PULSE RESPONSE OF THE CLOUDY ATMOSPHERE CALCULATED AT LARGE ZENITH ANGLES FOR OPTICAL PATH THICKNESSES FROM 1 TO 100 TAKING INTO ACCOUNT THE SPHERICAL GEOMETRY OF THE EARTH

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A Monte-Carlo based technique and computer algorithm are presented for calculating the pulse response functions of a multilayer spherical cloudy atmosphere (optical thicknesses from 1 to 100) to isotropic pulses from a subcloud point source.

Depending on the optical thickness of the cloud layer the relative systematic rms error of the algorithm varies from 2.5% to 25%.

Many authors[1–6] have contributed to our present-day understanding of the effects observed during the propagation of optical radiation through the atmosphere and the surface-atmosphere interaction, basing their calculations on the Monte-Carlo technique. As a rule, such studies pertain to plane-parallel (the Sun) or narrow-beam (laser) radiation sources. The present study attempts to calculate the pulse characteristics (PC) $J(t)$ – the responses of the transmitting medium (atmosphere) to a δ-pulse from an isotropic point source of radiation. This problem is solved for the following initial conditions:

– the atmosphere contains a homogeneous layer of continuous (stratiform) cloudiness of optical thickness τ, varying from 1 to 100;

– the reflectance of the Earth's surface is characterized by an angle-integrated reflection coefficient (i.e., the albedo A_e) and by Lambert's differential law;[7]

– the reference radiation wavelength is chosen at the border of the UV and the visible;

– the radiation source is located at the altitude H_s, below the cloud layer H, and the detector – at the altitude H_{det} and above the atmospheric "celling" H_{atm} (see Fig. 1);

– the sighting zenith angles z between the epicentral point 0 and the detectors D_1 (i = 1, 2, ..., see Fig. 1) vary from 0° to 85°.

The total scattering phase function for the cloudless atmosphere[8] is given by

$$\chi(\Theta) = \frac{\beta_M(h)\chi_M(\Theta) + \beta_a(h)\chi_a(\Theta)}{\beta(h)},$$

where Θ is the angle of scattering, $h \in H$; $\beta_M(h) = 0.0119 \cdot (\lambda_0/\lambda^4) \cdot \exp(-0.125\ h)$ is the molecular scattering coefficient, km^{-1} (Ref. 14); $\lambda_0 = 0.55$ μm, $\tau_0 = 0.3$ (for λ_0), and

$$\beta_a(h) = \left[\frac{3.91}{S_M} - 0{,}0119\right]\frac{\lambda_0}{\lambda}\exp\left[-\frac{\frac{3{,}91}{S_M} - 0.0119h}{\tau_0 - 0{,}095\ h}\right]$$

is the aerosol scattering coefficient for altitudes below H = 5 km.[9] Above 5 km the vertical profiles of the aerosol extinction are used[10]:

$$\beta(h) = \beta_M(h) + \beta_a(h);$$

and S_m is the meteorological visibility.

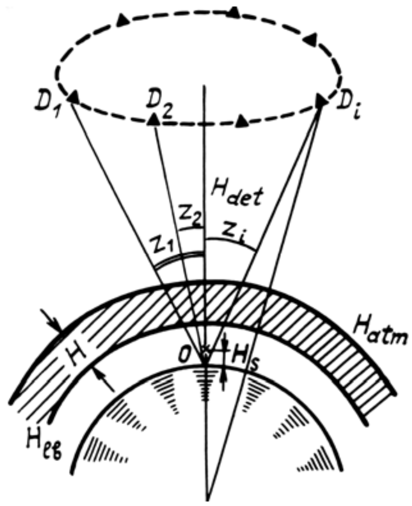

FIG. 1. Geometrical scheme of the calculations.

A stratified model of total scattering for the atmosphere divided into 17 vertical layers (layer 18 is outer space) is presented in Table I. Also the optical thicknesses of the individual layers are given there together with the cumulative values of the optical thickness, starting from the Earth's surface.

TABLE I.

Stratified model of cloudless atmosphere ($\lambda = 0.4$ μm) $S_M = 20$ km.

