Self-focusing of ultrashort laser pulse with super-Gaussian spatial profile of intensity

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The non-stationary self-focusing of femtosecond laser radiation with Gaussian and super-Gaussian spatial profiles of intensity in air in the single filamentation mode have been studied theoretically. The formalism of effective beam parameters is used; the analytical expressions are derived allowing calculating key self-focusing parameters of beams with non-Gaussian profile. Qualitative and quantitative similarity of the initial evolution stage of effective radius of such beams from the generalized evolutionary variable at distances up to global nonlinear focus is established.

Averaged description of wave beams on the base of formalism of effective (mean-square) parameters allows a unified representation of evolution of beams of different transverse profiles of intensity for the case of beam propagating through a linear medium.[1] Similar representation also exists at stationary laser beam self-focusing in a medium with cubic Kerr nonlinearity, at least up to the local collapse point z_N.[2] In this case, it is sufficient to calculate several parameters (R_{e0}, θ_D, P_c) for each specific type of beam, which depend only on optical parameters of the medium and transverse profile of electric field strength, to determine the value of stationary *effective radius* R_e of the beam at any point z of optical path by the universal evolutionary dependence:

$$R_e^2(z) = (1-\eta)(\theta_D z)^2 + R_{e0}^2\left(1-\frac{z}{F}\right)^2,$$

$$z < z_N \text{ at } \eta > 1. \qquad (1)$$

Here

$$R_e(z) = \left[\frac{1}{P(z)}\iint_{\mathbf{R}_\perp} d^2\mathbf{r}_\perp I(\mathbf{r}_\perp, z)\left|(\mathbf{r}_\perp - \mathbf{r}_{gr})\right|^2\right]^{1/2} \qquad (2)$$

is the effective (mean-square) radius (hereinafter, *instant* effective radius); $\mathbf{r}_{gr}$ is the radius-vector of the beam gravity center; $|\mathbf{r}_\perp| = \sqrt{x^2+y^2}$ is the transverse coordinate; $I = c/8\pi|U|^2$ is the radiation intensity; U is the strength of the wave electric field; $P = \iint_{\mathbf{R}_\perp} d^2\mathbf{r}_\perp I(\mathbf{r}_\perp, z)$ is the pulse power; $R_{e0} = R_e(z=0)$;

$$\theta_D = \left[\frac{1}{k_0^2}\frac{\iint_{\mathbf{R}_\perp} d^2\mathbf{r}_\perp \left|\nabla_\perp U(\mathbf{r}_\perp, z=0)\right|^2}{\iint_{\mathbf{R}_\perp} d^2\mathbf{r}_\perp \left|U(\mathbf{r}_\perp, z=0)\right|^2}\right]^{1/2} \qquad (3)$$

is the *diffraction divergence* of a collimated beam of the same transverse profile of the wave electric-field strength $U(\mathbf{r}_\perp)$; F is the initial curvature of wave front; $\eta = P_0/P_c$ is the self-focusing parameter, defined as the ratio of initial pulse power P_0 to the *self-focusing critical power*:

$$P_c = P_{cg} G[U(\mathbf{r}_\perp, 0)];$$

$$G[U(\mathbf{r}_\perp,0)] = \frac{\iint_{\mathbf{R}_\perp} d^2\mathbf{r}\left|U(\mathbf{r}_\perp,0)\right|^2 \iint_{\mathbf{R}_\perp} d^2\mathbf{r}_\perp\left|\nabla_\perp U(\mathbf{r}_\perp,0)\right|^2}{2\iint_{\mathbf{R}_\perp} d^2\mathbf{r}_\perp \left|U(\mathbf{r}_\perp,0)\right|^4}, \qquad (4)$$

where $P_{cg} = \lambda_0^2/(2\pi n_0 n_2)$ is the critical power for the beam with Gaussian profile; λ_0 is the radiation wavelength (carrier); n_0 is the linear index of refraction of the medium, n_2 is the nonlinear addition to it, connected with the Kerr effect; $k_0 = 2\pi n_0/\lambda_0$ is the wave number. The parameters defined by Eqs. (2)–(4) are the radiation propagation coefficients dependent on the laser beam profile.

