

# CONTRIBUTION OF PREVIOUS RANGING PULSES TO THE ERROR IN A LIDAR SIGNAL

G.V. Kolarov

*Institute of Electronics, Bulgarian Academy of Sciences, Sofia*

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*We discuss the systematic error in a lidar signal due to the additive effect of returns from previous laser pulses in the series, and determine the vertical and horizontal profiles of this error as a function of pulse repetition rate, atmospheric transparency, and the number of previous ranging pulses.*

The origin and characteristics of random errors in lidar signals have been discussed by many authors.<sup>1-4</sup> In spite of this, there is still no universally accepted method for analyzing the systematic errors in the signals due to inaccuracies in the return model. One source of such errors involves treating the additive effect of returns from previously transmitted pulses on the useful signal as being the response to a certain laser pulse. In general, this error is an increasing function of pulse repetition frequency, and may become quite large at high pulse repetition rates (such as those in nitrogen lasers, metal gas lasers, diode lasers, etc.) and require introduction of a correction.

We shall now analyze this error and estimate its value. The usual lidar equation

$$P(z) = Az^{-2}\beta(z)\exp \left[ -2 \int_0^z \alpha(z')dz' \right] \quad (1)$$

(where  $A$  is the instrumental constant and  $\beta$  and  $\alpha$  are the backscattering coefficient and extinction coefficient, respectively), gives the detected signal strength due to backscattering of the  $k$ th laser pulse as a function of range  $z$ . Signals from ranges  $z + nz_\Theta$ ,  $n = 1, 2, \dots$ , due to scattering of the  $k$ -1st,  $k$ -2nd, etc., laser pulses, where  $z_\Theta = c/2f_{laser}$  and  $f_{laser}$  is the pulse repetition rate of the laser. The strength of these additional returns is can also be described by Eq. (1) if  $z$  is replaced by  $z + nz_\Theta$ , so that the total strength from the "unique zone"  $z \leq z_\Theta$  is given by

$$P_\Sigma(z) = \sum_{n=0}^{\infty} P_n(z), \quad (2)$$

where

$$P_n(z) = Az_n^{-2} \beta(z_n) Y(0, z_n),$$

$z_n = z + nz_\Theta$  and  $Y(z_a, z_b)$  is the transparency of the round trip from  $z_a$  to  $z_b$ . The  $P_0$  term in Eq. (2) is the useful signal, while all of the other terms make up the error.

The relative error

$$\epsilon(z) = \sum_{n=1}^{\infty} P_n(z)/P_0(z) = \sum_{n=1}^{\infty} \left( \frac{z_n}{z} \right)^{-2} \frac{\beta(z_n)}{\beta(z)} Y(z, z_n). \quad (3)$$

where the first and the third factors on the right side of Eq. (3) decrease with increasing  $n$  and the ratio  $\beta(z_n)/\beta(z)$  shows a complex behavior as a function of  $n$  and  $z$  due to (for example) aerosol inhomogeneities outside the region  $(0, z_\Theta)$ . We should also note that  $(z_n/z)^{-2} \leq (1+n)^2$ , and  $Y(z, z_n) < 1$ .

It is somewhat difficult to analyze the full range of possibilities for the behavior of  $\alpha(z)$  and  $\beta(z)$ , so we shall consider two special cases: vertical ranging in a slightly turbid atmosphere (e.g., the model suggested by McClatchy,<sup>5</sup> which has a meteorological visibility range  $S_{met} = 23$  km) and horizontal ranging (in the horizontally-homogeneous model). In the latter case,

$$\epsilon(z) = \sum_{n=1}^{\infty} (1 + nz_\Theta/z)^{-2} \exp(-2\alpha nz_\Theta),$$

which reaches a peak value of

$$\epsilon_{max} = \epsilon(z_\Theta) = \sum_{n=1}^{\infty} (1 + n)^{-2} \exp(-2\alpha nz_\Theta),$$

at  $z = z_\Theta$  and exceeds 0.25 for sufficiently small  $\alpha$ .