N Layer	Layer boundary		Scattering coefficient km^{-1}	Optical thickness	
	Lower, km	Upper, km		Layer	Cumulative
1	0.0	0.2	0.1930	0.0386	0.0386
2	0.2	0.5	0.1580	0.0474	0.0860
3	0.5	1.0	0.1150	0.0575	0.1435
4	1.0	2.0	0.0663	0.0603	0.2098
5	2.0	3.0	0.0339	0.0339	0.2437
6	3.0	4.0	0.0200	0.0200	0.2637
7	4.0	5.0	0.0135	0.0135	0.2772
8	5.0	7.0	0.0125	0.0250	0.3022
9	7.0	9.0	0.0100	0.0200	0.3222
10	9.0	2.0	0.0082	0.0246	0.3468
11	12.0	5.0	0.0064	0.0192	0.3650
12	15.0	0.0	0.0046	0.0230	0.3890
13	20.0	5.0	0.0019	0.0095	0.3985
14	25.0	0.0	0.0008	0.0040	0.4025
15	30.0	5.0	0.0004	0.0020	0.4045
16	35.0	0.0	0.0001	0.0015	0.4060
17	50.0	0.0	0.000044	0.0003	0.4063
18	>80	—	0	0	—

The molecular scattering phase function[14] is given by

$$\chi_M(\Theta) = \frac{3}{8}\,(1 + \cos^2 \Theta).$$

The aerosol scattering phase function χ_a is read from a lookup table taken from Ref. 13 for the case of continental haze L.

The cloud layers are taken to be spherically concentric, their base height equal at H_{cb} and their thickness $h \in H$; they are introduced into a stratified, molecular, otherwise cloudless atmosphere. The cloud layer scattering coefficient is assumed to be constant.

The cloud layer scattering phase function is given in tabulated form following the C1 cloud model in Ref. 13.

The angular reflection law (Lambert's law) is given by

$$\chi_e(\Theta') = 2 \cos \Theta,$$

where Θ' is the reflection angle with respect to the surface normal.

The calculations are based on the Monte Carlo technique.[7] Following Ref. 7 the photon trajectories are modeled directly.

The value of PC for each grade t_j is calculated as mathematical expectation of the functional

$$J^*(t_j) = E^{-1}(D_1)\, M\left[\sum_{n=1}^{N} Q_n \varphi_n(\vec{r}_{n1}, \vec{r}_n)\Delta t_j\right],$$

where $E^{-1}(D_1)$ is the normalization factor; $E(D_1) = M\left[\sum_{n=1}^{N} Q_n \varphi_n(\vec{r}_{n1}, \vec{r}_n)\right]$ is the optical radiation flux density at the input to the detector D_1; $M\left[\sum_{n=1}^{N} Q_n \varphi_n(\vec{r}_{n1}, \vec{r}_n)\Delta\right] J(t_j)$ is the non-normalized PC; M is the mathematical expectation operator; N is the number of photon collisions; and Δ indicates whether the photon time of arrival falls into the given time step $[t_j, t_{j+1}]$. This indicator is defined as

$$\Delta = \begin{cases} 1, & \text{if } t_j \le t < t_j + \Delta t_j, \\ 0, & \text{otherwise.} \end{cases}$$

The time of arrival at the detector for the photon is given by

$$t = (L - R^*_{n1})/c,$$

where L is the total length of the photon trajectory, R^*_{n1} is the distance between the source and the detector ($L_{n1} \approx R_{n1}$); and c is the speed of light.

When the modified local estimate from the Monte Carlo technique is used,[7] the quantity $\varphi_n(\vec{r}_{n1}, \vec{r}_n)$ inside the mathematical expectation brackets, has the meaning of a statistical weight for the photon. This statistical weight is calculated at every point at which the photon is either scattered or reflected, under the condition that the detector D_1 is directly observable from each of these points.

The analytic form of representation of the photon statistical weights looks like this:

$$\varphi_n(\vec{r}_{n1}, \vec{r}_n) = \frac{\exp\,[-\tau(\vec{r}_{n1}, \vec{r}_n)]\;\chi(\mu)}{2\pi\,|\vec{r}_{n1} - \vec{r}_n|^2},$$

where n is the current number of photon collisions; $\vec{r}_n$ is the radius vector of the collision point; $\vec{r}_{n1}$ is the radius vector of the radiation test point; $\tau(\vec{r}_{n1}, \vec{r}_n)$ is the optical length of the segment $[\vec{r}_{n1}, \vec{r}_n]$; $x(\mu)$ is the corresponding scattering phase function $\mu = \cos\Theta'$; Θ' is the angle between the photon propagation direction prior to the collision and the vector $\vec{r}_{n1} - \vec{r}_n$.