Modify Eq. (1) to the following form:

$$R_e^2(z) = R_{e0}^2\left[(1-\eta)(z/L_{D*})^2 + \left(1-\frac{z}{F}\right)^2\right],$$

$$z < z_N \text{ at } \eta > 1, \qquad (5)$$

where the parameter of *generalized diffraction length* $L_{D*} = R_{e0}/\theta_D$ is introduced. For the beam with Gaussian initial transverse profile (GP)

$$U_g(\mathbf{r}_\perp, 0) = \exp\left(-\left|\mathbf{r}_\perp\right|^2 / 2R_0^2\right) \qquad (6)$$

with the radius R_0 (with respect to the level $1/e$ from the intensity maximum), we have $R_{e0} = R_0$;

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$\theta_D = \theta_{Dg} \equiv 1/(k_0 R_0)$, and $L_{D^*} = L_{Dg} \equiv k_0 R_0^2$. The resulting position of nonlinear focus of such beam z_N is defined with accounting for the combined effect of initial and induced focusings:

$$z_N = z_K F/(z_K + F)\,, \tag{7}$$

where $z_K = L_D/\sqrt{\eta - 1}$ is the coordinate of the point of the collimated beam transverse collapse.

The beam of super-Gaussian profile (SGP) on a circular aperture

$$U_{sg}(\mathbf{r}_\perp, 0) = \exp\left(-|\mathbf{r}_\perp|^{2q} / 2R_0^{2q}\right), \quad q = 1, 2, 3, \ldots, \tag{8}$$

considered below, is characterized by other values of critical power $P_c = P_{cg} q \cdot 2^{(1-q)/q}$ and diffraction length $L_{D^*} = L_D \sqrt{\frac{\Gamma(2/q)}{q^2}}$ (for reference: for SGP beams (7) $R_e(0) = R_0 \sqrt{\frac{\Gamma(2/q)}{\Gamma(1/q)}}$; $\theta_D = \theta_{Dg} \frac{q}{\sqrt{\Gamma(1/q)}}$). However, equation (5) keeps its generality for beams of such type also in relative coordinates $\bar{z}^* = z/L_{D^*}$ with accounting for corresponding change of relative beam power η.

Self-focusing of a short laser pulse, which is a wave packet restricted both in space and time, becomes of the dynamic character. If the temporal profile of pulse intensity is conventionally divided into successive layers, then each of them is characterized by its power $P_i = P(t_i)$, where t is the time. Hence, according to Eqs. (1) and (7), each layer has its own law of evolution of the effective radius $R_e(z; t_i)$ and the position of nonlinear stock $z_N(t_i)$, therefore, the closeness of this position to the path beginning depends on the larger P_i magnitude. Finally, propagation of a high-power short pulse in the self-focusing mode for an observer inside the laboratory coordinates is represented as a sequence of local foci of each time layer, propagating with the velocity of radiation propagation through the medium. This model of Kerr self-focusing is known in literature as the model of *moving foci* and first was suggested in Ref. 3.

Consider the beam *integral effective radius* R_{eg} characterizing the size of lumped radiant density zone[4]:

$$R_{eg}^2(z) = \left[\int_{-\infty}^{\infty} P(z,t)\,dt\right]^{-1} \int_{-\infty}^{\infty} P(z,t) R_e^2(z;t)\,dt =$$

$$= \frac{1}{E(z)} \int_{-\infty}^{\infty} dt \iint_{\mathbf{R}_\perp} d^2\mathbf{r}_\perp I(\mathbf{r}_\perp, z; t) \left|\mathbf{r}_\perp - \mathbf{r}_{gr}(t)\right|^2, \tag{9}$$

where E is the total pulse energy. Substituting Eq. (5) in Eq. (9), obtain the law of variation of squared integral effective radius at self-focusing:

$$R_{eg}^2(z) = R_{eg0}^2 \left[(1 - \eta^*)(z/L_{D^*})^2 + \left(1 - \frac{z}{F}\right)^2\right]. \tag{10}$$

Here

$$R_{eg0} = R_{eg}(z = 0);\quad \eta^* = \left[P_c \int_{-\infty}^{\infty} P(z,t)\,dt\right]^{-1} \int_{-\infty}^{\infty} P^2(z,t)\,dt$$

is the pulse-average focusing parameter. This equation forecasts the transverse beam collapse in general at the distance $\bar{z}_K^* = z_K / L_{D^*} = 1/\sqrt{\eta^* - 1}$. For the Gaussian temporal profile of the pulse

$$P(t) = P_0 \exp\{-\tau^2\},$$

where P_0 is the peak value; $\tau = t/t_p$ is the dimensionless time; t_p is the duration with respect to the level $1/e$, obtain $\eta^* = \eta_0/\sqrt{2}$ $(\eta_0 = \eta(t = 0))$. Thus, qualitative behavior of the beam integral effective radius at its non-stationary self-focusing can be described within the time-independent theory with Eq. (5), however, with another self-focusing parameter η^*.