In all cases,  $\epsilon$  depends on  $f_{laser}$  (and thus on  $z_\Theta$ ), so we have shown  $\epsilon$  as a function of  $z/z_\Theta$  in Figs. 1 and 2. The quantity  $\epsilon$  also depends on the  $\alpha(z)$  profile, especially  $\alpha(0)$  (and thus  $S_{met}$ ). Finally,  $\epsilon$  depends on  $\beta(z)$  as well.

In the vertical-ranging case (Fig. 1),  $\epsilon_{max}$  is 1.2, 9, and 17.5% for  $f_{laser} = 5, 15$ , and 30 kHz ( $z_\Theta = 30, 10$ , and 5 km); in the  $f_{laser} = 15$  kHz case, the maximum  $\epsilon$  occurs below  $z_\Theta = 10$  km, within the "unique zone"; this can be explained by the effect of the Junge aerosol layer in the stratosphere.

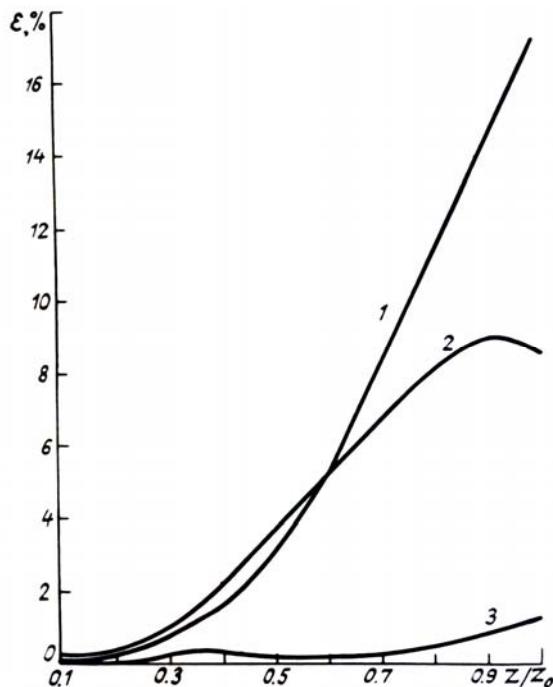


FIG. 1. Relative error  $\epsilon$ , %, as a function of relative range for vertical ground-based ranging at  $\lambda = 0.51 \mu\text{m}$ ,  $S_{\text{met}} = 23 \text{ km}$ , at  $f_{\text{laser}} = 30 \text{ kHz}$  (1), 15 kHz (2), and 5 kHz (3).