Our modification of the simple local estimate[7] consists in calculating the weight $\varphi_n(\vec{r}_{n1}, \vec{r}_n)$ with the averaging carried out over several detectors which are equidistantly positioned at the same height H_{n1} along a circle centered on the source vertical. Such a device reduces the variance of the systematic error by a factor of 1.3 to 1.7 in the calculations for detectors with sighting angles $z \ge 45°$.

The weight coefficients Q_n make it possible to account for absorption of the radiation by atmospheric aerosols without truncating the photon trajectories:

$$Q_n = \omega\, Q_{n-1},$$

where ω is the photon survival probability (assumed to be equal to 1 for these calculations). The same holds for absorption by the underlying surface:

$$Q_n = A_e\, Q_{n-1}.$$

The relative systematic rms error σ accumulated during calculations of the value $E(D_1)$ reaches 1–2% for the cloudless atmosphere, and increases when the cloud layer is introduced; its dependence on the layer optical thickness τ is given by the relation

$$\sigma \approx 2.5\sqrt{\tau_{\text{cloud}}},$$

(The number of photon trajectories Is approximately $2\cdot 10^3$–$4\cdot 10^3$). Here σ is in per cent; the sighting angle $z = 0°$.

To reduce the computer time expenditures for large τ, the algorithm was modified so that the initial direction of photon propagation was modeled by the density

$$f'(\mu) = \frac{f_0}{1 - d\mu},$$

here μ is the cosine of the outgoing direction of the particle; d is a dimensionless factor; f_0 is a normalizing factor, obtained from the normalization condition

$$\int_{-1}^{1} f'(\mu)\, d\mu = 1; \quad f_0 = \frac{d}{\ln\left(\frac{1+d}{1-d}\right)};$$

so that

$$f'(\mu) = \frac{d}{\ln\left(\frac{1+d}{1-d}\right)}\cdot\frac{1}{1-d\mu}.$$

The photon weight is then multiplied by the ratio of the distribution density actually used to the true density:

$$q = \frac{2d}{\ln\left(\frac{1+d}{1-d}\right)\cdot(1-d\mu)}.$$

The calculations performed using this modification yield results which agree with the basic calculation; however, the efficiency of the algorithm is increased only for those cases in which the detector is close to the zenith ($z \le 45°$).

The calculations yield PC of the atmosphere (both for cloudless and cloudy cases) at the detector for the following set of parameters: $\lambda = 0.4$ μm; $H_{det} = 4\cdot 10^4$ km, $H_{cb} = 1$ km, $H_s = 0.3$ km, $A_e = 0.3$, $S_m = 20$ km, $\tau = 0.3$.

Figure 2 shows the flux densities E which determine the signal detection levels at the detector as a function of the sighting angle for different values of τ_{c1} and also for vacuum (the dependence of E on z for vacuum is extremely weak since the distances R_n are changed very little for higher z).

It can be seen from Fig. 2a that for low optical thicknesses of the cloud layer ($1 \le \tau \le 10$) and sighting angles $z \le 45°$ the flux density at the detector exceeds the direct flux density in vacuum and in the cloudless atmosphere, and also exceeds the diffuse flux density In the cloudless atmosphere.

It also follows from Fig. 2b (where the dependence of E on the optical thickness τ is also shown) that for a cloud layer of $\tau \le 60°$ the flux densities are comparable (quite close) to the total (direct plus diffuse) flux densities under cloudless conditions (this is true for sighting angles $z \le 45°$).