This conclusion is confirmed by numerical calculations, carried out within the model of nonlinear Schrödinger equation (NSE) for Gaussian and super-Gaussian transverse profiles of intensity. As is known, this equation describes propagation of ultrashort laser radiation and takes into account a number of nonlinear effects, responsible for amplitude and phase self-modulation of light waves, in addition to beam diffraction and medium time dispersion. NSE in the most common form (see, e.g., Ref. 5) has the following structure:

$$\left\{\frac{\partial}{\partial z} - \frac{i}{2n_0 k_0}\nabla_\perp^2 + i\frac{k''_\omega}{2}\frac{\partial^2}{\partial t^2}\right\} U(\mathbf{r}_\perp, z; t) -$$

$$- ik_0\left(\tilde{n}_2 - n_p\right) U(\mathbf{r}_\perp, z; t) + \frac{\alpha_N}{2} U(\mathbf{r}_\perp, z; t) = 0. \tag{11}$$

Here $k''_\omega = \partial^2 k/\partial\omega^2$ (0.21 fs^2/cm at λ_0 = 800 nm) is the dispersion of envelope velocity of light pulse in air;

$$\tilde{n}_2 = \frac{n_2}{2}\left\{|U|^2 + \int_{-\infty}^{t} dt' \Lambda(t - t') |U(t')|^2\right\}$$

is the cubic nonlinearity of refraction coefficient with accounting for the instantaneous and inertial components of the Kerr effect; n_p is the variation of medium refractivity due to plasma generation in radiation channel; α_N is the nonlinear absorption coefficient, accounting for radiation energy loss at gas photoionization and plasma heating; Λ is the molecule response function.

Equation (11) was solved numerically for model beams of Gaussian (6) and super-Gaussian (8) transverse profiles of light field envelope with the following parameters: pulse width t_p = 60 fs, beam radius R_0 = 1.0 mm, carrier wavelength λ_0 = 800 nm. The radiation was considered as initially collimated ($F = \infty$), the self-focusing parameter η was equal to 10, which corresponded to the peak power $P_0 = 32$ HW ($P_c = 3.2$ HW) for GP beam, and according to Eq. (4) P_0 was about 2.4 times higher ($G \simeq 2.4$) for SGP beam with the geometry parameter q = 4. Initial radiation profiles were defined as ideal smooth functions; hence, only one axial light filament occurred while beam propagating.

Figure 1 shows the behavior of effective radius of laser beams with different intensity profiles normalized to its initial value, when propagating in air. Here the data are presented versus distance along the path.

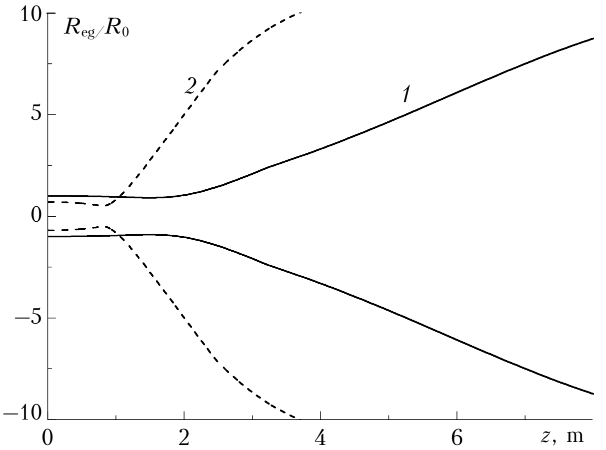

Fig. 1. Evolution of the effective radius of GP (*1*) and SGP (*2*) beams along the path with relative initial power η = 10.

The qualitative analysis of Fig. 1 allows separating three different special regions, reflecting different stages of radiation non-stationary self-focusing[5]: 1) the region of lateral contraction of a beam to the *global* nonlinear focus and formation of filament around it; 2) the region of sharp increase of the effective beam area after the nonlinear focus; and 3) the region of linear propagation of radiation passed through a nonlinear medium. As is seen from Fig. 1, the SGP beam forms the nonlinear focus much earlier and then has essentially higher angular divergence in comparison with the GP beam.

