, incident on the aperture plane form the source element $2 \pi r d r$, is identical to the radiation from the element $2 \pi r d r$ of the real (paraxial) image of the source being at the distance $\Delta s(\lambda)$.


Fig. 4. Chromatic aberration. The wavelength dependence of the position of the source real image relative to the screen.

Now we calculate the illumination generated by the image of the source on the lightguide aperture plane (screen). For wavelengths different from $\lambda_{0}$, the real image lies beyond the aperture plane, and the illuminated spot is much large than the real image. Therefore, the following equation for illumination can be used:

$$
\begin{equation*}
w_{\lambda}(\theta)=\frac{W_{\lambda \theta}}{(\Delta s)^{2}}\left(\frac{\alpha}{\alpha^{\prime}}\right)^{2} \frac{\cos ^{4} \vartheta}{\cos \theta}(1-4 \eta), \tag{7}
\end{equation*}
$$

where $\alpha$ and $\alpha^{\prime}$ are the angles formed by the paraxial ray passing through the source and its real image ${ }^{3} ; \vartheta$ is the angle, at which the light, emitted from the source at the angle $\theta$, is incident on the real image of this source; $\eta$ is the parameter proportional to the coma of the optical system:

$$
\begin{equation*}
\eta=\left(\delta s^{\prime} / \Delta s\right)+\delta s^{\prime \prime} . \tag{8}
\end{equation*}
$$

Here $\delta s^{\prime}$ is the spherical aberration at the axis of the optical system, $\delta s^{\prime \prime}$ is the aberration related to violation of the sine law. These aberrations can be written as

$$
\begin{gather*}
\delta s^{\prime}=A \sin ^{2} \theta,  \tag{9}\\
\delta s^{\prime \prime}=\frac{\alpha \sin \theta}{\alpha^{\prime} \sin \vartheta}-1=B \sin ^{2} \theta, \tag{10}
\end{gather*}
$$

where $A$ and $B$ are some coefficients.
In our case, the collecting lens has an identical curvature of the both sides, the source lies at the double focal length from the lens, the angle $\theta$ is small, and the lightguide aperture for the most part of wavelength values is much smaller than the illuminated spot. Therefore, we can take $\vartheta=\theta, \alpha=\alpha^{\prime}$, and $\eta \ll 1$. Then Eq. (7) can be written in the form

$$
\begin{equation*}
w_{\lambda}(\theta)=\frac{W_{\lambda \theta}}{(\Delta s)^{2}} \cos ^{3} \theta \tag{11}
\end{equation*}
$$

The radiation

$$
\begin{gather*}
W_{\lambda}=2 \pi \int_{0}^{\Theta} W_{\lambda \theta} \cos ^{3} \theta \tan \theta\left(1+\tan ^{2} \theta\right) \mathrm{d} \theta= \\
=\frac{2}{3} \pi W_{\lambda \theta}[1-\cos \Theta] \tag{12}
\end{gather*}
$$

falls into the lightguide aperture of the radius $r$. Here, $\Theta$ is the angular size of the aperture from the point of the real image of the source:

$$
\begin{equation*}
\Theta=\arctan \left(\frac{r}{\Delta s}\right) \tag{13}
\end{equation*}
$$

The correction function $k_{\lambda}$ in this case has the form shown in Fig. 5 by curve 1. The plateau near the wavelength $\lambda_{0}$ corresponds to the case that the whole radiation of the point source falls into the lightguide aperture.


Fig. 5. Correction curves.
For sources, the real image size of which at the wavelength $\lambda_{0}$ is larger than the aperture, the correction curve changes strongly and takes the form of a more blurred curve (Fig. 5, curve 2) because now the size of illuminated spots is much larger than the size of the lightguide aperture for any wavelength.

## Calculated results

The division of the detected radiation spectrum (see Fig. 3) by the correction curve (see Fig. 5, curve 2) yields the reconstructed spectral energy density (Fig. 6).