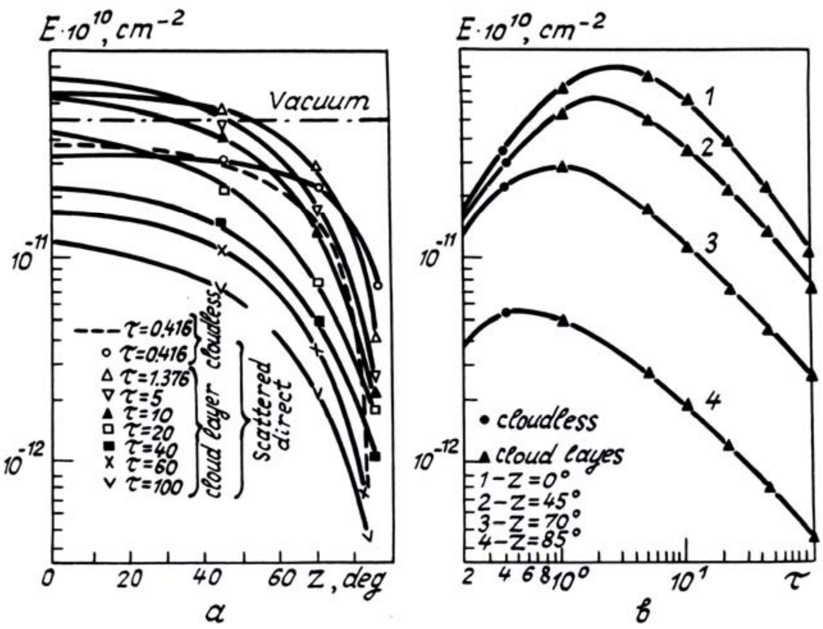

FIG. 2. Dependence of the PC normalization coefficients on sighting angle z and optical thickness τ

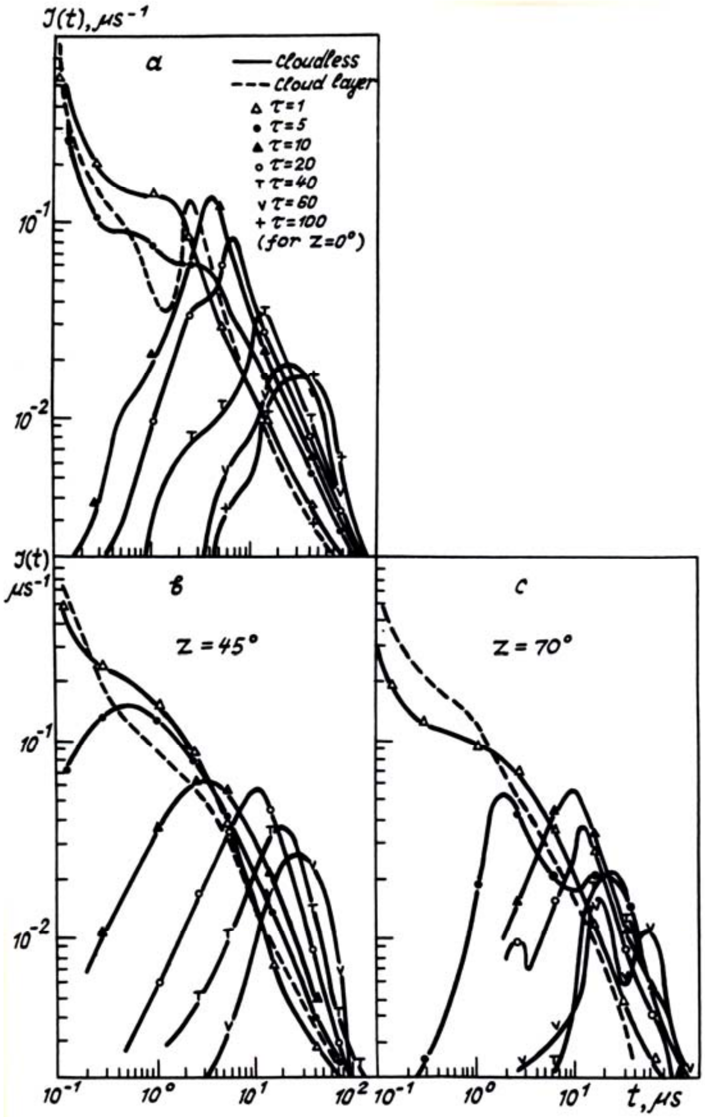

FIG. 3. Normalized pulse characteristics

Figure 3 shows the shape of the pulse characteristics as a function of the optical thickness for sighting angles $z = 0°$, $45°$, $70°$.