Figure 2 presents the same data as Fig. 1 but for normalized variables: the global effective radius $\bar{R}_{eg} = R_{eg}/R_{eg0}$ is related to its value in the beginning of the path $R_{eg0} = R_{eg}(z=0)$, and the distance $\bar{z}^*$ is calculated for each beam in accordance with its initial intensity profile.

In the considered case, this gave the ratio of generalized diffraction beam lengths $L_{D*}(\text{GP})/L_{D*}(\text{SGP}) \simeq 2.9$, i.e., the diffraction rate of SGP beam is about 3 times higher than that of GP one.

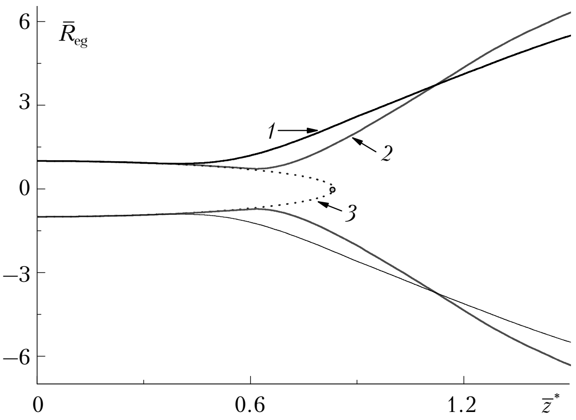

Fig. 2. The same data as in Fig. 1 in joint coordinates. Curve *3* corresponds to evaluation by Eq. (10), the point – to the calculated position of beam collapse in the general ($\bar{z}_K^*$) .

As is evident from Fig. 2, the initial stage of spatial evolution of the effective radius (lateral contraction) is similar for both beams in joint coordinates and is defined by Eq. (10). Contracting, each beam forms a global focal neck, the path position z_g and transverse size of which depend on radiation profile. It follows from comparison of Figs. 1 and 2, that the SGP beam is characterized by slower self-acting along the path, but has a narrower focal waist in comparison with the GP beam. This is a direct consequence of quasiuniform distribution of the SGP beam intensity near its axis. As a result, the SGP light beam is focused as a unit under the Kerr effect in contrast to GP beam with the more rapid increase in intensity in the centre as compared to its periphery.

Stronger lateral contraction of SGP beam results in its higher angular divergence after the global focus. However, the limit radiation divergence $\theta_{D\infty} = \lim_{z\to\infty} \theta_D(z)$, forming at the linear propagation stage, i.e., on passing through the layer of medium nonlinearity, the pulse has close values for GP and SGP, and essentially exceeds its initial diffraction-caused value (more than by 30 times for a given data).

It is important to estimate the coordinate of the global nonlinear beam focus position z_g, since the path maximum of mean-square radiation energy density $w_e(z) = E(z)/[\pi R_{eg}^2(z)]$ is realized at this point.[4]

Figure 3 shows the spatial evolution of the effective size of laser beams of different profiles. As is seen, the behavior of integral beam radius R_{eg} [Eq. (9)] best agrees with the behavior of instant effective radius R_e [Eq. (2)], calculated at the time point $\tau = 0$, i.e., in the center of temporal profile of the pulse. Evolution of the instant effective radius at other time cross sections of the pulse (e.g., transverse beam cross sections in time layers at leading and trailing fronts of the pulse shown in Fig. 3) demonstrates qualitatively different behaviors due to the lower power at edges and the plasma effect (see Ref. 6 for more details).