Fig. 6. Reconstructed spectrum; energy density of the incident radiation is $0.56(1)$ and $6.09 \mathrm{~J} / \mathrm{cm}^{2}(2)$.

It is seen from Fig. 6 that if the shortwave part in the initial spectrum is almost absent (see Fig. 3), although the blue-violet component of radiation is visually observed (see Fig. 2a), then in the reconstructed
spectrum (see Fig. 6) the presence of shortwave radiation becomes, to the contrary, noticeable.

For verification of the obtained data, it is necessary to check qualitatively the form of the obtained correction curve. This can be done approximately based on the following reasons.

Multiplication of the reconstructed spectrum (Fig. 6) by the spectral sensitivity of camera elements $f^{\text {R,G,B }}$ gives a set of integral values of "brightness" of the detected image in the red, green, and blue channels:

$$
F_{\text {calc }}^{\mathrm{R}, \mathrm{G}, \mathrm{~B}}=\int_{0}^{\infty} I_{\lambda}^{\mathrm{R}, \mathrm{G}, \mathrm{~B}} f^{\mathrm{R}, \mathrm{G}, \mathrm{~B}}(\lambda) \mathrm{d} \lambda .
$$

The decomposition of the image into pixels, the following separation of red, green, and blue components, followed by their separate summing for every channel, give a set of integral values of the image "brightness" in the photo in the red, green, and blue channels:

$$
F_{\mathrm{foto}}^{\mathrm{R}, \mathrm{G}, \mathrm{~B}}=\sum_{i j} G_{i j}^{\mathrm{R}, \mathrm{G}, \mathrm{~B}} .
$$

The ratio $F_{\text {calc }}^{\mathrm{R}, \mathrm{G}, \mathrm{B}} / F_{\text {foto }}^{\mathrm{R}, \mathrm{G}, \mathrm{B}}$, normalized to the ratio $F_{\text {calc }}^{\mathrm{R}} / F_{\text {foto }}^{\mathrm{R}}$ for the red level should show the reliability of correction. The selection of the red channel is not principal; it is caused by the fact that the red channel is least subjected to chromatic aberrations, because the entire optical system is adjusted to a wavelength of 800 nm (Fig. 4).

Ideally, the reconstructed spectrum should have the identical ratios of the calculated and photometric brightness for each color channel. The ratios between levels for the reconstructed and initial spectra are tabulated below.

Table. Reliability of the spectrum relative to the red level

| Color channel | Spectrum, \% |  |  |
| :--- | :---: | :---: | :---: |
|  | initial | reconstructed | ideal |
| Red, R | 100.0 | 100.0 | 100.0 |
| Green, G | 46.1 | 90.8 | 100.0 |
| Blue, B | 33.3 | 94.8 | 100.0 |

It is seen from the Table that if the level of the green channel in the initial spectrum was underestimated nearly by two times, and that of the blue channel was underestimated by three times due to chromatic aberrations, then the green and blue levels in the reconstructed spectrum differ from the ideal only by 9 and $5 \%$, respectively. This suggests that the obtained correction function is practicable.

## Conclusions

In this paper, we have studied the influence of chromatic aberrations on the measurements of radiation spectral characteristics.

The possibility of reconstruction of actual experimental data from parameters of the measurement optical system has been demonstrated.

## References

1. A.A. Zemlyanov, A.M. Kabanov, G.G. Matvienko, N.G. Ryzhov, A.N. Stepanov, S.V. Kholod, and S.B. Bodrov, in: Abstracts of Reports at the 8th Int. Conf. "Atomic and Molecular Pulsed Lasers" (Tomsk, 2007), p. 99.
2. N.P. Zakaznov, S.I. Kiryushkin, and V.I. Kuzichev, Theory of Optical Systems (Mashinostroenie, Moscow, 1992), 448 pp.
3. G.G. Slyusarev, Calculation of Optical Systems (Mashinostroenie, Leningrad, 1975), 640 pp.