The PC's are normalized by the condition

$$\int_0^\infty J^*(t)dt = 1$$

It then becomes possible to treat them as probability densities for the random time intervals spent by the individual photons in traveling from the source to the detector, minus the travel time of the direct beam. Then, like any probability density function, the PC is characterized by certain characteristic numerical values as shown in Fig. 4.

It can be seen from Figs. 4a and 4b that mathematical expectations m_t of PC's for a cloudy atmosphere remain practically unchanged while τ increases from 1 to 10 and slowly grow with further increase in τ.

The rms errors σ_t display a minimum around $\tau = 10$, and increase on both sides of this value. This fact follows from the shape of the PC probability density curve, as shown in Fig. 3.

The coefficient of variation $K_v = \dfrac{\sigma_t}{m_t}$ does not vary by more than 10% within our sighting angle range, which testifies to the statistical stability of the photon survival time distributions at the detector.

The eightieth percentiles of the distributions of t (for given z) (Fig. 4a), taken as functions of the optical thickness, do not display any significant scatter about the weighted average curves. This is also true of the ninetieth percentiles. However, the ninety-fifth percentiles considerably deviate from the weighted averages, particularly for $\tau \leq 10$. This agrees with the corresponding values of the relative systematic rms error of calculation for the flux densities E if we take into account the fact that $E = \int_0^\infty J(t)dt$.

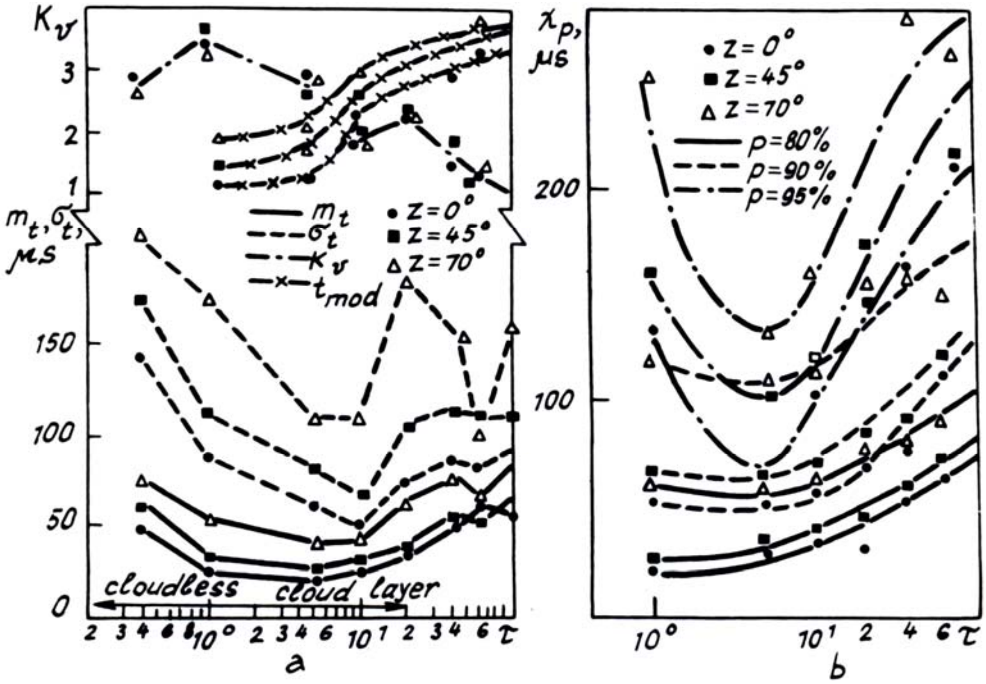

FIG. 4. The numerical characteristics m_t, σ_t, and K_v and the quantiles of the normalized pulse characteristics.

CONCLUSIONS

1. Our modification of the Monte Carlo local estimate has made it possible to calculate atmospheric PC's both for vertical and for inclined lines of sight at optical thicknesses up to 100 and zenith angles up to 85°. The variances of the results of the calculation are reduced by a factor of 1.3–1.7, and the computer time needed – by a factor of 3–5 as compared with the standard technique.

2. Analysis of the normalized pulse characteristics yields a qualitative picture of formation of a temporally non-stationary optical radiation field.

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