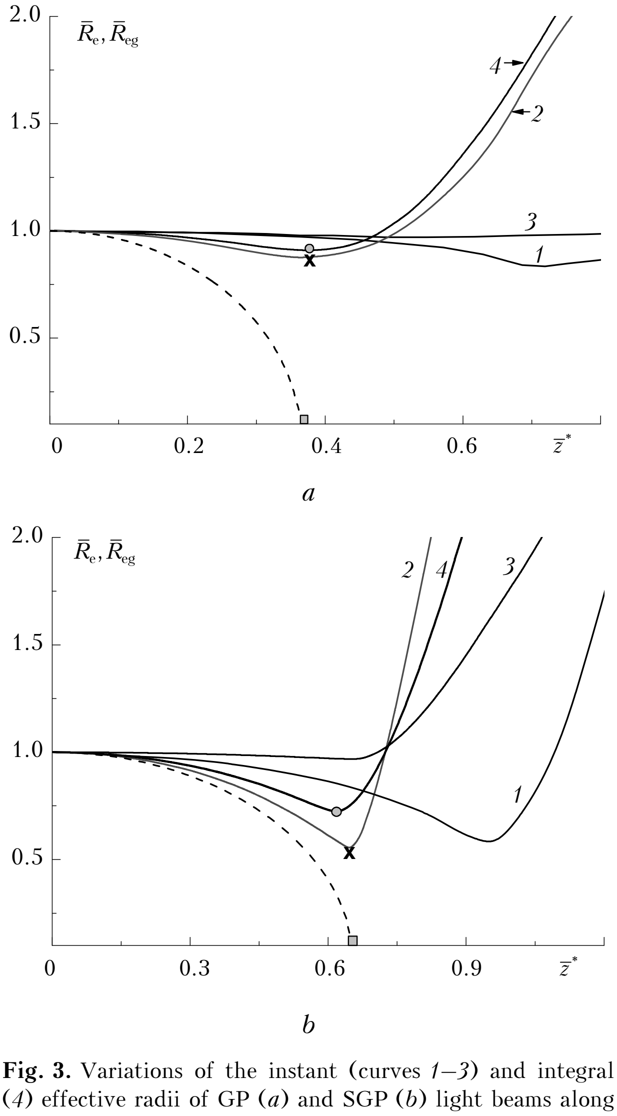

Fig. 3. Variations of the instant (curves *1–3*) and integral (*4*) effective radii of GP (*a*) and SGP (*b*) light beams along the propagation path at $\eta = 10$. Values of instant radius have been calculated at the time point $\tau = -1$ (*1*); 0 (*2*); and 1 (*3*). The dashed line corresponds to evaluation by Eq. (1).

Note that the position z_g of the center of global focal beam waist (the circle in Fig. 3) in this case sufficiently well matches the coordinate z_1 of the local nonlinear focus of central time cross section of the pulse (the cross in Fig. 3): $z_g \simeq z_1$. This allows one to roughly evaluate the distance of the global self-focusing of a pulse as a unit by means of changing the pulse for one time layer.

Using then the time-independent self-focusing theory (1) for this layer, we obtain an estimate of the coordinate of global nonlinear focus as a point of Kerr collapse z_N of the central layer (the rectangle in Fig. 3):

$$\bar{z}_g^* \simeq \bar{z}_N^* = \left[\frac{L_{D*}\sqrt{\eta_G - 1}}{kR_0^2} + \frac{1}{F}\right]^{-1}, \tag{12}$$

where the reduced self-focusing parameter η_G is calculated with accounting for a certain transverse intensity profile in the beam: $\eta_G = GP_0/P_{cg} = G\eta_0$ ($\eta_G \simeq 2.4\eta_0$ for SGP beam in the considered case). Then the size of focal waist of the beam $R_{eg}(z_g)$ is found from Eq. (10) by substitution $\bar{z}^* = \bar{z}_g^*$, and the normalized effective focal density of pulse energy as

$$\bar{w}_g = \frac{w(\bar{z}_g^*)}{w(0)} \simeq \frac{1}{\pi[(1-\eta^*)(\bar{z}_g^*)^2 + (1-\bar{z}_g^*/\bar{F})^2]}. \tag{13}$$

Here the initial focal distance of the beam is normalized to the generalized diffraction length $\bar{F} = F/L_{D*}$ to unify the expression.

The influence of initial parameters of a laser beam on the parameters of pulse self-focusing is evident from evaluating equations (10), (12), and (13). For example, a decrease in the beam power P_0 without change of its transverse profile is equivalent to a decrease in the self-focusing parameter η_G, which will result in corresponding disposal of the global nonlinear beam focus z_g, increase in the degree of beam lateral contraction, and increase in relative energy density in the focus $\bar{w}_g$. A decrease in the initial geometrical beam radius R_0 at keeping the peak pulse intensity at a previous level will affect similarly.

Thus, non-stationary self-focusing of ultrashort laser pulse in air with different transverse intensity distribution has been theoretically analyzed on the base of numerical solution of NSE. The formalism of averaged radiation parameters has been used. It was established that some evolution of the integral effective radii existed in the single filamentation mode at least for two different spatial profiles of light beam intensity, namely, Gaussian and super-Gaussian. It consists in their common functional dependence (10) on the generalized evolutionary variable at a distance up to the global nonlinear focus. Later on, such similarity is not observed, and spatial behavior of the effective beam radius after passing the nonlinear focus depends on the initial radiation profile.

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