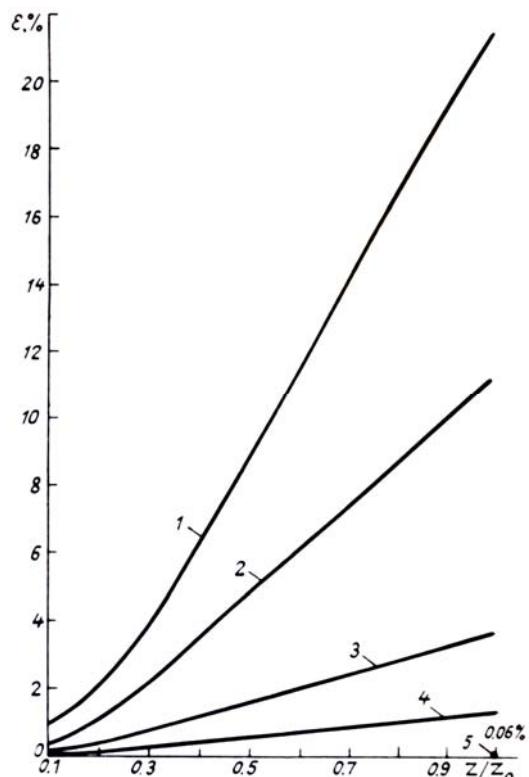


FIG. 2. Same as Fig. 1 for horizontal ranging in a homogeneous medium at  $\lambda = 0.51 \mu\text{m}$ . 1)  $S_{\text{met}} = 80 \text{ km}$ ,  $f_{\text{laser}} = 30 \text{ kHz}$ ; 2)  $S_{\text{met}} = 80 \text{ km}$ ,  $f_{\text{laser}} = 15 \text{ kHz}$  or  $S_{\text{met}} = 40 \text{ km}$ ,  $f_{\text{laser}} = 30 \text{ kHz}$ ; 3)  $S_{\text{met}} = 40 \text{ km}$ ,  $f_{\text{laser}} = 15 \text{ kHz}$ ; 4)  $S_{\text{met}} = 80 \text{ km}$ ,  $f_{\text{laser}} = 5 \text{ kHz}$ ; 5)  $S_{\text{met}} = 40 \text{ km}$ ,  $f_{\text{laser}} = 5 \text{ kHz}$ .

In the horizontal-ranging case (Fig. 2),  $\epsilon_{\text{max}}$  is 0.06, 3.6, and 11.1% for  $f_{\text{laser}} = 5$ , 15, and 30 kHz and  $\alpha = 0.01 \text{ km}^{-1}$  and 1.3, 11.1, and 21. 4% for  $\alpha = 0.05 \text{ km}^{-1}$ .

The error in  $\epsilon$  decreases with increasing  $\alpha$  at all pulse repetition rates, and even at the highest frequency, the observable  $\epsilon$  corresponds to  $\alpha \leq 0.4$ , i.e.,  $S_{\text{met}} \geq 10 \text{ km}$ .

In practice, when carrying out aerosol ranging at high pulse-repetition rates (and thus high pulse energies) the aerosol profile is measured using a series of ranging pulses. The return from the first pulse is not affected by the error being studied here, the return from the second pulse contains an error due to one previous pulse, that from the third pulse contains an error due to the two previous pulses, etc. The values of  $\epsilon$  discussed above correspond to the steady-state regime, i.e., these values are appropriate to pulses preceded by a sufficiently large number of pulses within the series. Table I shows the increase in error with the number of pulses, as well as the transition to the steady-state regime.

**CONCLUSIONS.** In contrast to the situation which holds in ordinary radar observations of concentrated objects, where matching the repetition rate to the operating range is used to avoid the appearance of false targets due to the lack of uniqueness in the range measurement, there is a positive error in lidar ranging of elastic scattering in frequency mode due to returns from the non-uniqueness zone due to previous pulses.

TABLE I.

Maximum range error  $\epsilon$  (%) as a function of pulse number in packet for horizontal ranging

| $\alpha$<br>$\text{km}^{-1}$ | $f_{\text{laser}}$<br>kHz | Return number |       |       |       |        |       |
|------------------------------|---------------------------|---------------|-------|-------|-------|--------|-------|
|                              |                           | 2             | 3     | 4     | 5     | 6      | 7     |
| 0.05                         | 30                        | 15.16         | 19.25 | 20.64 | 21.18 | 21.41  | 21.51 |
|                              | 15                        | 9.20          | 10.70 | 11.01 | 11.08 | 11.094 | 11.10 |
|                              | 5                         | 1.24          | 1.27  | 1.27  | 1.27  | 1.27   | 1.27  |
| 0.01                         | 30                        | 9.20          | 10.70 | 11.01 | 11.08 | 11.094 | 11.1  |
|                              | 15                        | 3.38          | 3.58  | 3.59  | 3.59  | 3.59   | 3.59  |
|                              | 5                         | 0.06          | 0.06  | 0.06  | 0.06  | 0.06   | 0.06  |

However, this error only becomes significant for pulse repetition rates greater than 5 kHz in vertical ranging; in the horizontal-ranging case, the error is only significant for these same frequencies and sufficiently small  $\alpha \leq 0.4 \text{ km}^{-1}$ , which occurs at ground level when  $S_{\text{met}} \geq 10 \text{ km}$  or even more commonly in observations from airplanes or high altitudes. Our calculations indicate that the relative error due to this effect may be greater than 20% under unfavorable (but realistic) conditions.

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